

Combined effects of perturbations, radiation and oblateness on the location of equilibrium points in the restricted three-body problem.

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Abstract

We have studied the effect of small perturbations in the coriolis and the centrifugal forces together with oblateness and radiation pressure forces of the primaries on the locations of equilibrium points in the restricted three-body problem. We have found that oblate-ness and radiation pressure forces affect the locations of equilibrium points. We have further seen that the positions of equilibrium points are not affected by the change in the coriolis force. They are only affected by the change in the centrifugal force. It is also observed that the triangular points form triangles with the primaries and lie on the line joining the primaries.

Key words: *equilibrium points, oblate-ness, perturbations, radiation, and restricted three- body problem.*

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1.0 Introduction

The classical restricted three- body problem consists of five equilibrium points, three of which are collinear and the remaining two are equilateral triangular points. The collinear points are denoted by L_1 , L_2 and L_3 and the triangular points by L_4 and L_5 .

Redizievskii [11] formulated the photogravitational restricted three-body problem from the classical problem when one of the interacting masses is an intense emitter of radiation. Bhartnagar and Hallan [9] studied the effect to small perturbations in the coriolis and the centrifugal forces on the location and stability of equilibrium points in the restricted problem. Sharma [8] examined the linear stability of triangular libation points of the restricted problem when the more massive primary is a source of radiation and oblate spheroid as well. Khasan [6] showed the existence of libration points (equilibrium points) and their stability in the photo- gravitational elliptic restricted three–body problem. Kunitsyn [3,4] studied of triangular and collinear points respectively in the photo-gravitational three-body problem. Dankowicz [1] gave an account for gravitational interactions with the asteroids and the sun and the radiation pressure from the sun.

The idea of the radiation pressure forces and oblate-ness of the raises a curiosity in our mind to study the combined effects of perturbations, radiation and oblate-ness of the primaries on the location of equilibrium points in the restricted three-body problem.

2.0 Equations of motions

Let m_1 and m_2 be the masses of the bigger and smaller primaries, m is the mass of the third infinitesimal body. We assume that both primaries are oblate spheroid and radiating as well. Let A_1 and A_2 denote the oblate-ness coefficients of the bigger and smaller primaries respectively such that $0 < A_i < 1$, $i = 1, 2$. We denote the radiation factors by q_1 and q_2 for the bigger and smaller primaries respectively such that $q_i = 1 - \delta_i$, $i = 1, 2$

Let (x, y) be the coordinates of the infinitesimal mass m , in a rotating coordinates system with the origin at 0. The line joining the primaries is taken as the x- axis and the line perpendicular to it being the y- axis. Let the origin be the barycentre of mass m_1 at $(x_1, 0)$ and mass m_2 at $(x_2, 0)$. Then in the dimensionless synodic coordinates system; the equations of motion of the infinitesimal mass under the influence of oblateness and radiation repulsive forces of the primaries are in [5],

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$$\ddot{x} - 2n\dot{y} = U_x \quad \ddot{y} + 2n\dot{x} = U_y \quad (2.1)$$

with

$$U = \frac{n^2}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} q_1 + \frac{\mu}{r_2} q_2 + \frac{1-\mu}{2r_1^3} A_1 q_1 + \frac{\mu}{2r_2^3} A_2 q_2 \quad (2.2)$$

$$r_1^2 = (x - \mu)^2 + y^2, \quad r_2^2 = (x + 1 - \mu)^2 + y^2 \quad (2.3)$$

and the mean motion n is given by

$$n^2 = 1 + \frac{3}{2}(A_1 + A_2) \quad (2.4)$$

We consider the perturbations in the coriolis and the centrifugal forces using the parameters ϕ and ψ respectively. The unperturbed value of each is unity. Then the equations of motion become

$$\ddot{x} - 2n\dot{y} - n^2\psi x = F_x, \quad \ddot{y} + 2n\dot{x} - n^2\psi y = F_y \quad (2.5)$$

where $F = \frac{1-\mu}{r_1} q_1 + \frac{\mu}{r_2} q_2 + \frac{(1-\mu)}{2r_1^3} A_1 q_1 + \frac{\mu}{2r_2^3} A_2 q_2$. We take

$$\begin{aligned} \phi &= 1 + \varepsilon, \quad \|\varepsilon\| \ll \pi \\ \psi &= 1 + \varepsilon', \quad \|\varepsilon'\| \ll \pi \end{aligned} \quad (2.6)$$

where ε and ε' are the small perturbations in the coriolis and the centrifugal forces respectively. Equations (2.5) can be put in the form

$$\ddot{x} - 2n\dot{y} = \Omega_x \quad \ddot{y} + 2n\dot{x} = \Omega_y \quad (2.7)$$

where

$$\Omega = \frac{1}{2} n^2 \psi (x^2 + y^2) + \frac{1-\mu}{r_1} q_1 + \frac{\mu}{r_2} q_2 + \frac{1-\mu}{2r_1^3} A_1 q_1 + \frac{\mu}{2r_2^3} A_2 q_2 \quad (2.8)$$

