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A PSEUDO GRAPH ARISING FROM ADDITIVE AND MULTIPLICATIVE INVERSES MODULO N

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Abstract

In this paper, different kind of interplay between the theory of congruences and the theory of graphs is presented. Using the properties of numbers particularly the theory of congruences, the edge pair, the trivial edge pair, the subgraphs $\Gamma(\oplus)$ and $\Gamma(\odot)$ corresponding to additive inverse and multiplicative inverse modulo n were respectively defined in the work. We start by constructing the subgraphs $\Gamma(\oplus)$ and $\Gamma(\odot)$ for n = 2, 3, …, 15 as illustrative example then generalized for any given positive integer n ≥2. We found that the components of $\Gamma(\oplus)$ has only one loop at the vertex 0 for odd n and two loops (trivial and non-trivial) for even n. While the components of $\Gamma(\odot)$ has exactly two loops at the vertices1 and n−1 for odd n. Moreover, for any given n≥2, the subgraph $\Gamma(\oplus)$ is a graph associated to the algebraic structure ($\mathbb{Z}_n$, $\oplus$) and the subgraph $\Gamma(\odot)$ is a graph associated to the algebraic structure ($\mathbb{U}_n$, $\odot$), where $|\mathbb{U}_n| = \phi(n)$.The following results were established for the new class of graph obtained called pseudo graph of additive and multiplicative inverse modulo n Γ(mod n) : (i) It always has loops at the vertices 0, 1 and n−1for any n ≥2.(ii) It consists of at least three loops for any positive integer n>2. (iii) The vertices a and n−a are always adjacent for each non-zero $a\in\mathbb{Z}_n$. (iv) If n is an even positive integer then the vertex n/2 forms a loop edge in Γ(mod n). (v) It has no centre.(vi) It has two maximal components for odd n and three maximal components for even n. Finally, the size, degree of vertex, chromatic number, independent sets and independence number of the graph were determined.

***Keywords*:** Pseudo graph, additive and multiplicative inverse modulo *n*, component graph.

1. Introduction

Graph theory is intimately related to many branches of mathematics, including group theory, probability, combinatorics, matrix theory, Numerical analysis and topology. The studies of algebraic structures using the properties of graphs become an existing research topic in the last twenty years, leading to many fascinating results and questions [1]. Graph theory serves as a mathematical model for any system involving a binary relation. There is strong connection between theory of numbers and theory of graphs. One such connection available in the literature is cyclic graphs. In line with this, cycles of length 0 modulo 4 in graphs were studied by [2]. They have shown that a graph G contains such a cycle provided it has any of the following three properties: (1) G has minimum degree at least 2 and at most two vertices of degree 2, (2) G is not 3-colourable, and (3) G is a subdivision of a graph of order p≥5 with at least 3p−5 edges. The notion of congruence is intrinsic in number theory. In recent years studying graphs through congruence is of charismatic and an independent interest of number theorist [3]. According to [4], a graph is said to be representable modulo n if its vertices can be labeled with distinct integers between 0 and $n-1$ inclusive such that two vertices are adjacent if and only if their labels are relatively prime to n. They defined the representation number of a graph G to be the smallest n representing G, also reviewed known results and investigate representation number for several new classes. In particular, they relate the representation number of the disjoint union of complete graphs to the existence of complete families of mutually orthogonal Latin squares. There are many papers on connection of number theory and graph theory, for instance [5] studied a digraph whose set of vertices
H = {0, 1, … ,$n-1$} and for which there is a directed edge from $a\in$H to $b\in$H if $a^2\equiv b$(mod n) and examined when the digraph is semiregular. They established necessary and sufficient conditions for the existence of isolated fixed points. Moreover, conditions for the number of components and length of cycles were presented and two new necessary and sufficient conditions for compositeness of Fermat

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numbers were also introduced. In [3] their Authors discussed the graph G(n) arising from exponential congruences and shown that the graph has 2^r component, where r is the distinct number of primes factors of n. Furthermore, it was proved that the component G(p^2) is a tree with root at zero, and if n is a Fermat's prime, then the component of the simple graph is complete.

2. Preliminaries

In this section we give definition of some basic terminologies and relevant results needed for the understanding of this paper. Since this work is a connection between number theory and graph theory, we start with basic definitions and properties from number theory followed by some related terminologies and relevant results on graph theory. For the definitions of these basic terms and results given here refer to [6], [7], [8] and [9] .

Definition 2.1 (Congruence Modulo *n*): Let n be some fixed positive integer. Two integers a, $b\in\mathbb{Z}$ are said to be congruent modulo n written $a\equiv b(\text{mod } n)$ if and only if $n|(a-b)$. The positive integer n is called the modulus of the congruence.

