Transactions of the Nigerian Association of Mathematical Physics
Volume 12, (July – Sept., 2020 Issue), **pp101 – 108**
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OPTIMIZATION OF ECONOMIC ORDER QUANTITY (EOQ) UNDER STOCHASTIC DEMAND

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Abstract

One of the most frequent decisions faced by operations managers is "how much" or "how many" items are they to make or buy in order to satisfy external or internal requirements for the item. Replenishment in many cases is made using the economic order quantity (EOQ) model. The model considers the tradeoff between ordering cost and storage cost in choosing the quantity to use in replenishing items in inventories. This paper demonstrates an approach to optimize the EOQ of an item under a periodic review inventory system with stochastic demand. The objective is to determine in each period of the planning horizon, an optimal EOQ so that the long run profits are maximized for a given state of demands. Using dynamic programming over a finite planning horizon with equal intervals, the decision of how much quantity to order or not to order is made. We use a numerical example to demonstrate the existence of an optimal state, economic order quantity, as well as corresponding profits.

Keywords: Dynamic programming, inventory management, optimization, EOQ, Markov chain, stochastic process.

1.0 INTRODUCTION

There is no question that uncertainty plays a role in most inventory management situations. The retail merchant wants enough supply to satisfy customer demands, but ordering too much increases holding costs and the risk of losses through obsolescence or spoilage. An order too small increases the risk of lost sales and unsatisfied customers. The water resources manager must set the amount of water stored in a reservoir at a level that balances the risk of flooding and the risk of shortages. The operations manager sets a master production schedule considering the imprecise nature of forecasts of future demands and the uncertain lead time of the manufacturing process. These situations are common, and the answers one gets from a deterministic analysis very often are not
satisfactory when uncertainty is present. The decision maker faced with uncertainty does not act in the same way as the one who operates with perfect knowledge of the future. In this paper we deal with inventory model in which the stochastic nature of demand is explicitly recognized.

In the dynamic external business environment, when the risk becomes a natural and unavoidable factor, stochastic optimization can demonstrate a high efficiency in the process of solving the task of determining the optimal production, inventory levels and replenishment. Stochastic formulation of the problem accurately reflects the economic reality in terms of medium-term planning period. Moreover, the use of stochastic methods significantly exceeds the efficiency of deterministic models in the formation of the optimal production plan, improving financial and economic results of the company's business including profit figures[1].

In inventory management, the zeal for manufacturing industries to plan for optimal production levels that sustain random demand leaves a lot to be desired. Normally, when production exceeds quantity demanded, inventory carrying costs accumulate which affect profit margins of the manufacturer. Similarly, production levels below demand impose shortage costs and loss of good will from potential customers. Both cases drastically reduce profit margins unless proper planning and coordination are put in place to establish optimal production levels in a given manufacturing industry. In an effort to achieve this goal, two major problems are usually encountered:

(i) Determining the economic order quantity (EOQ) of the item.

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(ii) Determining the optimal profits associated with the economic order quantity (EOQ) when demand is uncertain [2].

A stochastic order quantity and reorder point model in comparison with a corresponding deterministic EOQ model was analyzed by [3]. The research result indicated that at large quantities, the difference between deterministic and stochastic models is small and the relative increase of the cost incurred by using the quantity determined by the EOQ instead of the optimal from the stochastic model does not exceed one eighth and vanishes when ordering costs are significant relative to other costs. The formulation of a two-stage model that minimized the cost of stochastic demand was carried out by [4]. The first stage dealt with moving inventory from the plant to the warehouses based on forecasted demand. The second stage was moving the inventory from the warehouses to the customers when they send an order. Using an experimental case, the model indicated that having two warehouses per customer was more efficient than having one warehouse per customer.

A periodic review system under stochastic demand using a single product was examined by [5].The simple solution procedure gave an almost optimal solution where results were extended to the joint replenishment problem for multiple items and the simple heuristic developed provided promising results. A stochastic EOQ type models with discounting using Gaussian processes in the context of the classical EOQ model was formulated by [6]. The Numerical properties of the order quantities that minimize expected costs for various model parameters were examined.

