

## APPLICATIONS OF INHOMOGENEOUS INCIDENT WAVE FREQUENCY ON THE NON-LINEAR SPATIAL ABSORPTION OF LOW-FREQUENCY ALFVEN WAVES IN HIGH-ENERGY PLASMA PHYSICS

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### Abstract.

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*A non-linear absorption of an Alfvén wave falling on the boundary of incompressible plasma occupying a half-space is investigated. This absorption is due to dissipative effects of such factors as magnetic and hydrodynamic viscosities, thermal conductivities of electrons and ions, cluster of electrons, temperature relaxation etc. The dependence of the absorption on the spatial inhomogeneity of plasma density and the frequency of the incident Alfvén wave is also examined under some boundary conditions. It is shown that the parameters of an Alfvén wave penetrating dissipative plasma stabilize in time and satisfy a certain boundary value problem for a system of ordinary differential equations on a half-line. A numerical solution of this boundary value problem allows us to find some important relationships characterizing the absorption of Alfvén waves in high-photon energy plasma physics.*

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**Keywords:** Alfvén wave, spatial absorption, Electromagnetic Hydrodynamics, Plasma Physics.

### 1. Introduction

For a plane Alfvén wave falling on the boundary of homogeneous incompressible plasma occupying a half-space, the nonlinear spatial absorption due to dissipative effects was previously considered in [1], the non-linearity of the absorption process being primarily determined by the dependence of the transport coefficients on the temperatures of electrons and ions. That investigation was based on the equations of electromagnetic hydrodynamics (EMHD) of plasma [2,3] taking into account its electron-ion structure. The EMHD equations are written out in section 2 of this work. In section 3, we indicate some exact solutions of the EMHD equations. These solutions, called plane Alfvén waves, differ from the Alfvén waves of the classical MHD, but turn into the latter in the longwave limit. It has been shown in [1] that: (i) with bremsstrahlung of electrons taken into account the parameters of the absorbed Alfvén wave in dissipative plasma stabilize in time to a quasi-stationary regime; (ii) the Alfvén wave penetrates plasma to a finite depth; (iii) the parameters of the stabilized Alfvén wave can be found by solving a certain boundary value problem for a system of ordinary differential equations on a half-line. In this work, the dependence of the said absorption on the inhomogeneous density of the incompressible plasma and the frequency,  $\omega$  of the incident Alfvén wave was examined.

Interest is mainly focused on two types of inhomogeneity with the Gaussian distribution  $\rho(x) = 1 + A \exp(-(x-x_0)^2/D)$ ,  $D > 0$ ,  $x_0 > D > 0$ , namely: (i) the case of  $A > 0$  corresponding to a hump; (ii) the case of  $-1 < A < 0$  corresponding to a hollow. In section 4, an initial boundary value problem describing the decay of an Alfvén wave in nonhomogeneous plasma, as well as a boundary value problem on a half-line for the parameters of the time-stabilized absorbed Alfvén wave was formulated. Section 5 contains the results of numerical analysis of plasma parameters in the solar corona, which allows us to study the process of its anomalous heating by Alfvén waves arising in the photosphere [4]. It is shown that the basic relations characterizing the absorption in homogeneous dissipative plasma still hold in the nonhomogeneous case. Moreover, for the depth to which the Alfvén wave penetrates dissipative plasma and the maximal temperatures of electrons and ions, the dependence on the parameters  $A$ ,  $D$ ,  $\omega$ , where  $A$  is the height of the hump,  $D$  is its width, and  $\omega$  is the frequency of the incident wave was found.

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**2. Equations of Electromagnetic Hydrodynamics (EMHD) of Plasma.**

To study the dynamics of two-fluid plasma, we use the equations of electromagnetic hydrodynamics (EMHD). The EMHD equations of incompressible plasma have the form

$$\nabla U = 0, \frac{\partial P}{\partial t} + U \cdot \nabla P = 0, \frac{\partial \rho U}{\partial t} + \nabla \Pi = \nabla P$$

Thus,  $E + \frac{c^2 \lambda_+ \lambda_- \mu E}{4\pi P} = \frac{J}{\sigma} - \frac{1}{c} [U, H] + \frac{1}{\rho} \nabla W$  (1)

$$\frac{1}{c} \frac{\partial H}{\partial t} + \mu E = 0, \nabla E = 0, J = \frac{c}{4\pi} \mu H, \nabla H = 0$$