Definition 2.2 (Inverse Modulo *n*) : Let $a\in\mathbb{Z}$ and $n\in\mathbb{N}$ then $\gcd(a, n) = 1$ if and only if there exist some integer x such that $ax \equiv 1(\text{mod } n)$. We call x the inverse of a modulo n.

Theorem 2.1 (Existence of multiplicative inverse modulo *n*): If the integers a and n are coprime then there is a multiplicative inverse of a modulo n. For a proof refer to [9].

Definition 2.3 (Reduced Residue System Modulo *n*): A reduced residue system modulo n is a set $\mathbb{U}_n= \{a_i\in\mathbb{Z}_n : \gcd(a, n) =1 \text{ for } i = 1, 2, \ldots,n-1\}$.

Definition 2.4 (Euler phi-function): The number of elements in the reduced residue system modulo n is denoted by $\phi(n)$ and this function is called Euler phi-function or totient function.

Theorem 2.2: The number $\phi(n)$ is the number of positive integers less than or equal to n that are relatively prime to n. For a proof refer to [9].

Definition 2.5 (Graph): A graph Γ is a mathematical structure (V, E), where V is the set of vertices viewed as points and E is the set of edges viewed as line joining the points. With $e\in E$ we associate two vertices u and v such that $e = (u, v)$.

Definition 2.6 (loop edge): An edge that connects a vertex to itself is called a loop edge. That is an edge which has only one endpoint.

Definition 2.7 (Degree of Vertex): The degree of a vertex v in a graph denoted by $\delta(v)$ is the number of edges incident (connecting) to it except that a loop at a vertex contributes twice to the degree of that vertex. That is in an in an undirected graph, the degree of a loop is considered as two. A vertex of degree zero is called isolated and a vertex of degree one is said to be pendant. If the degree of the vertex is odd the vertex is then called an odd vertex. On the other hand if the degree of the vertex is even then the vertex called an even vertex.

Definition 2.8 (Centre of a Graph): A vertex is said to be the centre of graph if every vertex of the graph has an edge with that vertex.

Definition 2.9 (Size of graph): The size of a graph is the number of edges in the graph.

Definition 2.10 (Null graph): A graph whose edge set is empty is called a null graph. In other words, a null graph does not contain any edge in it.

Definition 2.11(Trivial graph): A graph with only one vertex is called a trivial graph.

Definition 2.12 (Subgraph): A subgraph of a graph Γ is a graph whose sets of vertices and edges are respectively subsets of the set of vertices and edges of Γ.

Definition 2.13 (Graph Union): The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ denoted by $G_1\cup G_2$ is the graph with vertex set $V_1\cup V_2$ and edge set is $E_1\cup E_2$.

Definition 2.14 (Regular graph): A graph in which all vertices are of equal degree is called a regular graph. If the degree of each vertex is k then the graph is called a graph of degree k.

Definition 2.15 (Directed and undirected graph): Directed graph or digraph is a graph in which the edges have a direction and is usually indicated with an arrow on the edge. An undirected graph is a graph whose edges do not have direction.

Definition 2.16 (Complete Graph): A complete graph on n vertices denoted by K_n is a simple graph with n vertices where each pair of distinct vertices is connected by an edge.

Definition 2.17 (Connected Graph): A connected graph is a graph in which there is an edge between every pair of vertices. Equivalently a graph is connected when it has exactly one connected component. An undirected graph that is not connected is called disconnected. A connected component is a maximal connected subgraph of G, where each vertex belongs to exactly one connected component, as does each edge.

Definition2.18 (Simple Graph): A graph is simple if it has no loops or multiple edges. Multiple edges refer to more than one edge connecting same pair of vertices. That is a graph is simple if no two edges have the same endpoints.

Definition 2.19 (Pseudograph): A graph with loop edges as well as multiple edges connecting the same pair of vertices called a Pseudograph.

Definition 2.20 (Graph Colouring): A colouring of a graph is an assignment of colours to the vertices of a graph so that no two adjacent vertices have the same colour.

Definition 2.21 (Chromatic number): Is the minimum number of colours required to properly colour a graph Γ and it is denoted by $\chi(\Gamma)$.

Definition 2.22 (Independent Set): A non-empty subset S of set of vertices V is called an independent set of Γ if there is no pair of vertices in S that are adjacent. In other words, an independent set of Γ is a set of vertices in Γ that are assigned a particular color.

Definition 2.23 (Independence Number): The number of elements (vertices) in maximum independent set is called independence number of a graph Γ and is denoted by $\alpha(\Gamma)$.