The dynamic lot-size problem under stochastic and non-stationary demand over the planning horizon was investigated [7]. The problem was solved by three popular meta-heuristic methods from the fields of evolutionary computation and swarm intelligence. A proposed formulation and solution procedure for inventory planning with the Markov decision process (MDP) models was done by [8]. They formulated the Markov decision model by identifying the chain's state space and the transition probabilities, specifying the cost structure and evaluating its individual component, and then use a policy-improvement algorithm to obtain the optimal policy. A model that would help decision makers about uncertainty in the supply chain was developed by [9]. The method reduced the number of variables and that made the model more robust. The network design of a supply chain with stochastic demand and risk pooling was examined by [10]. The model developed maximized the reduction of total cost as the variability and holding cost increased because the number of warehouses and inventory cost decreased. A replenishment inventory model, which analyzed a perishable inventory system based on item aging and retrieval behavior was developed by [11]. The study has profound insights especially in terms of the randomness of demand. A stochastic extension of the inventory-location model by including the probability of different scenarios based on demand and cost was presented by [12]. This extension was referred to as the stochastic location model with risk pooling (SLMRP). The model proved to be cost effective compared to deterministic models by having low regret values. An optimal policy for a stochastic inventory model of deteriorating items with time dependent selling price was determined by developed [13]. The rate of deterioration of the items was constant over time and the selling price decreased monotonically at a constant rate with deterioration of items. An optimization model for determining the economic production lot size which minimizes production and inventory costs of a periodic review production-inventory system was developed by [14] under stochastic demand. The development of an optimization model for determining the EOQ that minimizes inventory costs of multiple items under a periodic review system with stochastic demand was done by [15].The model demonstrates ordering decision using dynamic programming to establish the existence of optimal quantity and ordering policies. The inventory of a stochastic system whose goal is to optimize the quantity and profits associated with ordering and holding cost of the item was considered by [16]. A joint location inventory replenishment problem involving a chain of supermarkets at designated locations was considered by [17]. Associated with each supermarket is stochastic stationary demand where inventory replenishment periods are uniformly fixed for the supermarkets. Considering inventory positions of the supermarket chain, they formulated a finite state Markov decision process model where states of a Markov chain represent possible states of demand for milk powder product. The unit replenishment cost, shortage cost, demand and inventory positions were used to generate the total inventory cost matrix, representing the long run measure of performance for the Markov decision process problem. The problem was to determine for each supermarket at a specific location of an optimal replenishment policy so that the long run inventory costs are minimized for the given states of demand. An internet cafe faced with an optimal choice of band width for internet users under stochastic stationary demand was considered by [18]. The choice was made over uniformly time horizons with the goal of optimizing profits. Considering customer demand, price and operating costs of internet service, they formulated a finite state Markov decision process model where states of a Markov chain represented possible states of demand for internet service. A profit matrix was generated, representing the long run measure of performance for the Markov decision process problem. In this paper, we modify the work of [16] which considered the inventory of as to chastic system whose goal is to optimize the quantity and profits associated with ordering and holding cost of an item. At the beginning of every period, a major decision has to be made. That is, whether to order additional units of item or not to order, and make use of available units in inventory until the next period. In each case, our aim is to determine an optimal economic order quantity so that the long run profits are maximized for a given state of demand. We modify the model used in computation of demand transition and the profit matrix so as to reflect what is actually happening in real life.

2.0 MODEL PARAMETERS

$i, j =$ States of demand
$f =$ Favorable state
z = Ordering policy
$I^z =$ Inventory matrix
$P_s =$ Selling price per unit
$Q^z =$ Demand transition matrix
$O_i^z =$ Economic order quantity
$e_i^z =$ Expected total profits
$C_h =$ Holding cost per unit
$C_s =$ Shortage cost per unit
$g_n(i, p) =$ Optimal expected total inventory cost
$D_{ij}^z =$ Quantity demanded in state i, j for z policy

$n, N =$ Stages going from n=1 to N
$u =$ Unfavorable state
$D^z =$ Demand matrix
$P^z =$ Profit matrix
$P_{ij} =$ Profit in state i, j
$I_{ij}^z =$ Quantity in inventory in state i, j for z policy
$C_p =$ Cost price per unit
$C_h =$ Holding cost per unit
$C_o =$ Ordering cost per unit

3.0 MODEL DEVELOPMENT

In formulating the model, an inventory system of a single item is considered. The demand during each time period over a fixed planning horizon is classified under two states: favorable state (f) or unfavorable state (u). The transition probabilities over the planning horizon from one demand state to another could be described by means of a Markov decision process, as such the demand during each period is assumed to depend on the demand of the preceding period. To obtain an optimal course of action, a decision of ordering additional units of item denoted by (z =1) or not to order additional units denoted by (z=0) has to be made during each period over the planning horizon, where z is a binary decision variable. The maximum expected profits are put together at the end of the period to obtain optimality.