Where  $\Pi$  is the momentum flux tensor,  $P$  is the viscous stress tensor,  $W$  is the ‘‘hall terms’’ tensor. Thus,

$$\Pi = \Pi^h + \Pi^p + \Pi^c, P = P^c + P^p + P^c,$$

$$W = (\lambda_- - \lambda_+) (\Pi^p + \Pi^c) + (\lambda_- P_+ - \lambda_+ P_-) I_3 + \lambda_+ \lambda_- (JU + UJ^*) - \Pi_*^{(U)} - \Pi_*^{(C)} \quad (2)$$

$$\Pi^h = \rho UU + P_\varepsilon I_3, \Pi^p = \frac{H^2}{8\pi} I_3 - \frac{HH}{4\pi}, \Pi^c = \lambda_+ \lambda_- \frac{JJ}{\rho}.$$

Here and in what follows, the subscripts  $\pm$  refer to ions and electrons,  $\lambda_\pm = \frac{m_\pm}{e_\pm}$ ,  $\lambda_\varepsilon = \lambda_+ + \lambda_-$ ,  $P_\varepsilon = P_+ + P_-$ ,  $m_\varepsilon = m_+ + m_-$ ,  $\rho_\varepsilon = \rho_+ + \rho_-$ ,  $U = \frac{\rho_+ \rho_+ + \rho_- \rho_-}{\rho}$ ,  $I_3$  is the three-dimensional identity tensor,  $\sigma$  is the conductivity of plasma. The viscous stress tensors (with plasma incompressibility taken into account) have the form

$$\Pi_*^{(U)} = 2\mu_\varepsilon D^U, \Pi_*^{(U)} = 2\mu_* D^U, \Pi_*^{(C)} = 2\mu^* D^U - (2/3)U^* J \cdot \nabla(1/\rho) I_3,$$

$$\Pi_*^{(C)} = 2\mu_* D^U - (2/3)U^* J \cdot \nabla(1/\rho) I_3,$$

$$\Pi_\pm = 2U_\pm D_\pm \mp \left(\frac{2}{3}\right) \mu_\pm \lambda_\pm J \cdot \nabla(1/\rho) I_3 \quad (3)$$

Where  $D^U = \det U$ ,  $D^C = \det \left(\frac{j}{\rho}\right)$ ,  $D_\pm = \det V_\pm$  are strain tensors,  $\mu_\varepsilon = \mu_+ + \mu_-$ ,  $\mu_* = \lambda_- \mu_+ - \lambda_+ \mu_-$ ,  $\mu^* = \lambda_-^2 \mu_+ + \lambda_+^2 \mu_-$ ,  $\mu_\pm$  are hydrodynamic viscosities of electrons and ions. Here, the second viscosities of electrons and ions are assumed to be equal to zero. For plane flows considered below, we have  $j \cdot \nabla \left(\frac{1}{\rho}\right) = 0$  and the expressions (3) become much simpler.

Taking into account the dependence (see below) of the plasma conductivity  $\sigma$  the hydrodynamic viscosities  $\mu_\pm$ , and the other transport coefficients on the temperatures  $T_\pm$  of electrons and ions, we should supplement system (1) – (3) with the equations for the temperatures [5],

$$\rho_\pm C_p^\pm \left[\frac{\partial T_\pm}{\partial t} + \Delta T_\pm V_\pm\right] = \Delta(\chi_\pm \Delta T_\pm) + \text{tr}(\Pi_\pm D_\pm) + \frac{m_\pm}{m_\varepsilon \sigma} j^2 \pm b(T_- - T_+) - P_T^\pm \quad (4)$$

Where  $C_p^\pm = K/[(y - 1)m_\pm]$  are heat capacities at constant pressure,  $k$  is the Boltzmann constant,  $\chi_\pm$  are thermal conductivities of electrons and ions.  $Q = \pm b(T_- - T_+)$  is the heat transmitted to each other by plasma components in elastic collisions,  $P_T^\pm$  are the losses in bremsstrahlung of electrons and ions (the losses of ions are assumed to be equal to zero in what follows). Here, we should also take into account the expressions of hydrodynamic parameters of electrons and ions in terms of  $\rho$ ,  $U$ ,  $J$ :

$$V_\pm = U_\pm \left(\frac{\lambda_\pm}{\rho}\right) J, \rho_\pm = (\lambda_\pm / \lambda_\varepsilon) \rho \quad (5)$$

In view of the dependence of  $\sigma, \mu_\pm, \chi_\pm, b, P_T^\pm$  on the other plasma parameters, especially, the temperatures  $T_\pm$  (see below), equations (1) – (5) form a closed determined system of equations for  $\rho_\pm, T_\pm, U, H, E$ .