3. Result and Discussion

3.1 Component graphs of additive and multiplicative inverses modulo *n*

In this subsection, we introduced and defined graphs associated to the non-empty subsets of $\mathbb{Z}_n$ ($n \geq 2$) called component graphs of additive inverse modulo n and multiplicative modulo n

3.1.1 Definitions and notations

Definition 3.1.1 (Edge pair): Given a positive integer $n \geq 2$ and $a, b \in \mathbb{Z}_n$, the pair (a, b) is defined to be an edge pair if either of the congruences $a \oplus b \equiv 0 \pmod{n}$ or $a \odot b \equiv 1 \pmod{n}$ is true. If $a \oplus a \equiv 0 \pmod{n}$ or $a \odot a \equiv 1 \pmod{n}$ then (a, a) is defined as a trivial edge pair.

Note

(i) $\mathbb{Z}_n$ stands for the set of integers modulo n, i.e $\{0, 1, \ldots, n-1\}$.

(ii) The symbols $\oplus$ and $\odot$ stand for addition modulo n and multiplication modulo n respectively.

(iiI) In number theory terminology, if $a \oplus b \equiv 0 \pmod{n}$ then b is called the additive inverse of a modulo n. Similarly, if $a \odot b \equiv 1 \pmod{n}$ then b is called the multiplicative inverse of a modulo n.

We now define additive and mulyiplicative inverses modulolo n graphs.

Definition 3.1.2: A graph of additive inverse modulo n is defined as a subgraph $\Gamma(\oplus)$ whose set of vertices is a non-empty subset $W(\oplus)$ of $\mathbb{Z}_n$ and the set of edges is a subset

$E(\oplus) = \{(a, b) : a \oplus b \equiv 0 \pmod{n}\}$ of E. A graph of multiplicative inverse modulo n is defined as a subgraph $\Gamma(\odot)$ whose set of vertices is a non-empty subset $W(\odot)$ of $\mathbb{Z}_n$ and the set of edges is a subset $E(\odot) = \{(a, b) : a \odot b \equiv 1 \pmod{n}\}$.

3.2 Construction and Analysis of Γ(⊕) and Γ(⊙)

We start by considering the component graphs for $n = 2, 3, \ldots, 15$ as illustrative example then generalize for any given positive integer $n \geq 2$.

Using **Definition 3.1.1 and Theorem 2.1** we computed $E(\oplus)$ and $E(\odot)$ as follows:

$n = 2$, $E(\oplus) = \{(0, 0), (1, 1)\}$, $E(\odot) = \{(1, 1)\}$.

$n = 3$, $E(\oplus) = \{(0, 0), (1, 2)\}$, $E(\odot) = \{(1, 1), (2, 2)\}$.

$n = 4$, $E(\oplus) = \{(0, 0), (1,3), (2, 2)\}$, $E(\odot) = \{(1, 1), (3, 3)\}$.

$n = 5$, $E(\oplus) = \{(0, 0), (1, 4), (2, 3)\}$, $E(\odot) = \{(1, 1), (2, 3), (4, 4)\}$.

$n = 6$, $E(\oplus) = \{(0, 0), (1, 5), (2, 4), (3, 3)\}$, $E(\odot) = \{(1, 1), (5, 5)\}$.

$n =7$, $E(\oplus) = \{(0, 0), (1, 6), (2, 5), (3, 4)\}$, $E(\odot) = \{(1, 1), (2, 4), (3, 5)\}$.

$n = 8$, $E(\oplus) = \{(0, 0), (1, 7), (2, 6), (3, 5), (4, 4)\}$, $E(\odot) = \{(1, 1), (7, 7)\}$.

$n = 9$, $E(\oplus) = \{(0, 0), (1, 8), (2, 7), (3, 6), (4, 5)\}$, $E(\odot) = \{(1, 1), (2, 5), (4, 7), (8, 8)\}$.

$n = 10$, $E(\oplus) = \{(0, 0), (1, 9), (2, 8), (3, 7), (4, 6), (5, 5)\}$, $E(\odot) = \{(1, 1), (3, 7), (9, 9)\}$.

$n = 11$, $E(\oplus) = \{(0, 0), (1, 10), (2, 9), (3, 8), (4, 7), (5, 6)\}$, $E(\odot) = \{(1, 1), (2, 6), (3, 4), (10, 10)\}$.

$n = 12$, $E(\oplus) = \{(0, 0), (1, 11), (2, 10), (3, 9), (4, 8), (5, 7), (6, 6)\}$, $E(\odot) = \{(1, 1), (5, 5), (7, 7), (11, 11)\}$.

$n = 13$, $E(\oplus) = \{(0, 0), (1, 12), (2, 11), (3, 10), (4, 9), (5, 8), (6, 7)\}$, $E(\odot) = \{(1, 1), (2, 7), (3, 9), (4, 10), (5, 8), (6, 11), (12, 12)\}$.

$n = 14$, $E(\oplus) = \{(0, 0), (1, 13), (2, 12), (3, 11), (4, 10), (5, 9), (6, 8), (7, 7)\}$, $E(\odot) = \{(1, 1), (3, 5), (9, 11), (13, 13)\}$.