The change in demand is modeled by means of Markov Chain with transition matrix and demand matrix

$$Q_{ij}^z = \begin{pmatrix} Q^z{}_{ff} & Q^z{}_{fu} \\ Q^z{}_{uf} & Q^z{}_{uu} \end{pmatrix} \qquad D^z = \begin{pmatrix} D^z{}_{ff} & D^z{}_{fu} \\ D^z{}_{uf} & D^z{}_{uu} \end{pmatrix}$$

Also, the inventory and profit matrices are as follows

$$I^z = \begin{pmatrix} I^z{}_{ff} & I^z{}_{fu} \\ I^z{}_{uf} & I^z{}_{uu} \end{pmatrix} \qquad P^z = \begin{pmatrix} P^z{}_{ff} & P^z{}_{fu} \\ P^z{}_{uf} & P^z{}_{uu} \end{pmatrix}$$

We consider the work of [16] where he demonstrated the demand fluctuation of iron sheets in a hardware company in Uganda. The company wants to avoid ordering when demand is low or ordering when demand is high and hence, seek an analytic decision support in terms of optimal ordering policy. After observing some weaknesses in the paper, we make the following observations and modifications:

(i) In [16] the computation of demand transition matrix Q_{ij}^z was done using the number of customers over all states. However, since demands are available and we are considering demand transitions, then we think it will be more appropriate to use demand rather than number of customers in computing Q_{ij}^z since different customers have different demands.

(ii) Also, in [16] selling price P_s was used in the computation of profit matrix P^z. This also is not appropriate, since selling price is different from profit. Thus, in this paper, we are using profit in place of the selling price used, because profit is not equal to selling price but profit is equal to selling price minus cost price.

(iii) Also, in [16] every item incurs ordering, stockholding and shortage costs. irrespective of whether the item was held in stock before selling it to customers or sold to the customers directly before stocking. This is not what happens in reality. We modify it in such a way that any item held in stock before selling to the customers incurs ordering and stockholding costs only. Whereas if the item is sold directly before being held in stock, it incurs ordering and shortage costs only.

3.1 Dynamic programming formulation

As already stated, demand can be in favorable state (f) or unfavorable state (u), the problem of finding an optimal EOQ may be expressed as a finite period dynamic programming model.

Let $g_n(i)$ denote the expected total profit accumulated during periods n,n+1,………,Ngiven that the state of the system at the beginning of period n is $i \in \{f,u\}$

The recursive equation relating g_n and g_{n+1} is as follows

$$g_n(i) = \max_z\{Q_{if}^z(P_{if}^z + g_{n+1}(f)), Q_{iu}^z(P_{iu}^z + g_{n+1}(u))\} \qquad (1)$$

$i \in \{f,u\}$

$n = 1, 2, \ldots\ldots\ldots N$

for

$$g_{N+1}(f) = g_{N+1}(u) = 0 \qquad (2)$$

The justification to the recursive relationship is by noting the cumulative total profit

$$P_{ij}^z + g_{n+1}(j) \qquad (3)$$

which results from getting to state $j \in \{u, f\}$ at the initial period n+1 from state $i \in \{f,u\}$ at the beginning of period n occurs with the probability Q_{ij}^z.

It shows that $e_i^z = Q^z(P^z)^T$ (4)

The dynamic recursive equation becomes

$$g_n(i) = \max_z\{e_i^z + Q_{if}^z g_{n+1}(f) + Q_{iu}^z g_{n+1}(u)\} \qquad (5)$$

or $g_N(i) = \max_z\left[e_i^z\right]$ (6)

4.0 COMPUTATION OF DEMAND TRANSITION MATRIX, PROFIT MATRIX AND EOQ

Given the ordering policy, $z \in \{0,1\}$ the demand transition probability from state i to j could be justifiably taken as the quantity demanded when the demand is initially in state i and then changing to state j divided by the total quantity demanded over all states. This is therefore given as

$$Q_{ij}^z = \frac{D_{ij}^z}{[D_{if}^z + D_{iu}^z]} \qquad (7)$$

$i \in \{f,u\}$

$z \in \{0,1\}$

If the demand is greater than the inventory at hand, the profit matrix P^z can be computed as follows: Profit = selling price – cost price

