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Power series like relation of power law and coupled creep constrained grain boundary cavitation under strain gradient plasticity analysis.

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Abstract

The continuum damage theory of Kachanov and Rabotnov has limitations since the mechanical properties of a material (especially plastic deformation and fracture) are determined by its microstructure. When a solid deforms at high temperature its microstructure may in some sense be altered- holes and cracks may nucleate and grow inside the solid by various mechanism controlled by diffusion and by power law creep or by a combination of these mechanisms. Considering a coupled diffusion power law creep mechanism using a mechanistic model approximate analytical equations for the growth rate under multi-axial stress states are developed. These results are related to the power law mechanistic results in a power series like form, which are used to analyze the crack, tip fields for the coupled mechanism using a strain gradient plasticity analysis. The Kachanov-Rabotnov results and the HRR results are shown to be special cases of these results.

Key words: diffusion, power law, creep, microstructure, strain gradient plasticity,

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1.0 Introduction

When a solid deforms at high temperature or when load bearing metallic components operate at temperatures in excess of approximately one third of the melting temperature consideration must be given to the effect of creep deformation and rupture.

Rupture can be the consequence of thinning induced by large strains but it can also occur at small strains as the result of growth of damage in the metal. Holes or cracks may nucleate and grow inside the solid; its grain size may increase or decrease; precipitate or depressoids within it may coarsen or dissolve; a substructure introduced by prior working may be destroyed or the microstructure may in some sense be altered.

Also under the action of loads which are often considerably less than the fracture stress, and especially at elevated temperature many solids particularly ceramics gradually deform plastically, or creep until instabilities are created which lead to sudden, and often catastrophic, failure. Clearly the design of components for engineering application must incorporate, and thus allow for this creep behaviour so that excessive plastic strain or premature fracture is avoided during the anticipated lifetime of the structure.

Creep fracture is a complicated phenomenon [1,9]. Under creep conditions fracture most commonly occur by the growth and coalescence of voids, which lie on grain boundaries. Voids usually grow by diffusion when they are small but as they become larger power law creep takes over as the dominant creep mechanism. Thus for a general treatment coupled mechanism like diffusion coupled with power law should be considered.

Cocks and Ashby [4] and Oyesanya [24] have discussed the interesting creep fracture phenomenon under power law mechanism and coupled diffusion power law mechanism. As a micro-mechanical concept the result were compared with Rabotnov-Kachanov equations of continuum damage theory [15] and the inadequacy of continuum theory was highlighted.

A relationship between power law mechanism and coupled mechanism involving power law will be very useful as we can now use results for the power law to predict the results for coupled mechanism. In this paper multi-axial constitutive relation is proposed for steady state creep of polycrystalline materials undergoing grain boundary conditions under coupled mechanism of surface diffusion and power law. We assume that the overall stresses are in a range such that the void growth is creep constrained.

We show that there exists a power series like relation between results for power law and coupled mechanism. We apply these results to study the influence of cavitations in the stress and the strain rate fields at the tip of a macroscopic crack. We use the strain gradient plasticity analysis since we recognize that continuum

damage theory models lose their validity after a macro-scale crack arises.

2.0 Models for the void growth and assumptions

The results of Cocks and Ashby used in this paper are based on a mechanistic model shown in Figure 1. We isolate a cylindrical element of material of diameter l (the void spacing) and grain size d centered on a grain boundary void of diameter $2r_h$. The surfaces of this are subjected to the stress fields $\sigma_1, \sigma_2, \sigma_3$ as shown in Figure 2a, b, c. The simple mechanism of surface diffusion and power law are shown in Figure 2.

Under surface diffusion mechanism the void becomes flatter and more crack-like and the void can be idealized as a penny shaped crack of constant width and semi-circular tip. Under this analysis we assume that

(i) When no cavities are present the average steady state creep rate for the power law is

$$\dot{\varepsilon}_{ss} = A(\xi)\,\sigma^n \exp\left(-\frac{Q}{RT}\right) \tag{2.1}$$

(ii) The steady state creep rate for the surface diffusion is

$$\dot{\varepsilon}_{ss} = \frac{\alpha\,\sigma_e \Omega D_0}{L^2 kT} \exp\left[-\left(\frac{Q + P\Delta V}{kT}\right)\right] \tag{2.2}$$

$$\begin{aligned} \sigma_e &= \left[\tfrac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]\right]^{1/2} \\ \dot{\varepsilon}_{ss} &= \left[\tfrac{2}{9}\left[(\varepsilon_1-\varepsilon_2)^2 + (\varepsilon_2-\varepsilon_3)^2 + (\varepsilon_3-\varepsilon_1)^2\right]\right]^{1/2} \end{aligned} \tag{2.3}$$

and Ω is the atomic volume, D_0 the surface diffusion coefficient, k the Boltzman constant, R the universal gas constant, T absolute temperature, Q the activation energy, V the activation volume. Grain boundaries slide, so that the increase in volume of the slab containing the voids is taken up by a relative rigid body displacement of the grains on either side.