$n = 15$, $E(\oplus) = \{(0, 0), (1, 14), (2, 13), (3, 12), (4, 11), (5, 10), (6, 9), (7, 8)\}$, $E(\odot) = \{(1, 1), (2, 8), (4, 4), (7, 13) (11, 11), (14, 14)\}$.

In constructing the graphs we use a dot for each vertex and a line from a to b for each edge (a, b) and circle for edge loop (a, a). Figure 1(a) to 14(a) and Figure 1(b) to 14(b) are the subgragraphs $\Gamma(\oplus)$ and $\Gamma(\odot)$respectively.

$n = 2$

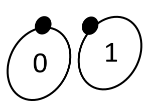


Figure 1(a)

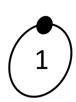


Figure. 1(b)

$n = 3$

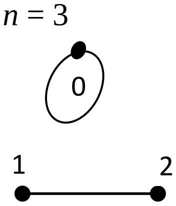


Figure 2(a)

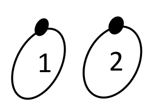


Figure 2(b)

$n = 4$

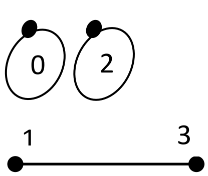


Figure 3(a)

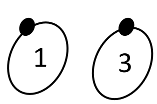


Figure 3(b)

$n = 5$

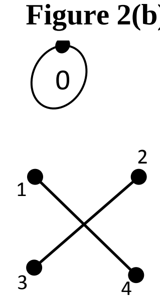
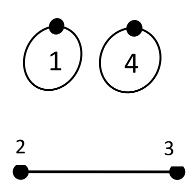


Figure 4(a)



Figure

$n = 6$

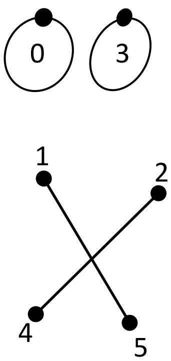


Figure 5(a)

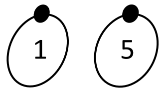
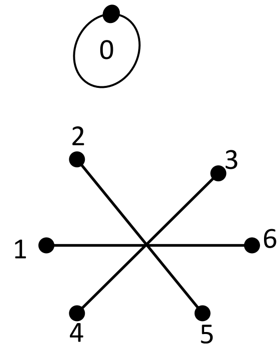


Figure 5(b)

$n = 7$



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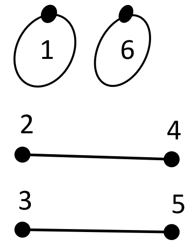
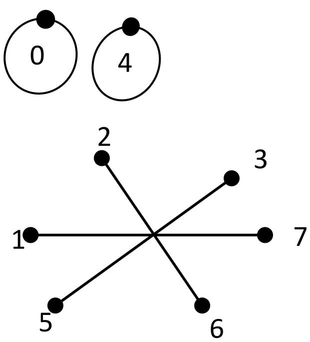


Figure 6(b)

$n = 8$



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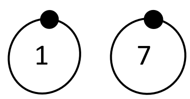
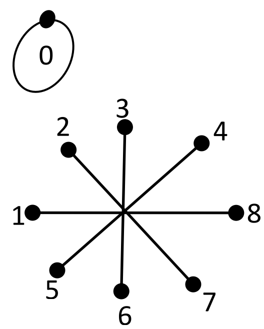


Figure 7(b)

$n = 9$



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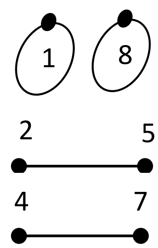
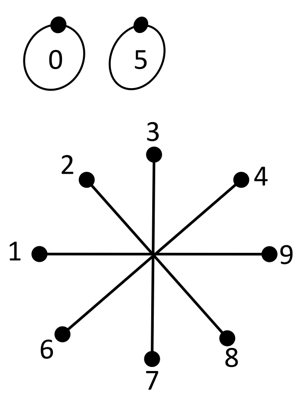


Figure 8(b)

$n = 10$



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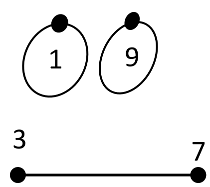

Figure 9(b)

$n = 11$

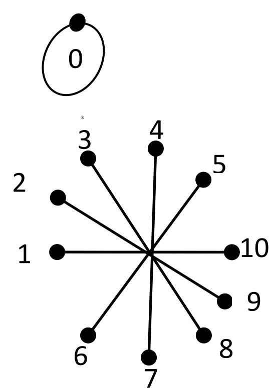

Figure 10(a)