$$P_{ij} = P_s - C_p \qquad (8)$$

Therefore,

$$P^z = P_{ij}(D^z) - (C_o + C_h)I^z - (C_o + C_s)[D^z - I^z] \qquad (9)$$

On the other hand, if the demand is less than or equal to the inventory at hand, then

$$P^z = P_{ij}D_{ij}^z - (C_o + C_h)D_{ij}^z \qquad (10)$$

$$\Rightarrow P^z = \begin{Bmatrix} P_{ij}D_{ij}^z - (C_o + C_h)I_{ij}^z - (C_o + C_s)[D_{ij}^z - I_{ij}^z] & if \;\; D_{ij}^z > I_{ij}^z \\ P_{ij}D_{ij}^z - (C_o + C_h)D_{ij}^z \;\; if \;\; D_{ij}^z \le I_{ij}^z & \end{Bmatrix} \qquad (11)$$

$\forall \; i, j \in \{f,u\}$,

$z \in \{0,1\}$

The justification for equation (11) is that $D_{ij}^z - I_{ij}^z$ units must be ordered immediately to meet the excess demand. The units ordered should attract shortage cost but not holding cost, otherwise when demand is less than or equal to on hand inventory, no order will be placed. In that case there will be no shortage cost but holding cost. The economic order quantity when demand is in state $i \in \{f,u\}$ with ordering policy $z \in \{0,1\}$ is

$$O_i^z = (D_{if}^z - I_{if}^z) + (D_{iu}^z - I_{iu}^z) \quad \text{provided } D_{ij}^z > I_{ij}^z \qquad (12)$$

$i, j \in \{f,u\}$, $z \in \{0,1\}$

Otherwise, $O_i^z = 0$

5.0 COMPUTING THE ECONOMIC ORDER QUANTITY FOR THE TWO PLANNING HORIZONS

In this paper we consider a two-period (N=2) planning horizon as in [16] since we are modifying the analysis in his paper.

5.1 Optimization during period 1 (First planning horizon)

The ordering policy during period 1 with favorable demand is

$$z = \begin{cases} 1, & if\ e_f^1 > e_f^0 \\ 0, & if\ e_f^1 \le e_f^0 \end{cases} \tag{13}$$

The associated total profit and EOQ are

$$g_1(f) = \begin{cases} e_f^1, & if\ z = 1 \\ e_f^0, & if\ z = 0 \end{cases} \tag{14}$$

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{15}$$

with the proviso in (12)

Also ordering policy during period 1 with unfavorable demand is

$$z = \begin{cases} 1, & if\ e_u^1 > e_u^0 \\ 0, & if\ e_u^1 \le e_u^0 \end{cases} \tag{16}$$

The associated total profits and EOQ are

$$g_1(u) = \begin{cases} e_u^1, & if\ z = 1 \\ e_u^0, & if\ z = 0 \end{cases} \tag{17}$$

and $O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases}$ (18)

with the proviso in (12)

5.2 Optimization during period 2 (Second planning horizon)

Recalling that a_i^z represent the accumulated profits at the end of period 1 as a result of decisions made using recursive equation (1) , it follows that:

$$a_i^z = e_i^z + Q_{if}^z g_1(f) + Q_{iu}^z g_1(u) \tag{19}$$

Therefore, the ordering policy when the demand is favorable is

$$z = \begin{cases} 1, & if\ a_f^1 > a_f^0 \\ 0, & if\ a_f^1 \le a_f^0 \end{cases} \tag{20}$$

The associated total profits and EOQ are

$$g_2(f) = \begin{cases} a_f^1, & if\ z = 1 \\ a_f^0, & if\ z = 0 \end{cases} \tag{21}$$

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases} \tag{22}$$

with the proviso in (12)

Also, the ordering policy when demand is unfavorable is

$$z = \begin{cases} 1, & if\ a_u^1 > a_u^0 \\ 0, & if\ a_u^1 \le a_u^0 \end{cases} \tag{23}$$

The associated total profit and EOQ are

$$g_2(u) = \begin{cases} a_u^1, & if\ z = 1 \\ a_u^0, & if\ z = 0 \end{cases} \tag{24}$$

and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); \ if\ z = 1 \\ 0;\ if\ z = 0 \end{cases} \qquad (25)$$

with the proviso in (12)