3.0 Location of equilibrium points

The equilibrium positions exist at the points where $\Omega_x = 0, \Omega_y = 0$. That is

$$\left. \begin{aligned} x \left[n^2 \psi - \frac{1-\mu}{r_1^3} q_1 - \frac{\mu}{r_2^3} q_2 - \frac{3}{2} \frac{(1-\mu)}{r_1^5} A_1 q_1 - \frac{3}{2} \frac{\mu}{r_2^5} A_2 q_2 \right] \\ + \frac{\mu(1-\mu)}{r_1^3} q_1 - \frac{\mu(1-\mu)}{r_2^3} q_2 + \frac{3}{2} \frac{(1-\mu)}{r_1^5} A_1 q_1 - \frac{3}{2} \frac{\mu(1-\mu)}{r_2^5} A_2 q_2 = 0 \\ \text{and} \\ y \left[n^2 \psi - \frac{1-\mu}{r_1^3} q_1 - \frac{\mu}{r_2^3} q_2 - \frac{3}{2} \frac{(1-\mu)}{r_1^5} A_1 q_1 - \frac{3}{2} \frac{\mu}{r_2^5} A_2 q_2 \right] = 0 \end{aligned} \right\} \quad (3.1)$$

3.1 Locations of triangular points

To locate the triangular points we consider the second equation of (3.1) for $y \neq 0$ so that

$$n^2 \psi - \frac{1-\mu}{r_1^3} q_1 - \frac{\mu}{r_2^3} q_2 - \frac{3}{2} \frac{(1-\mu)}{r_1^5} A_1 q_1 - \frac{3}{2} \frac{\mu}{r_2^5} A_2 q_2 = 0 \quad (3.2)$$

Using (3.2) in the first equation of (3.1), we obtain

$$\frac{q_1}{r_1^3} - \frac{q_2}{r_2^3} + \frac{3}{2} \frac{A_1 q_1}{r_1^5} + \frac{3}{2} \frac{A_2 q_2}{r_2^5} = 0 \quad (3.3)$$

After re-writing equation (3.2) and making use of equation (3.3), we have

$$n^2 \psi - \frac{q_1}{r_1^3} - \frac{3}{2} \frac{A_1 q_1}{r_1^5} = 0 \quad (3.4)$$

Combing (3.3) and (3.4) we get

$$n^2 \psi - \frac{q_2}{r_2^3} - \frac{3}{2} \frac{A_2 q_2}{r_2^5} = 0 \quad (3.5)$$

We can obtain r_1 and r_2 from equations (3.4) and (3.5), and coordinates of triangular points by solving equation (2.3) for x and y . The exact coordinates of L_4 and L_5 are given by

$$x = \mu - \frac{1}{2} + \frac{r_2^2 - r_1^2}{2}, \quad y = \pm \left[\frac{r_1^2 + r_2^2}{2} - \frac{1}{4} - \frac{(r_2^2 - r_1^2)^2}{2} \right]^{\frac{1}{2}} \quad (3.6)$$

When the primaries are neither radiating nor oblate spheroid i.e. $A_i = 0, q_i = 1$ ($i = 1, 2$), the solutions of (3.4) and (3.5) are $r_i = \psi^{\frac{1}{3}}$. Therefore, we may assume that the solutions of (3.4) and (3.5) are

$$r_i = \frac{1}{\psi^{\frac{1}{3}}} + \alpha_i \quad (3.7)$$

$$|\alpha_i| \ll 1, \quad (i = 1, 2) \quad (3.8)$$

are very small. If we restrict ourselves to only linear terms in $\alpha_i, A_i, 1 - q_i$ and coupling terms in $A_1 q_1, A_2 q_2, \psi \alpha_i, A_i \psi$ we obtain the expressions for α_1 and α_2 respectively as

$$\alpha_1 = \frac{1}{3\psi^{\frac{4}{3}}} \left[-\frac{3}{2} \psi (A_1 + A_2) - \psi (1 - q_1) + \frac{3}{2} A_1 q_1 \psi^{\frac{5}{3}} \right] \text{ and}$$