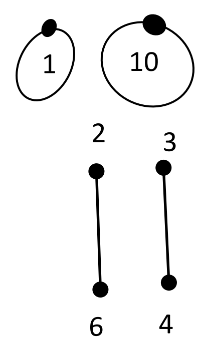

Figure 10(b)

n = 12

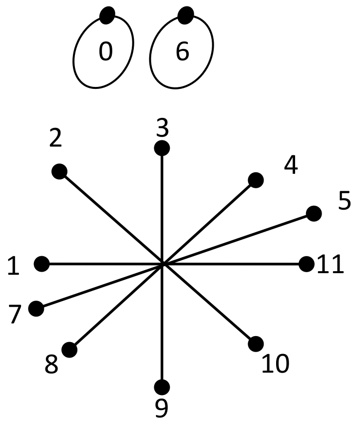

Figure 11(a)

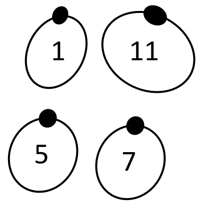

Figure 11(b)

n =13

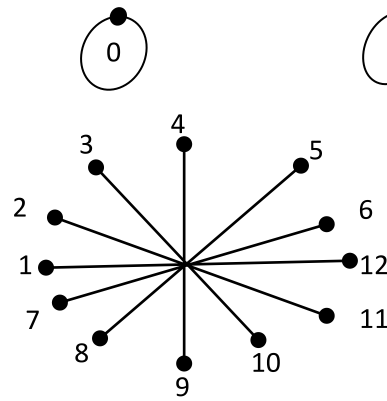

Figure12(a)

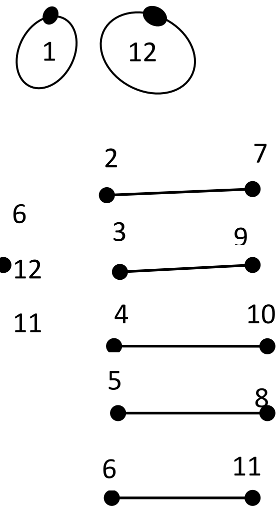

Figure 12(b)

n = 14

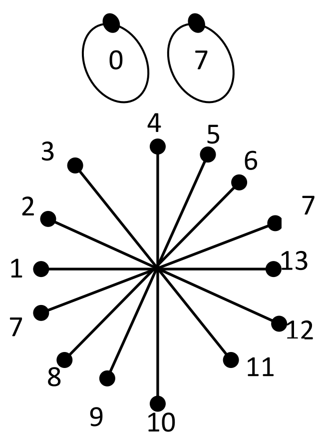

Figure 13(a)

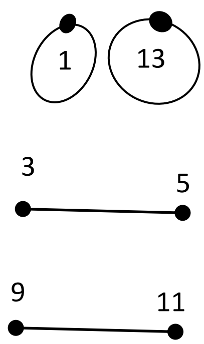

Figure 13(b)

n = 15

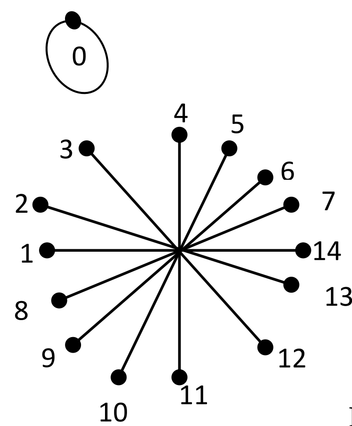

Figure 14(a)

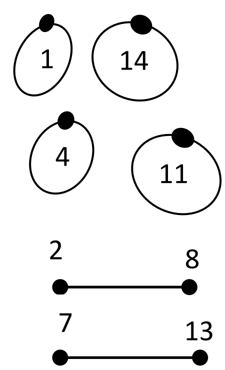

Figure 14(b)

3.3 Observations

In the graph Γ(⊕), (i) For odd n, it consists of two components, the trivial loop (0, 0) and complete simple graph having $(n-1)/2$ edges. (ii)There are two loops only for n=2.(ii) For even n>2, it consists of three components (two loops (0, 0), (1, 1) and simple graph) where the simple graph is complete with $(n-2)/2$ edges in the corresponding graph.

In Γ(⊙), (i) There is trivial loop only for $n = 2$. (ii) For $n = 3$, it consists of only the two trivial loops and for odd $5 \leq n < 15$ it has two trivial loops and simple graph having at least one edge. (iii) For even n>2, the graph has two trivial loops and simple subgraph having at least one line. for $n \geq 10$.

Note: We can observe that for $n = 12$, the graph consists of loops only.

Following the above observations, we have the following propositions.

Proposition 3.1: The components of Γ(⊕) has only one loop at the vertex 0 for odd n and two loops (trivial and non-trivial) for even n. While the components of Γ(⊙) has exactly two loops at the vertices1 and $n-1$ for odd n.