6.0 NUMERICAL EXAMPLE

Consider a sample of customers with the following demand patterns and inventory levels over state transitions collected in past 10 weeks in respect of favorable and unfavorable demand of a particular item. For ordering policy z=1, the data is in Table 1 and for ordering policy z=0, the data is in Table 2 below:

Table 1: demand and inventory at state transitions ordering policy z=0

i, j	D_{ij}^1	I_{ij}^1
ff	40	37
fu	10	30
uf	60	30
uu	20	5

Table 2: demand and inventory for ordering policy z=1

i, j	D_{ij}^0	I_{ij}^0
ff	25	10
fu	15	20
uf	80	40
uu	40	10

We can break up the tables as follows:
When additional units are ordered, z=1

$$D^1 = \begin{array}{c|cc} & f & u \\ \hline f & 40 & 10 \\ u & 60 & 20 \end{array}$$

$$I^1 = \begin{array}{c|cc} & f & u \\ \hline f & 37 & 30 \\ u & 30 & 5 \end{array}$$

When additional units are not ordered, z=0

$$D^0 = \begin{array}{c|cc} & f & u \\ \hline f & 25 & 15 \\ u & 80 & 40 \end{array}$$

$$I^0 = \begin{array}{c|cc} & f & u \\ \hline f & 10 & 20 \\ u & 40 & 10 \end{array}$$

In each of the cases, the unit selling price (P_s) is ₦3000, the ordering cost (C_o) is ₦200, the holding cost (C_h) is ₦50 and shortage cost (C_s) is ₦100 and cost price (C_p) is ₦2000

6.1 Computation of Model Parameters

$$P_{ij} = 3000 - 2000 = 1000$$

The demand transition matrix and profit matrix are computed using equations (7) and (11) when additional units are ordered, z=1 we get

$$Q^1 = \begin{pmatrix} \dfrac{D_{ff}^1}{D_{ff}^1 + D_{fu}^1} & \dfrac{D_{fu}^1}{D_{ff}^1 + D_{fu}^1} \\ \dfrac{D_{uf}^1}{D_{uf}^1 + D_{uu}^1} & \dfrac{D_{uu}^1}{D_{uf}^1 + D_{uu}^1} \end{pmatrix}$$

$$Q^1 = \begin{pmatrix} \dfrac{40}{50} & \dfrac{10}{50} \\ \dfrac{60}{80} & \dfrac{20}{80} \end{pmatrix}$$

$$\Rightarrow \quad Q^1 = \begin{pmatrix} 0.80 & 0.20 \\ 0.75 & 0.25 \end{pmatrix}$$

$$P^1 = 1000\begin{pmatrix} 40 & 10 \\ 60 & 20 \end{pmatrix} - 250\begin{pmatrix} 37 & 30 \\ 30 & 5 \end{pmatrix} - 300\begin{pmatrix} 3 & 0 \\ 30 & 15 \end{pmatrix}$$

$$\Rightarrow \quad P^1 = \begin{pmatrix} 40000 & 10000 \\ 60000 & 20000 \end{pmatrix} - \begin{pmatrix} 9250 & 7500 \\ 7500 & 1250 \end{pmatrix} - \begin{pmatrix} 900 & 0 \\ 9000 & 4500 \end{pmatrix}$$

$$\Rightarrow \quad P^1 = \begin{pmatrix} 29850 & 2500 \\ 43500 & 14250 \end{pmatrix}$$

When additional units are not ordered z=0

$$Q^0 = \begin{pmatrix} \dfrac{D^0_{ff}}{D^0_{ff} + D^0_{fu}} & \dfrac{D^0_{fu}}{D^0_{ff} + D^0_{fu}} \\ \dfrac{D^0_{uf}}{D^0_{uf} + D^0_{uu}} & \dfrac{D^0_{uu}}{D^0_{uf} + D^0_{uu}} \end{pmatrix}$$

$$Q^0 = \begin{pmatrix} \dfrac{25}{40} & \dfrac{15}{40} \\ \dfrac{80}{120} & \dfrac{40}{120} \end{pmatrix}$$

$$\Rightarrow \quad Q^0 = \begin{pmatrix} 0.63 & 0.38 \\ 0.67 & 0.33 \end{pmatrix}$$

$$P^0 = 1000\begin{pmatrix} 25 & 15 \\ 80 & 40 \end{pmatrix} - 250\begin{pmatrix} 25 & 15 \\ 80 & 40 \end{pmatrix}$$

$$\Rightarrow \quad P^0 = \begin{pmatrix} 25000 & 15000 \\ 80000 & 40000 \end{pmatrix} - \begin{pmatrix} 6250 & 3750 \\ 20000 & 10000 \end{pmatrix}$$

$$\Rightarrow \quad P^0 = \begin{pmatrix} 18750 & 11250 \\ 60000 & 30000 \end{pmatrix}$$