The transport coefficients  $\mu_\pm, \chi_\pm, \sigma, b$  are obtained from approximate solutions of kinetic equations [2] and, in what follows, are taken equal to [6–8]

$$\mu_+ = 3.44 \times 10^{-18} \left(\frac{m_+}{m_-}\right)^{\frac{1}{2}} T_+^{\frac{5}{2}}, \mu_- = 1.85 \times 10^{-18} T_-^{\frac{5}{2}}, \sigma = 0.906 \times 10^7 T_-^{\frac{3}{2}},$$

$$b = 9.044 \times 10^{38} \left(\frac{m_-}{m_+}\right)^3 \rho^2 T_-^{-\frac{3}{2}}, \chi_- = 3.657 \times 10^{-5} T_-^{\frac{5}{2}},$$

$$\chi_+ = 6.436 \times 10^{-5} (m_- / m_+)^{1/2} T^{5/2}, P_T^+ = 0, P_T^- = 8.044 \times 10^{32} n_+ n_- T^{\frac{1}{2}} \text{erg} / (\text{s} \cdot \text{cm}^3)$$

where  $T_\pm$  are measured in K. In the calculated results given below, the electron viscosity is assumed to be equal to zero. More detailed formulas for the dissipative coefficients can be found in [9].

**3. Alfvén Waves in Electromagnetic Hydrodynamics (EMHD)**

In the dissipation-free planar case, the EMHD equations admit the following exact solutions [9]:

$$U_\pm = \mu(t) \exp(ikx), H_\pm = h(t) \exp(ikx), E_\pm = e(t) \exp(ikx), T_\pm = \text{constant}, \rho = \text{constant}, U_x = 0 \quad (6)$$

called planar Alfvén waves. In (6), we have  $k > 0$  and use the complex notation:  $U_{\pm} = U_y + iU_z$ ,  $H_{\pm} = H_y + iH_z$ ,  $E_{\pm} = E_y + iE_z$ ,  $J_{\pm} = J_y + iJ_z$  and  $e(t)$  can be explicitly expressed through  $u(t)$  and  $h(t)$  :

$$e(t) = \frac{iH_x u(t) + ikAV_A}{c \frac{\omega \rho}{1+r^2}}, \quad r = \frac{kc}{\omega \rho}, \quad \Lambda = \sqrt{\frac{\lambda_+}{\lambda_-}} - \sqrt{\frac{\lambda_-}{\lambda_+}}, \quad = \frac{H_x}{\sqrt{4\pi\rho}} \tag{7}$$

The functions  $u(t)$ ,  $h(t)$  have the form

$$U(t) = C_1 e^{i\omega_+ t} + C_2 e^{i\omega_- t}, \quad h(t) = (4\pi\rho)^{1/2} (kV_A)^{-1} \{C_1 \omega_+ e^{i\omega_+ t} + C_2 \omega_- e^{i\omega_- t}\} \tag{8}$$

Here  $\omega_{\rho} = (4\pi\rho)^{1/2} (\lambda_+ \lambda_-)^{-1/2}$  is plasma frequency,  $C_1, C_2$  are arbitrary complex constants. The transverse component of the current density  $J_{\pm} = J_y + iJ_z$  varies in the same way,  $J_{\pm} = J(t) \exp(ikx)$ , and  $J_x = 0$ , where :

$$J(t) = \frac{-kc}{(4\pi)h(t)}. \text{ Finally,}$$

where

$$-\omega_c^+ < \omega_{\pm} = \omega_{\pm}(k) = \frac{KV_A}{2} \left\{ \frac{r\Lambda}{1+r^2} \pm \left[ \frac{r\Lambda}{(1+r^2)} + \frac{4}{(1+r^2)} \right]^{1/2} \right\} \tag{9}$$

$\omega_c^-$  It is convenient to invert formula (9):

$$\text{for } H_x > 0, \quad K = K(\omega) = \frac{\omega \rho}{c} \cdot \omega \left( \frac{V_A^2 \omega_{\rho}^2}{c^2} + \frac{\omega \rho V_A \Lambda \omega}{c} - \omega^2 \right)^{1/2} \tag{10}$$

0, and

$$-\omega_c^- < \omega < \omega_c^+ \text{ for } H_x < 0$$

#### 4. Physical meaning of the Spatial Absorption Problem.

Consider a planar Alfvén wave running from left to right in the domain  $x \leq 0$  and arriving at the dissipative plasma boundary  $x = 0$ , the plasma filling up the half-space  $x \geq 0$ . Further propagation of the Alfvén wave in the domain  $x \geq 0$  is accompanied by its absorption, which is the subject of this study. From the results of the preceding section, it follows that for the longitudinal magnetic field  $H_x < 0$ , the frequency  $\omega$  of the incident Alfvén wave varies within the interval  $-\omega_c^- = H_x / (\lambda_- c) < \omega < 0$ , and if  $H_x > 0$ , then it varies on the interval  $-\omega_c^+ = -H_x / (\lambda_+ c) < \omega < 0$ .