Proposition 3.2: (a) For any given $n \geq 2$, the subgraph Γ(⊕) is a graph associated to the algebraic structure ($\mathbb{Z}_n$, ⊕) and the subgraph Γ(⊙) is a graph associated to the algebraic structure ($\mathbb{U}_n$, ⊙), where $| \mathbb{U}_n | = \phi(n)$

3.4 Pseudo graph of additive and multiplicative inverses modulo *n*

In this subsection, a new graph is obtained as a result of combining the subgraphs Γ(⊕) and Γ(⊙) provided in **subsection 3.2** above. This graph is a pseudo graph consists of two or three components depend on whether n is odd or even. Let us start by given the definition and then construct and analyse the graphs for $n = 2, 3, \ldots, 15$.

Definition 3.4.1: The graph denoted by Γ(mod n) is a pseudo graph arising from additive and multiplicative inverses modulo n. We assign to each $a, b \in \mathbb{Z}_n$ an edge (a, b) of the graph Γ(mod n) if the congruences; $a \oplus b \equiv 0 \pmod{n}$ or $a \odot b \equiv 1 \pmod{n}$ holds, where $V = \mathbb{Z}_n$ is the set of vertices and $E \subseteq \mathbb{Z}_n \times \mathbb{Z}_n$ is the set of edges of Γ(mod n). Then the graph has a loop at a vertex a if and only if $a \oplus a \equiv 0 \pmod{n}$ or $a \odot a \equiv 1 \pmod{n}$. Since 0 is an additive identity in $\mathbb{Z}_n$, hence (0, 0) is always a loop in Γ(mod n) and for any given even positive integer $n \geq 2$, $(n/2, n/2)$ is also a loop. Similarly, 1 is a multiplicative inverse in $\mathbb{Z}_n$ and for any given positive integer $n \geq 2$, (1, 1) and $(n-1, n-1)$ are always loops in Γ(mod n). Thus we call (0, 0) , (1, 1) and $(n-1, n-1)$ trivial loops.

3.4.1Construction of Γ(mod n)

The union of Γ(⊕) and Γ(⊙) gives the Pseudo graph Γ(mod n) where $\mathbb{Z}_n$ is the set of vertices and E(⊕)∪E(⊙) is the edge set. We construct the Pseudo graph Γ(mod n) for $n = 2, 3, \ldots, 15$ as follows (Figure 15 to Figure 28):