(iii) Surface diffusion controls the early part of void growth while power law creep takes over as the voids become larger and the net section stress rises so that the coupled rate is simply the sum of the rates of two individual mechanisms.

3.0 Constitutive law analysis

As shown in [3.4] for only surface diffusion mechanism the rate of change of area fraction f_h of holes on grain boundary is given by

$$\frac{1}{\dot{\varepsilon}_0}\frac{d\,f_h}{d\,t} = \frac{\Psi_0 f_h^{1/2}}{(1-f_h)^3}\left(\frac{\sigma_1}{\sigma_0}\right)^3 \tag{3.1}$$

The strain rate is given for this mechanism by

$$\frac{1}{\dot{\varepsilon}_0}\frac{d\varepsilon}{dt} = \frac{4\Psi_0 f_h^{1/2}\Phi_s}{(1-f_h)^3 d\sigma_0}\left(\frac{\sigma_1}{\sigma_0}\right)^2 \tag{3.2}$$

where
$$\Psi_0 = \frac{1}{\sqrt{2}}\frac{D_s\delta_s\Omega\sigma_0^3}{kTl\Phi_s^2\dot{\varepsilon}_0} \tag{3.3}$$

Here δ_s is the thickness of layer of material in which surface diffusion takes place Φ_s is the surface free energy σ_0 and ε_0 are creep constants. For pure power law

$$\frac{1}{\dot{\varepsilon}_0}\frac{df_h}{dt}=\beta\left(\frac{\sigma_e}{\sigma_0}\right)^n\left[\frac{1}{(1-f_h)^n}-(1-f_h)\right] \tag{3.4}$$

$$\frac{1}{\dot{\varepsilon}_0}\frac{d\varepsilon}{dt}=\left[1+\frac{2r_h\beta}{d}\left(\frac{1}{(1-f_h)^n}-1\right)\right]\left(\frac{\sigma_e}{\sigma_0}\right)^n \tag{3.5}$$

where
$$\beta=sinh\left\{-2\frac{(n-\frac{1}{2})p}{(n+\frac{1}{2})\sigma_e}\right\} \tag{3.6}$$

is a parameter that measures effect of strain rate on void growth; r_h is the radius of growing voids. By assumption (iii) of section 2.0 the rate of change of area fraction of holes for the coupled diffusive power law is given by

$$\frac{df_h}{dt}=\frac{\Psi_0 f_h^{1/2}}{(1-f_h)^3}\left(\frac{\sigma_1}{\sigma_0}\right)^3+\beta\left(\frac{\sigma_e}{\sigma_0}\right)^n\left[\frac{1}{(1-f_h)^n}-(1-f_h)\right] \tag{3.7}$$

That is equation (3.1) plus equation (3.4) which for convenience is written

$$\frac{1}{\dot{\varepsilon}_0}\frac{d\,f_h}{d\,t}=\left(b+P_s\frac{f_h^{1/2}}{(1-f_h)^{3-n}}\right)\frac{\sigma_e^r}{(1-f_h)^n} \tag{3.8}$$

where $\sigma_e^r=\left(\frac{\sigma_e}{\sigma_0}\right)^r$, $\quad P_s=\Psi_0\left(\frac{\sigma_1}{\sigma_0}\right)^3$ *and* $b=\beta\left[1-(1-f_h)^{n+1}\right]$ (3.9)

Note that $r = 0$ depicts pure diffusion mechanism and $r = n$ pure power law mechanism with index n. Equation (3.8) for f_h large ($f_h \to 1$), $b \to \beta$ gives

$$\frac{d\,f_h}{dt}=\frac{\beta\sigma_e^r}{(1-f_h)^n} \tag{3.10}$$

and for small $f_h << 1$ we have

$$\frac{d\,f_h}{d\,t}=f_h^{1/2}\left(f_h^{1/2}(n+1)\beta+\frac{2\,d\,l\,P_s}{d^2}\left[1-(n-3)f_h\right]\frac{\sigma_e^r}{(1-f_h)^n}\right) \tag{3.11}$$