Below, we limit ourselves to the most interesting first case. The plasma in the region  $x \geq 0$  is assumed to be magnetized, stationary, and isothermal, with a given density distribution  $\rho(x)$ . Thus, at the initial instant in the domain  $x \geq 0$ , we have :

$$U_{\perp}|_{t=0} = 0, \quad U_x|_{t=0}, \quad H_{\perp}|_{t=0}, \quad T_{\pm}|_{t=0} = T^0, \quad \rho|_{t=0} = \rho(x) \tag{11}$$

Where  $\rho(x)$  tends to a finite limit  $\rho > 0$  as  $x \rightarrow \infty$ , with the constant  $\rho$  and the longitudinal magnetic field  $H_x = \text{constant}$ , being the same as those in the domain  $x \leq 0$  from which the Alfvén wave comes. The values of the dissipative plasma parameters on the boundary  $x = 0$  coincide with the values of the Alfvén wave parameters on the same boundary, and therefore, in view of (6) – (10), we have the following boundary conditions for  $x = 0$

$$U_{\perp}|_{x=0} = U_0 \exp(i\omega t), \quad H_{\perp}|_{x=0} = U_0 \frac{\sqrt{4\pi\rho}}{KV_A} \omega \exp(i\omega t), \\ E_{\perp}|_{x=0} = \frac{iU_0}{1+(kc/\omega\rho)^2} \left( \frac{H_x}{c} + \Lambda \sqrt{\lambda_+ \lambda_-} \omega \right), \quad J_{\perp}|_{x=0} = -U_0 \frac{\omega c \rho}{KV_A} \exp(i\omega t) \tag{12}$$

At infinity, the dissipative plasma parameters coincide with the unperturbed plasma parameters for  $t = 0$  :  $U_{\perp}|_{x=+\infty} = 0$ ,  $H_{\perp}|_{x=+\infty} = 0$ ,  $T_{\pm}|_{x=+\infty} = T^0$ ,

$$E_{\perp}|_{x=+\infty} = 0, \quad J_{\perp}|_{x=+\infty} = 0 \tag{13}$$

Let us write system (1) – (5) in dimensionless form in the case of plane symmetry

( $\partial/\partial y = \partial/\partial z = 0$ ), using the complex notation:  $U = U_{\pm} = U_y + iU_z$ ,  $H = H_{\pm} = H_y + iH_z$ ,  $E = E_{\pm} = E_y + iE_z$ ,  $J =$

$J_{\pm} = J_y + iJ_z$ . Taking into account the relation  $J_x = 0$ , we have

$$\rho \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left( H_x H + \mu_{\varepsilon} \frac{\partial u}{\partial x} + \xi U_* \frac{\partial j}{\partial x \rho} \right) = 0, \quad \frac{\partial H}{\partial t} + i \frac{\partial E}{\partial x} = 0, \quad J = i \frac{\partial H}{\partial x}, \quad \rho = \rho(x),$$

$$E - \xi^2 \frac{\partial^2 E}{\partial x^2} = \xi \varepsilon \frac{j}{\sigma} + i H_x U + \frac{\xi \varepsilon}{\rho \partial x} [\Lambda H_x H - U_* \frac{\partial u}{\partial x} - \xi \mu^* \frac{\partial j}{\partial x \rho}] \tag{14}$$

$$\frac{\partial T_{\pm}}{\partial t} = \frac{2Z_{\pm}(\gamma-1)}{\rho} \left\{ \frac{\partial}{\partial x} \left( \chi_{\pm} \frac{\partial T_{\pm}}{\partial x} \right) + \frac{m_{\pm} \xi \varepsilon}{m_{\varepsilon} \sigma} |j|^2 + \frac{\varepsilon}{\xi} C_0 \rho^2 \frac{T_{\pm} - T_{\pm}^0}{T_{\pm}^{3/2}} + \frac{T_{\pm}^{5/2}}{R_{\pm}} \left[ \left| \frac{\partial u}{\partial x} \right|^2 + \frac{\lambda_{\mp}}{\lambda_{\pm}} \xi^2 \left| \frac{\partial j}{\partial x \rho} \right|^2 \right] \pm 2\xi \sqrt{\frac{\lambda_{\mp}}{\lambda_{\pm}}} \text{Re} \left( \frac{\partial u}{\partial x} \frac{\partial j}{\partial x \rho} \right) \right\} -$$

$$\frac{(1 \mp 1) \xi T_{\pm}}{2 \varepsilon} (\gamma - 1) Z^2 \rho T_{\pm}^{-1/2} \tag{15}$$

Where  $\mu_{\varepsilon} = \mu_+ + \mu_-$ ,  $\mu^* = (\lambda_- / \lambda_+)^{1/2} \mu_+ - (\lambda_+ / \lambda_-)^{1/2} \mu_-$