$n = 2$

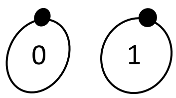

Figure 15

$n = 3$

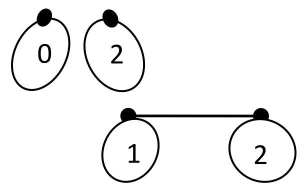

Figure 16

$n = 4$

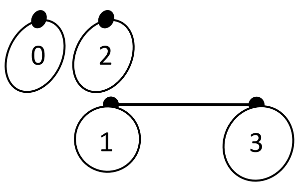

Figure 17

$n = 5$

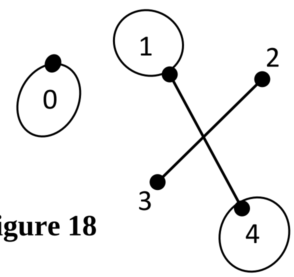

Figure 18

$n = 6$

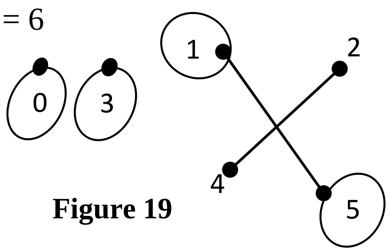

Figure 19

$n = 7$

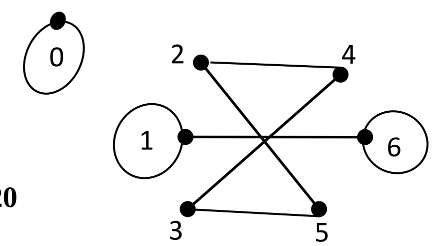

Figure 20

$n = 8$

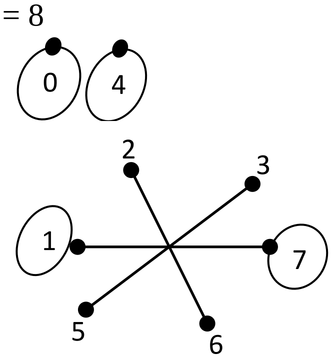

Figure 21

$n = 9$

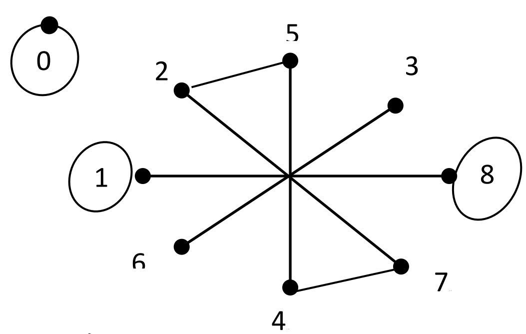

Figure 22

$n = 10$

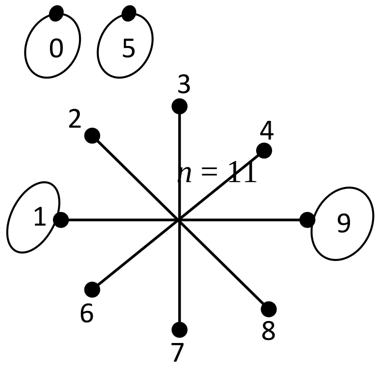

Figure 23

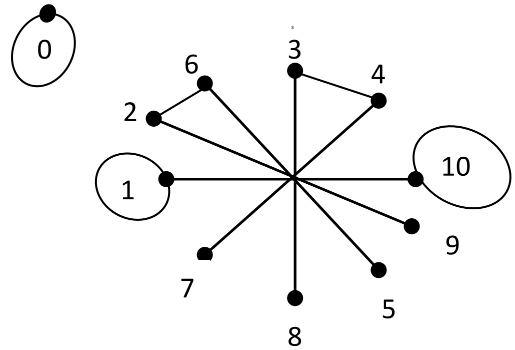

Figure 24

$n = 12$

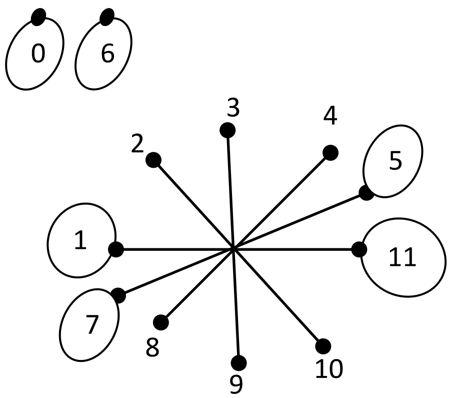

Figure 25

$n = 13$

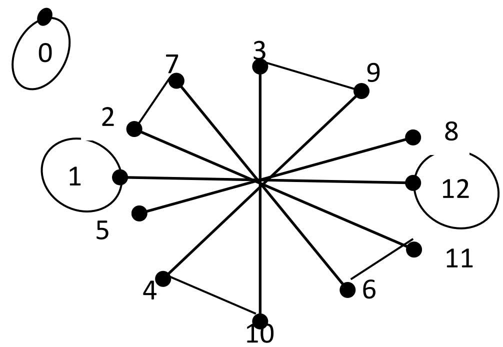

Figure 26

$n = 14$

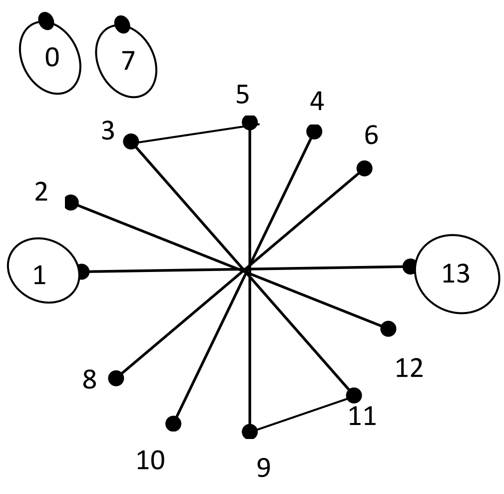

Figure 27

$n = 15$

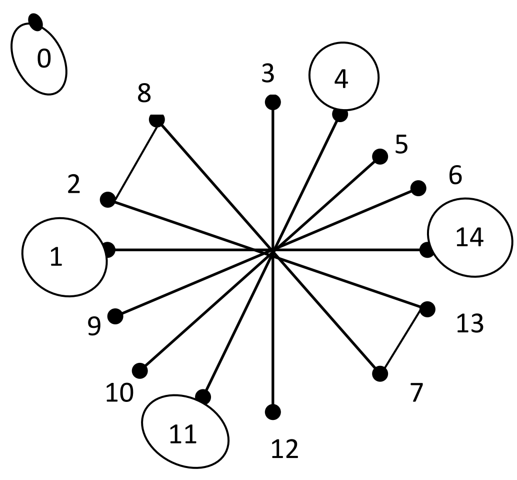

Figure 28

Remark: All multiple edges between a pair of vertices are considered to be a single edge between them and any number of loops at a particular vertex is considered to be a single loop at that vertex.