$$\alpha_2 = \frac{1}{3\psi^{\frac{4}{3}}} \left[-\frac{3}{2} \psi (A_1 + A_2) - \psi (1 - q_2) + \frac{3}{2} A_2 q_2 \psi^{\frac{5}{3}} \right],$$

putting these values of α_1 and α_2 in (3.7), we get

$$r_1 = \frac{1}{\psi^{\frac{1}{3}}} \left[1 - \frac{A_1 + A_2}{2} - \frac{1 - q_1}{3} + \frac{A_1 q_1}{2} \psi^{\frac{2}{3}} \right] \text{ and } r_2 = \frac{1}{\psi^{\frac{1}{3}}} \left[1 - \frac{A_1 + A_2}{2} - \frac{1 - q_2}{3} + \frac{A_2 q_2}{2} \psi^{\frac{2}{3}} \right] \quad (3.9)$$

Substituting these values of r_1 and r_2 into equations (3.6) we obtain

$$x = \mu - \frac{1}{2} + \frac{1}{3} (1 - q_1) - \frac{1}{3} (1 - q_2) + \frac{1}{2} (A_2 q_2 - A_1 q_1) \psi^{\frac{2}{3}} \quad (3.10)$$

and

$$y = \pm \frac{\sqrt{4 - \psi^{\frac{2}{3}}}}{2\psi^{\frac{1}{3}}} \left[1 - \frac{2}{4 - \psi^{\frac{2}{3}}} \left\{ A_1 + A_2 + \frac{1}{3} (1 - q_1) + \frac{1}{3} (1 - q_2) - \frac{1}{2} (A_1 q_1 + A_2 q_2) \psi^{\frac{2}{3}} \right\} \right]$$

These points are denoted by L_4 and L_5 , and are known as triangular liberation points.

3.2 Locations of collinear points

The collinear positions are the solutions of equations (3.1) when $y = 0$. That is

$$n^2 \psi x + \frac{(1 - \mu)(\mu - x)}{r_1^3} q_1 + \mu \frac{(\mu - x - 1)}{r_2^3} q_2 + \frac{3(1 - \mu)(\mu - x)}{2r_1^5} A_1 q_1 + \frac{3\mu(\mu - x - 1)}{2r_1^5} A_2 q_2 = 0, \quad y = 0 \quad (3.11)$$

The collinear points are the solutions of equation (3.11), and their abscissas are the roots of the equation

$$f(x) = n^2 \psi x - \frac{(1 - \mu)(x - \mu)}{|x - \mu|^3} q_1 - \frac{\mu(x + 1 - \mu)}{|x + 1 - \mu|^3} q_2 - \frac{3(1 - \mu)(x - \mu)}{2|x - \mu|^5} A_1 q_1 - \frac{3\mu(x + 1 - \mu)}{2|x + 1 - \mu|^5} A_2 q_2 = 0 \quad (3.12)$$

Now, since $\frac{df(x)}{dx} > 0$ in each of the open intervals $((-\infty, \mu - 1), (\mu - 1, \mu)$ and (μ, ∞) ; it follows that the function is strictly increasing in each of the interval.

Also $f(x)$ approaches $-\infty$ as x approaches $-\infty$ or $(\mu-1)+0$ or $\mu+0$ and $f(x)$ approaches ∞ as x approaches ∞ or $(\mu-1)-0$ or $\mu-0$ or ∞ . Therefore there exist one and only one value of x in each of the above intervals such that $f(x) = 0$. Further, we see that $f(\mu-2) < 0$, $f(0) > 0$ and $f(\mu+1) > 0$. Hence, there are only three real roots of equation (3.12) with one lying in each of the open intervals $(\mu-2, \mu-1)$, $(\mu-1, 0)$ and $(\mu, \mu+1)$. This shows the locations of the three collinear points L_1 , L_2 and L_3 . The first is located to the left of the second primary, the second is between the two primaries and the third collinear liberation point is to the right of the first primary. To find the position of L_1 we put $r_1 = \mu - x$, $r_2 = \mu - x - 1$, in equation (3.11) we have

$$n^2 \psi x + \frac{1-\mu}{(\mu-x)^2} q_1 + \frac{\mu}{(\mu-x-1)^2} q_2 + \frac{3(1-\mu)}{2(\mu-x)^4} A_1 q_1 + \frac{3}{2} \frac{\mu}{(\mu-x-1)^4} A_2 q_2 = 0 \quad (3.13)$$