$\mu^* = \left( \frac{\lambda_-}{\lambda_+} \right) \mu_+ + \left( \frac{\lambda_+}{\lambda_-} \right) \mu_-$ ,  $\mu_{\pm} = \frac{T_{\pm}^2}{R_{\pm}}$ ,  $\sigma = \sigma_0 T_{\pm}^{3/2}$ ,  $\chi_{\pm} = C_{\pm} \left( \frac{\xi}{\varepsilon} \right) T_{\pm}^{5/2}$ , and  $\sigma_0, C_0, R_{\pm}, C_{\pm}$  have the form

$$\sigma_0 = 2.59, \quad C_0 = \frac{m_-}{m_+}, \quad R_{\pm} = 1.108 \left( \frac{\xi}{\varepsilon} \right) \frac{m_-^{1/2}}{m_+}$$

Finally,  $Z_+ = Z, Z_- = 1$  and  $\xi, \zeta, \xi_T$  are similarity numbers which, for  $Z=1$ , have the form

$$\xi = \frac{\ell_c}{L_0} = c \frac{\sqrt{\lambda_+ \lambda_-}}{\sqrt{4\pi\rho_0 L_0}}, \zeta = 15 \frac{(4\pi\rho_0)^{5/2} c e^3}{H_0^2 m_+^3}, \xi_T = 21.825 \frac{\rho_0^{3/2}}{H_0^2} \frac{10^7 e^5}{c^2 h m_+ m_-},$$

where  $\lambda_c = c/w_p$  is skin length,  $L_0, \rho_0, H_0, \dots$ , are the respective characteristic scales of length, density, magnetic field strength, etc. Moreover, when passing to dimensionless parameters, it has been assumed that  $t_0 = L_0/v_0, v_0 = v_A = H_0/\sqrt{4\pi\rho_0}, E_0 = V_0 H_0/c, j_0 = cH_0/(4\pi L_0), T_0 = V_A^2 \lambda_e e/(2k)$ .

Our aim is to describe steady quasi-stationary states of the absorbed wave of frequency  $\omega$ . We seek these states as solutions of system (14) having the form  $T_{\pm} = T_{\pm}(x), U = U(x) \exp(i\omega t), H = H(x) \exp(i\omega t), E = E(x) \exp(i\omega t), J = J(x) \exp(i\omega t)$ . Substituting these expressions into (14) and dividing the resulting relations by  $\exp(i\omega t)$ , we obtain the following system of ordinary differential equations for the amplitudes  $T_{\pm}(x), U(x), E(x)$ :

$$i\rho\omega U + \frac{H_x}{\omega} E'' - (\mu_\varepsilon U)' - \frac{i\xi}{\omega} \left( \mu_* \left( \frac{E''}{\rho} \right)' \right) = 0$$

$$E - \left( \frac{\xi^2}{\rho} - \frac{i\xi\xi}{\omega\sigma} - \frac{\xi\Lambda H_x}{\omega\rho} \right) E'' - iH_x U + \frac{\xi}{\rho} (\mu_* U)' - \frac{i\xi^2}{\omega\rho} \left( \mu_* \left( \frac{E''}{\rho} \right)' \right) \tag{16}$$

$$(\chi_{\pm} T'_{\pm})' + \frac{m_{\pm}}{m_e} \frac{\xi\zeta}{\omega^2\sigma} |E''|^2 \pm \frac{\zeta}{\xi} C_0 \rho^2 \frac{T_- - T_+}{T_{\pm}^{3/2}} + \frac{T_{\pm}^{3/2}}{R_{\pm}} \left[ |U'|^2 + \frac{\lambda_{\mp} \xi^2}{\lambda_{\pm} \omega^2} \left| \frac{E''}{\rho} \right|^2 \pm 2 \frac{\xi}{\omega} \sqrt{\frac{\lambda_{\mp}}{\lambda_{\pm}}} \operatorname{Re} \left( iU' \left( \frac{E''}{\rho} \right)' \right) \right] - \frac{(1 \mp 1) \xi_T}{4} \frac{\xi}{\xi} Z^2 \rho T_{\pm}^{-1/2} = 0$$

where  $\rho(x)$  is a given function and the prime symbol indicates differentiation in  $x$ . Taking into account the above expressions of  $\mu_\varepsilon, \mu_*, \mu^*, \sigma$  in terms of  $T_{\pm}$ , we see that (15) is a nonlinear system of ordinary differential equations on  $[0, +\infty)$  whose solutions allow us to calculate the functions  $H(x) = \frac{-E'}{\omega}$  and  $J(x) = \frac{-iE''}{\omega}$ . System (15) is supplemented by the following dimensionless boundary conditions at the points  $x=0$  and  $x=+\infty$

$$T_{\pm}(0) = T^0, U(0) = U_0, E(0) = \frac{iU_0}{1+K^2\xi^2} (H_x + \xi\Lambda\omega), E'(0) = \frac{-\rho U_0 \omega^2}{KH_x}, T_{\pm}(+\infty) = T^0, U(+\infty) = 0, E(+\infty) = 0, E'(+\infty) = 0 \tag{17}$$