6.2 Computation of expected total profit

The matrices Q^1 and P^1 yield profits in Naira when additional units are ordered, that is for z=1

$$e^1 = \begin{pmatrix} 0.8 & 0.2 \\ 0.75 & 0.25 \end{pmatrix}\begin{pmatrix} 29850 & 2500 \\ 43500 & 14250 \end{pmatrix}$$

$$\Rightarrow \quad e^1_f = 29850(0.8) + 2500(0.2) = 24,380$$

$$\Rightarrow \quad e^1_u = 43500(0.75) + 14250(0.25) = 36,187.5$$

The matrices Q^0 and P^0 yield profits in Naira when additional units are not ordered, z=0

$$e^0 = \begin{pmatrix} 0.63 & 0.38 \\ 0.67 & 0.33 \end{pmatrix}\begin{pmatrix} 18750 & 11250 \\ 60000 & 30000 \end{pmatrix}$$

$$\Rightarrow \quad e^0_f = 18750(0.63) + 11250(0.38) = 16,087.5$$

$$\Rightarrow \quad e^0_u = 60000(0.67) + 30000(0.33) = 50,100$$

6.3The optimal ordering policy and EOQ

For week 1, it shows that z=1 is the optimal ordering policy for favorable state since ₦24,380 is greater than ₦16,087.5 with associated total profits of ₦24,380and EOQ of 40-37=3.0units.Also it shows that z=0 is the optimal ordering policy for unfavorable state since ₦50,100 is greater than ₦36,187.5 with associated profits of ₦50,100 with EOQ of 0 units since demand is unfavorable.

The accumulated profits for favorable and unfavorable demand in the second week are computed using the equation $a^z_i = e^z_i + Q^z_{if}\, g_1(f) + Q^z_{iu}\, g_1(u)$ for $z = \{1, 0\}$ and $i = \{f, u\}$

For favorable demand, we have:

$$a^1_f = 24380 + (0.8)(24380) + (0.2)(50100) = 53,904$$

$$a^0_f = 16087.5 + (0.63)(24380) + (0.38)(50100) = 50,484.9$$

It shows that z=1 is the optimal ordering policy for favorable demand since ₦53,904is greater than ₦50,484.9 with accumulated profit of ₦53,904 and an EOQ of 40-37=3units.

However, the accumulated profits for unfavorable demand are as follows:

$a_u^1 = 36188 + (0.75)(24380) + (0.25)(50100) = 66,998$

$a_u^0 = 50100 + (0.67)(24380) + (0.33)(50100) = 82,967.6$

It shows that z=0 is the optimal ordering policy since ₦82,967.6is greater than ₦66,998 with accumulated profits of ₦82,967.6 and an EOQ of 0 units.

7.0 CONCLUSION

In this paper, we present an inventory model which determines the economic order quantity for a given item with stochastic demand. With the aid of dynamic programming, the decision to order or not to order additional units is modeled as a multi-period decision problem. We demonstrate the working of the model with a numerical example. The model is a modification of an earlier model by [16]. In [16] the computation of demand transition matrix Q_{ij}^z was done using the number of customers over all states. However, since demands are available and we are considering demand transitions, then we used demand rather than number of customers in computing Q_{ij}^z since different customers have different demands.

Also, in [16] selling price P_s was used in the computation of profit matrix P^z but since selling price is different from profit, we changed that in this paper and used profit in place of the selling price. This is so because profit is not equal to selling price but profit is equal to selling price minus cost price. Also, in [16] every item incurs ordering, stockholding and shortage costs irrespective of whether the item was held in stock before selling it to customers or sold to the customers directly before stocking. This is not what happens in reality, therefore we modified it in such a way that any item held in stock before selling to the customers incurs ordering and stockholding costs only. Whereas if the item is sold directly before being held in stock, it incurs ordering and shortage costs only. From the results of our numerical example, we found out that if demand for an item favors a business, then the owner will order for more items in order to meet up with the increase in demand which will in turn increase the profit. Also, when the demand for the item is unfavorable, the business owner will not order new items, since keeping large inventory incurs more holding costs.

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