On putting $r_2 = \mu - x - 1 = \xi$, equation (3.13) becomes

$$\left[1 + \frac{3}{2}(A_1 + A_2) \right] \psi(\mu - 1 - \xi) + \frac{1-\mu}{(\xi+1)^2} q_1 + \frac{\mu}{\xi^2} + \frac{3(1-\mu)}{2(\xi+1)^4} A_1 q_1 + \frac{3}{2} \frac{\mu}{\xi^4} A_2 q_2 = 0$$

substituting a for $\left[1 + \frac{3}{2}(A_1 + A_2) \right] \psi$ it yields

$$\begin{aligned} & a\xi^9 - a(\mu-5)\xi^8 - a(4\mu-10)\xi^7 - [a(6\mu-10) + (1-\mu)q_1 + \mu q_2] \xi^6 \\ & - [a(4\mu-5) + 2(1-\mu)q_1 + 4\mu q_2] \xi^5 - \left[a(\mu-1) + (1-\mu)q_1 + 6\mu q_2 + \frac{3}{2}(1-\mu)A_1 q_1 + \frac{3}{2}\mu A_2 q_2 \right] \xi^4 \\ & - [4\mu q_2 + 6\mu A_2 q_2] \xi^3 - [\mu q_2 + 9\mu A_2 q_2] \xi^2 - 6\mu A_2 q_2 \xi - \frac{3}{2}\mu A_2 q_2 = 0 \end{aligned} \quad (3.14)$$

This is a ninth-degree algebra equation in ξ with parameter μ . Descartes' sign rule indicates that equation (3.13) has at least one positive root. Solving equation for ξ (using small parameter method) we find that there is one real root $\xi = 0$ for $\mu = 0$. Equation (3.13) may be written as

$$(1-\mu) \left[-a + q_1(\xi+1)^{-2} + \frac{3}{2} A_1 q_1 (\xi+1)^{-4} - a\xi \right] = \mu \left[-q_2 \xi^{-2} - \frac{3}{2} A_2 q_2 \xi^{-4} + a\xi \right]$$

and simplified to

$$\frac{\mu}{1-\mu} = \frac{\xi^4 \left[a(\xi+1)^5 - q_1(\xi+1)^2 - \frac{3}{2} A_1 q_1 \right]}{(\xi+1)^4 \left[q_2 \xi^2 - a\xi^5 + \frac{3}{2} A_2 q_2 \right]} \quad (3.15)$$

In order to obtain a series solution for ξ in powers of the quantity $v = \left[\frac{\mu}{1-\mu} \right]^{\frac{1}{4}}$, we assume

$$\xi = c_1 v + c_2 v^2 + c_3 v^3 + \dots + c_9 v^9 + \dots \quad (3.16)$$

and which when substituted in (3.14) the first four coefficients were obtained to be

$$c_1 = \left[\frac{3A_2q_2}{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)} \right]^{-14}, \quad c_2 = \frac{-(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{1}{2}}}{\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{3}{2}}}$$

$$c_3 = \frac{-3(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{1}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{7}{4}}} + \frac{q_2 + 9A_2q_2}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{3}{4}}(3A_2q_2)^{\frac{1}{4}}} + \frac{3(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{1}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{4}}} s$$

$$+ \frac{(25a - 10q_1)(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{3}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{11}{4}}} - \frac{(10a - q_1)(3A_2q_2)^{\frac{3}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{4}}}$$

$$c_4 = \frac{2q_2 + 3A_2q_2}{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)} - \frac{3(q_2 + 9A_2q_2)(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{1}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{9}{4}}} + \frac{(q_2 + 9A_2q_2)^2}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{4}}(3A_2q_2)^{\frac{3}{4}}}$$

$$+ \frac{3(q_2 + 9A_2q_2)(a + 2q_1 + 6A_1q_1)}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{7}{4}}(3A_2q_2)^{\frac{1}{4}}} - \frac{(25a - 10q_1)(q_2 + 9A_2q_2)(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{1}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{13}{4}}}$$

$$\frac{(10a - q_1)(q_2 + 9A_2q_2)(3A_2q_2)^{\frac{1}{4}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{7}{4}}} - \frac{3(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{1}{5}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{2}}} + \frac{(q_2 + 9A_2q_2)(3A_2q_2)^{\frac{1}{2}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{3}{2}}}$$