Where  $k(\omega)$  is obtained from (10) in dimensionless form:

$$k = k(\omega) = |\omega| \rho^{\frac{1}{2}} (H_x^2 + H_x \Lambda \xi \omega - \xi^2 \omega^2)^{-1/2}$$

### 5. Analysis of calculated Results .

In order to find a solution of the boundary value problem (16), (17), we use the stabilization method [9] for  $\xi = 1, \xi_T = 3, \zeta = 3 \times 10^3$  which corresponds to the typical parameters of the solar corona:  $\rho_0 = 10^{-12} g/cm^3, H_0 = 1G, Z = 1$ . We take  $L_0 = \ell_c = 1cm$  as the characteristic length, where  $\ell_c = c/\omega_p$  is the skin length.

Then we obtain the following characteristic values of the other parameters:  $V_A = \frac{H_0}{\sqrt{4\pi\rho}} = 2.8 \times 10^5 cm/s$  (velocity),  $t_0 =$

$$\frac{L_0}{V_A} = \frac{c\sqrt{\lambda_+ \lambda_-}}{H_0} = 3.45 \times 10^5 s, T_0 = \frac{V_A^2 \lambda_e e}{2k} = 10^3 K,$$

$$r_D = (KT_0)^{\frac{1}{2}} / (2\pi^{1/2} \rho_0 n_0^{1/2}) \approx 2 \times 10^{-4} cm \text{ (Debye radius)}, n_0 = \frac{\rho_0}{m_+}.$$

In particular, we have  $r_D \ll L_0$ , which is much smaller than the grid step  $h = 10^{-2} \div 10^{-3} cm$  of the difference scheme used in the stabilization method. Moreover, it is assumed that the incident Alfvén wave has fixed amplitude  $|U_0| = 0.1$  and

$$H_x = -1. \text{ The wave frequency } \omega \text{ varies on the interval } -16 \cong -\left(\frac{\lambda_+}{\lambda_-}\right)^{\frac{1}{2}} < \omega < 0.$$

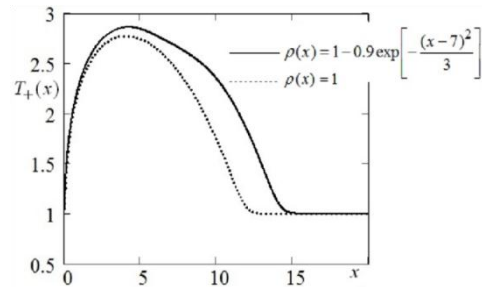
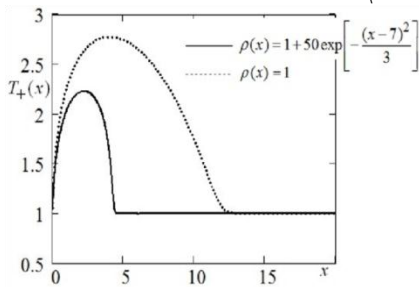
In the basic case,  $\rho(x) \cong 1, \omega = -30$ , it follows from [5,6] that the heating depth is  $d \cong 12.6$  and the maximal temperatures of electrons and ions have the values  $T_- = 3.35, T_+ = 2.77$

Suppose now that in the domain  $x \geq 0$  the dissipative plasma density has a hump:

$$\rho(x) = 1 + A \exp[-(x - x_0)^2/D], x_0 = 7, D = 3, A = 50.$$

Figure 1 shows that, in contrast to the basic case, the penetration depth of the Alfvén wave has decreased about 3 times ( $d = 4.5$ ), and the maximal temperatures of ions and electrons  $T_+ = 2.23, T_- = 2.9$  have become smaller by about 15%. Suppose now that the density function has a hollow:  $\rho(x) = 1 + A \exp[-(x - x_0)^2/D], x_0 = 7, D = 3, A = -0$ .

Figure 2 shows that now, in contrast to the basic case, the penetration depth has increased by about 20% ( $d \cong 15.1$ ), and the maximal temperatures  $T_+ = 3.55$ ,  $T_- = 2.74$  have also become larger. These results can be explained by the role of the bremsstrahlung of electrons whose density is equal to  $\rho_T \sim \rho^2 \sqrt{T_-}$ . In the case of the hump-shaped inhomogeneity, the bremsstrahlung substantially increases on the interval  $[x_0 - \sqrt{2D}, x_0 + \sqrt{2D}] = [4.559, 9.45]$  and, according to the law of conservation of energy (see [5,6]), the thermal energy of electrons and ions in the domain  $x \geq 0$  occupied by dissipative plasma causes a decrease of the penetration depth and the maximal values of the temperatures  $T_{\pm}$ . The same behavior can still be expected (figure 1 and figure 2 confirm this), if bremsstrahlung grows (drops) only in some part of the half-space  $x \geq 0$  occupied by dissipative plasma. The above considerations can also explain why in the case of the hollow-shaped density, the heating of electrons increases both with respect to the depth and the maximal temperatures. The above analysis suggests that with a decrease of the hump height  $A$ , the penetration depth of the Alfvén wave, as well as the maximal temperatures of electrons and ions increase. This is confirmed by the results obtained for inhomogeneous densities of the form  $\rho(x) = 1 + A \exp[-(x - 8)^2/D]$  with various  $A$  (see table 1).