3.4.2Analysis of Γ(mod n)

In this subsection, we analysed the graphs obtained above. The following theorems are the results obtained.

Theorem 3.4: The pseudo graph of additive and multiplicative inverse modulo n always has loops at the vertices 0, 1 and $n-1$for any $n \geq 2$.

Proof

Let Γ(mod n) be a pseudo graph of additive and multiplicative inverse modulo n. For any $n \geq 2$, 0 is an additive identity and1 is a multiplicative inverse in $\mathbb{Z}_n$, hence, clearly i.e $0 \oplus 0 \equiv 0 \pmod{n}$ and $1 \odot 1 \equiv 1 \pmod{n}$. Thus Γ(mod n) always has loop at the vertices 0 and 1. Similarly, by the definition of congruence $(n-1)(n-1) \equiv 1 \pmod{n}$ since $n | n(n-2)$. Hence there is a loop at $n-1$. It follows that (0, 0), (1, 1) and ($n-1$, $n-1$) are always loop edges in Γ(mod n).

Corollary 3.5: The Pseudo graph Γ(mod n) consists of at least three loops for any positive integer $n>2$.

Proof

This follows from **Theorem 3.4**

Theorem 3.5: In the pseudo graph of additive and multiplicative inverse modulo n, the vertices a and $n-a$ are always adjacent for each non-zero $a \in \mathbb{Z}_n$.

Proof: Let Γ(mod n) be a pseudo graph of additive and multiplicative inverse modulo n and a be non-zero element of $\mathbb{Z}_n$. Observe that the additive of any non-zero element of $\mathbb{Z}_n$ is $n-a$ since $a \oplus (n-a) \equiv 0 \pmod{n}$ i.e $n|n$. Thus (a, $n-a$) is an edge in Γ(mod n).

Theorem 3.6: If n is an even positive integer then the vertex $n/2$ forms a loop edge in Γ(mod n).

Proof

Let Γ(mod n) be a pseudo graph of additive and multiplicative inverse modulo n and suppose n is even. Then $n/2$ is an element in $\mathbb{Z}_n$ and it is self additive inverse. i.e. $n/2 \oplus n/2 = n \equiv 0 \pmod{n}$. Thus ($n/2$, $n/2$) is a loop edge in Γ(mod n). Hence for any even n, Γ (mod n) has loop at $n/2$.

Theorem 3.7: The Pseudo graph Γ(mod n) has no centre.

Proof

Clearly in Γ(mod n) there exist no vertex that is connected to every other vertex for any $n \geq 2$.

Theorem 3.8: The Pseudo graph Γ (mod n) has two maximal components for odd n and three maximal components for even n, hence Γ (mod n) is disconnected by **Definition 2.17**.

Proof

The vertex 0 is not connected to any other vertex but itself; hence the loop (0, 0) is always a maximal component. For odd n, all the non-zero vertices are connected in another component. On the other hand, for even n, the loop ($n/2$, $n/2$) forms another independent component and the third component connects all other vertices in the graph.

We now investigate some general graph properties of Γ(mod n)

3.4.2.1 Degree of vertex in Γ(mod n)

(i) For any positive integer $n \geq 2$, the vertex 0 has degree two while each of the vertices 1 and $n-1$ has degree three. When n is even the vertex $n/2$ is of degree two.

(ii) The degree of every vertex in Γ(mod n) is at most three. Hence the graph Γ(mod n) is not regular.

3.4.2.2 Size of Γ(mod *n*)

The size of Γ(mod n) is half of the total sum of degrees of vertices in the graph. Hence it is a non-null and non-trivial graph.

3.4.2.3 Chromatic number of Γ(mod *n*)

The Pseudo graph Γ (mod n) is 2-colourable. Hence χ(Γ (mod n)) = 2.

3.4.2.4 Independent sets in Γ(mod *n*)

Since χ(Γ (mod n)) = 2, then there are exactly two independent sets of Γ (mod n) for each n. For odd n, the independent sets are of order $n/2 + 1$ and $n/2-1$. While for even n, the independent sets are having order $n/2$.

3.4.2.5 Independence number of Γ(mod *n*)

For odd n, α(Γ (mod n)) = $n/2 + 1$ and α(Γ(mod n)) = $n/2$ for even n.

4. Conclusion

In this study, we introduced new class of graphs using the congruences $a \oplus b \equiv 0 \pmod{n}$ and $a \odot b \equiv 1 \pmod{n}$ for $a, b \in \mathbb{Z}_n$; $n \geq 2$. We combined the graphs to obtain a new class of graph which we call the pseudo graph of additive and multiplicative inverses modulo n. The number of loops and maximal components in the pseudo graph were determined. Moreover, we investigate the graph properties such as size, degree of vertex, chromatic number, independent sets and independence number of the graph.

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