$$- \frac{3(a + 2q_1 + 6A_1q_1)(3A_2q_2)}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^2} + \frac{(25a - 10q_1)(a + 2q_1 + 6A_1q_1)(3A_2q_2)^{\frac{3}{2}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^{\frac{7}{2}}} - \frac{(10a - q_1)(3A_2q_2)^{\frac{3}{2}}}{2\left\{2\left(a - q_1 - \frac{3}{2}A_2q_2\right)\right\}^2} +$$

$$\frac{(a + 2q_1 + 6A_1q_1)^3(3A_2q_2)}{\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^4} - \frac{9(a + 2q_1 + 6A_1q_1)^2(3A_2q_2)}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^3} + \frac{3(q_2 + 9A_2q_2)(a + 2q_1 + 6A_1q_1)}{2\left\{2\left(a - q_1 - \frac{3}{2}A_1q_1\right)\right\}^2} +$$

$$\begin{aligned}
& \frac{9(a+2q_1+A_1q_1)^2(3A_2q_2)^{\frac{1}{2}}}{2\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{2}}} - \frac{(25a-10q_1)(a+2q_1+6A_1q_1)(3A_2q_2)}{2\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^4} - \frac{3(10a-10q_1)(a+2q_1+6A_1q_1)(3A_2q_2)}{2\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{2}}} \\
& \frac{(25a-10q_1)(a+2q_1+6A_1q_1)^3(3A_2q_2)^{\frac{3}{2}}}{\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^{\frac{11}{2}}} + \frac{(25a-10q_1)(a+2q_1+6A_1q_1)(3A_2q_2)}{4\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^3} - \frac{(25a-10q_1)(q_1+9A_2q_2)}{4\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^2} \\
& \frac{3(25a-10q_1)(a+2q_1+6A_1q_1)(3A_2q_2)^{\frac{1}{2}}}{\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{2}}} - \frac{(25a-10q_1)^2(a+2q_1+6A_1q_1)(3A_2q_2)}{4\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^4} + \frac{(10a-q_1)(25a-10q_1)(3A_2q_2)}{4\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^{\frac{5}{4}}} + \\
& \frac{3(10a-q_1)(a+2q_1+6A_1q_1)(3A_2q_2)}{4\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^3} - \frac{5a(3A_2q_2)}{\left\{2\left(a-q_1-\frac{3}{2}A_1q_1\right)\right\}^2}
\end{aligned}$$

The values of c_1, c_2, c_3, c_4 obtained are in terms of A_1, A_2, q_1, q_2 , and ψ , and when substituted into equation (3.16), and using the fact that

$$\frac{\mu}{1-\mu} = \mu \left(\frac{1}{1-\mu} \right) = \mu (1 + \mu + \mu^2 + \dots) \approx \mu,$$

The abscissa of the first collinear point (L_1) is given by

$$x_1 = \mu - 1 - \xi \quad (3.17)$$

Similarly, we can find the positions of L_2 and L_3 .

4.0 Discussions

- i. If the primaries are neither oblate nor radiating, i.e. $A_1 = 0, A_2 = 0, q_1=1, q_2 = 1$, then equations (3.9) and (3.10) correspond to the results obtained by Bhartnagar and Hallan [9].
- ii. When $A_2 = 0, q_2 = 1, \psi = 1, \psi = 1$, equation (3.9) and (3.10) is the same as that of Sharma [8].
- iii. If there are no perturbations in coriolis and centrifugal forces, i.e. $\psi = 1, \phi = 1$, equations (3.9) and (3.10) tally with the results of Singh and Ishwar [5].
- iv. In equations (3.9) and (3.10) $r_1 \neq r_2$ implies that the triangular points (L_4 and L_5) form simple triangles with the primaries different from [2], [3] and [4] contrary to the classical case in which they form equilateral triangles.
- v. Appearance of A_1, A_2, q_1, q_2 and ψ in equation (3.9) and (3.16) due to oblateness, radiation the centrifugal force indicate that these factors affect the locations of triangular and collinear points. Equations (3.1) are independent of, ϕ , also indicate that the coriolis force does not affect their positions.
- vi. Equation (2.4) shows that the mean motion is affected by oblateness only.

5.0 Conclusion

We have shown the existence of five equilibrium points, $L_i, i=1, \dots, 5$. The point L_4 and L_5 form simple triangles with the primaries contrary to the classical or other problems. The points L_1, L_2 and L_3 remain collinear and lie on the line joining the primaries. We have seen that oblateness of radiation pressure force affect the location of the equilibrium points but are not affected by the change in the coriolis force. They are only affected by the change in the centrifugal force. Only oblateness affects the mean motion. Hence the location of equilibrium points different as obtained by others.

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