**Figure 1.** The influence of a hump-shaped density on the penetration depth of an Alfvén wave. The temperature of ions for  $\omega = -30$ . **Figure 2.** The influence of a hollow-shaped density on the penetration depth of an Alfvén wave. The temperature of ions for  $\omega = -30$ .

**Table 1.** The penetration depth  $d$  and the maximal temperatures versus the hump height  $A$ ;  $\rho(x) = 1 + A \exp[-(x - 8)^2/D]$

$A$	100	50	25	10	5	2.5	2	1.5	1	0.5	0.1	0
$d$	6.4	6.5	6.7	7.2	7.5	7.9	8.1	8.5	9.4	11.2	12.3	12.5
$T_+$	2.5	2.52	2.54	2.57	2.59	2.62	2.6	2.6	2.66	2.71	2.76	2.77
$T_-$	3.23	3.24	3.25	3.26	3.26	3.27	3.26	3.28	3.29	3.31	3.34	3.35

We omit the graphs of  $T_{\pm}(x)$  for the cases represented in table 1, since they are all of the same type, like those in figure 1 and figure 2, and their qualitative behavior is determined by the values of  $d$ ,  $T_{\pm}$  (note that the heating depths for electrons and ions,  $d_e$  and  $d_i$ , slightly differ, but  $d_i \geq d_e$ ,  $d_i \cong d_e$ ; in table 1,  $d = \max(d_i, d_e) = d_i$ ). As a compensation, we offer the graphs of the temperatures for  $A = 50, 25, 2, 0$  in figure 3.

The above considerations allow us to make some predictions. For instance, if  $D$  is decreased, then the penetration depth of an Alfvén wave, as well as the maximal temperatures of ions and electrons, should increase, because of the contraction of the domain,  $[x_0 - \sqrt{2D}, x_0 + \sqrt{2D}]$  in which the bremsstrahlung grows, and therefore, the losses in bremsstrahlung become smaller. This conjecture is supported by calculations. Thus, for  $A = 2, x_0 = 8, D = 0.5$ , we obtain  $d = 8.4, T_+ = 2.65, T_- = 3.28$ , and the above prediction based on table 1 is true.

Now, consider the influence of the frequency  $\omega$  of an Alfvén wave on its absorption. Figure 4 shows the profiles of  $T_{\pm}(x)$  in the case of hump-shaped density,  $\rho(x) = 1 + A \exp[-(x - 8)^2/D]$ ,  $x_0 = 8, A = 50, D = 1$ , and  $\omega = -50, \omega = -35, \omega = -9$ . Supplementing these results with more detailed data from table 2, we conclude that the maximal penetration depth of an Alfvén wave and the maximal temperatures of electrons and ions are observed at midrange frequencies near the value  $-0.5(\lambda_+/\lambda_-)^{1/2}$ , the midpoint of the interval  $(-\lambda_+/\lambda_-)^{1/2}, 0$  of admissible frequencies. For high frequencies,  $\omega = -(\lambda_+/\lambda_-)^{1/2}$ , or low frequencies,  $\omega = 0$ , the Alfvén wave practically does not heat dissipative plasma, which is penetrated only by the decaying electromagnetic field. These results are explained by the sharp drop of the electric field at the left boundary point  $x = 0$ .

According to the boundary conditions (16), for  $\omega = -5$  (low frequency), the absolute value of  $E|_{x=0}$  is more than 3 times smaller than that off  $E|_{x=0}$  for  $\omega = -30$  (middle frequencies) and almost 3 times smaller than that for  $\omega = -55$  (high frequencies). This, together with equations (14), leads to an approximately threefold drop of the electric (and therefore, magnetic) field and the current density inside the domain occupied by dissipative plasma. A special role is played by the drop of the current density inside the domain  $x \geq 0$ , which, for low frequencies,  $\omega \cong -5$ , is strengthened by a six-fold current density drop at the boundary  $x = 0$ . For high frequencies,  $\omega = -55$ , the current density drop inside the domain  $x \geq 0$  is partially compensated by a two-fold increase of the current density at the boundary  $x = 0$ . In any case, the almost three-fold drop of the current density inside the domain  $x \geq 0$  leads to an approximately 10-fold decrease of the Joule heat of plasma, which explains the sharp (almost by an order of magnitude) drop of the temperatures of electrons and ions, as well as practically no heating of dissipative plasma by the Alfvén wave. Note that in the extreme case  $\omega = 0$ , the current density on the boundary vanishes,  $|H| \sim 0.1$ , and the absolute value of  $\square|_{\square=0}$  drops almost 1000 times in comparison with  $\square|_{\square=0}$  for  $\square = -30$ . In the

other extreme case,  $\eta = -(\eta_+/\eta_-)^{1/2}$  we have  $H = 0$ ,  $E = 0$  on the boundary, and absolute value of  $\eta|_{\eta=0}$  is about 2 times greater than that of  $\eta|_{\eta=0}$  for  $\eta = -30$ .

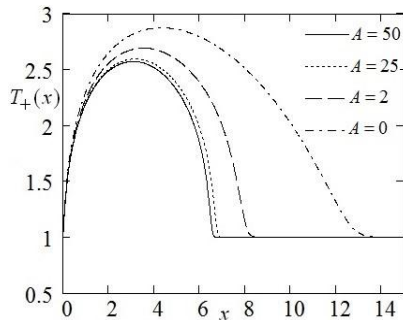


Figure 3: The dependence of absorption of an

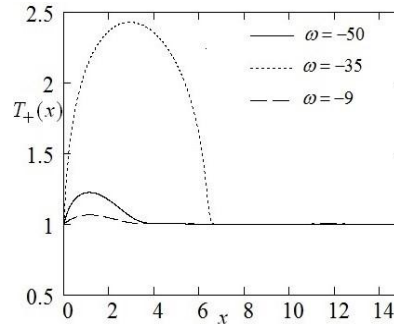


Figure 4: The dependence of absorption of an Alfvén

wave on the height  $A$  of the density hump of Wave on its frequency  $\omega$ . Ion temperature distribution

temperature distribution for  $\omega = -25$ , for  $\rho(x) = 1 + 50\exp[-(x - 8)^2]$

$$\rho(x) = 1 + A\exp[-(x - 8)^2]$$

Table 2. The dependence of the Alfvén wave penetration depth  $d$  and maximal temperatures of electrons and ions on the frequency  $\omega$ :  $\rho(x) = 1 + 50\exp[-(x - 8)^2]$

$-\omega$	60	55	50	45	35	30	25	20	10	9	8	1
$d$	0.96	1.3	3.7	6.3	6.5	6.6	6.6	6.6	6.3	2.9	0	0
$T_+$	1.04	1.05	1.22	1.94	2.43	2.52	2.57	2.57	1.91	1.06	1	1
$T_-$	1.2	1.22	1.27	2.15	3.01	3.24	3.34	3.31	2.07	1.07	1	1

6. Conclusion.

Our main statement is that the inhomogeneity of plasma density and the incident wave frequency have a substantial effect on the penetration depth of an Alfvén wave, as well as the heating of electrons and ions of dissipative plasma. More precisely, hump-shaped density distribution hinders the passage of an Alfvén wave, decreasing its penetration depth and the temperatures of electrons and ions of dissipative plasma, while a hollow-shaped density distribution increases the penetration depth and the temperatures of electrons and ions. On the other hand, Alfvén waves of high ( $\omega \sim \omega_c$ ) and low ( $\omega \sim 0$ ) frequencies practically have no heating effect on plasma, and the absorption of the wave reduces to the decay of the electromagnetic field. The greatest heating effect on plasma, both in relation to the penetration depth and maximal temperatures of electrons and ions, is observed for midrange frequencies ( $\omega \sim \omega_c/2$ ).

Note that in the nonlinear heat equation for  $T_+$ , at the initial time instant, when  $T_+ \sim 1$ , the terms corresponding to bremsstrahlung and temperature relaxation have the same order which is much smaller ( $\sim 10^3$  times) than the Joule heat. With the growth of  $T_+$ , the value of the Joule heat,  $\sim T_+^{-3/2}$ , as well as the relaxation term  $\sim T_+^{-1/2}$ , experience a fast decrease, while the magnitude of bremsstrahlung  $\sim T_+^{1/2}$  is increasing. As a result, in a short time these three terms acquire the same order of magnitude and none of these can be neglected! It should be added that the strong nonlinearity of the thermal conductivity coefficient  $\eta \sim T_+^{5/2}$  substantially reduces the role of the Joule heat. Note also that the above results have been obtained under the assumption that  $\eta = 0$ , since the theoretical value of electron viscosity given in section 2 appears to be greater than the experimental one at least by 2 to 3 orders of magnitude. Absorption of Alfvén waves for real values of electron viscosity  $\eta$  requires further investigation.

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