For pure power law $P_s = 0$ and

$$\frac{d\,f_h}{d\,t}=b\left(\frac{\sigma_e}{1-f_h}\right)^n \tag{3.12}$$

where $b \to \beta$ for f_h large. This shows that the f_h large results of (3.10) corresponds to the pure power law mechanism which is just the continuum damage theory result with $f_h = \omega$ as damage parameter. For multi-axial case of power law mechanism the normalized strain rate is given by

$$\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_0}=\frac{3}{2}\left(\frac{\bar{\sigma}}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0(1-f_h)^n} \tag{3.13}$$

the rate of change of area fraction of holes is given by

$$\frac{df_h}{dt}=\frac{3}{2}\left(\frac{\bar{\sigma}}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0}\left[\frac{1}{(1-f_h)^n}-(1-f_h)\right] \tag{3.14}$$

where s_{ij} is the deviatoric stress. Equation (3.13) shows that for $f_h <<1$ we have

$$\frac{df_h}{dt}=\dot{\varepsilon}_{ij}(\varepsilon_0)^{-1}\left[1-(1-f_h)^{n+1}\right]\approx(n+1)f_h\dot{\varepsilon}_{ij}(\varepsilon_0)^{-1} \tag{3.15}$$

that is
$$\frac{df_h}{dt}=\begin{matrix}\dot{\varepsilon}_{ij}, & f_h \text{ large}\\ (n+1)f_h\dot{\varepsilon}_{ij}, & f_h \text{ small}\end{matrix} \tag{3.16}$$

For multi-axial case of the coupled diffusion power law we have in [23, 24].

$$\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_0} = \tfrac{3}{2}\left(\frac{\sigma}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0(1-f_h)^n}\left[b+\frac{2\,d\,l\,P_s f_h^{1/2}}{d^2(1-f_h)^{3-n}}\right] \qquad (3.17)$$

and

$$\frac{d\,f_h}{d\,t} = \tfrac{3}{2}\varepsilon_0\left(\frac{\sigma}{\sigma_0}\right)^{n-1}\frac{s_{ij} f_h^{1/2}}{\sigma_0(1-f_h)^n}\left[(n+1)\,\phi\, f_h^{1/2}+\frac{2\,d\,l\,P_s f_h^{1/2}}{d^2(1-f_h)^{3-n}}\right] \qquad (3.18)$$

that is

$$\frac{d\,f_h}{d\,t} = (n+1)f_h\dot{\varepsilon}^p_{ij} + \left((n+1)f_h\right)^2;\quad \dot{\varepsilon}_{ij} = (n+1)f_h\left[\dot{\varepsilon}^p_{ij} + (n+1)f_h\dot{\varepsilon}^d_{ij}\right] \qquad (3.19)$$

Thus $(n+1)f_h$ is a factor of change by (3.14) and (3.15). It thus follows that for multi-axial case

$$\dot{f}_h = \begin{cases} \dot{\varepsilon}^p_{ij} + (n+1)f_h\dot{\varepsilon}^d_{ij} & f_h \text{ large} \\ (n+1)f_h\,[\dot{\varepsilon}^p_{ij} + (n+1)f_h\dot{\varepsilon}^d_{ij}\,], & f_h \text{ small} \end{cases} \qquad (3.20)$$

where $\dot{\varepsilon}^p_{ij}$ is for power law mechanism and $\dot{\varepsilon}^d_{ij}$ is the strain rate for surface diffusion mechanism. From (3.16) and (3.20) and Figure 3 we deduce a global definition for f_h small. For the purpose of our exposition we define: f_h small implies that $f_h \in (0, (n+1)^{-1})$ for n-power law.

Following Hutchinson [13] in the absence of micro-cracks the potential function can be taken as

$$\Phi_0 = \tfrac{3}{2}\varepsilon_{ij}\,\frac{s_{ij}}{n+1}\left(\frac{\sigma_e}{\sigma_1}\right)^{n-1}\frac{s_{ij}}{\sigma_o} \qquad (3.21)$$

for which the strain rate is given by $\dot{\varepsilon}_{ij} = \tfrac{3}{2}\dot{\varepsilon}_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0}$ (3.22)

Substituting (3.22) in (3.21) gives $\Phi_0 = \frac{s_{ij}}{n+1}\dot{\varepsilon}_{ij}$ (3.23)

From the damage theory [15].

$$\dot{\varepsilon}_{ij} = \tfrac{3}{2}\dot{\varepsilon}_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0}(1-\omega)^{-n} \qquad (3.24)$$

so that for $\omega = 0$ (no damage) $\dot{\varepsilon}_{ij} = \tfrac{3}{2}\dot{\varepsilon}_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0}$ (3.25)

For $\omega \neq 0$, $\omega \ll 1$ (3.24) can be written as

$$\dot{\varepsilon}_{ij} = \tfrac{3}{2}\dot{\varepsilon}_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0}\left(1+n\,\omega+n(n+1)\omega^2+\Lambda\right) \qquad (3.26)$$

That is $\dot{\varepsilon}_{ij} = n\varepsilon_{ij}\left\{\tfrac{1}{n}+\omega\left[1+(n+1)\omega\right]\right\}+hot$ (3.27)

One expects that in the presence of micro-cracks and damage

$$\Phi = \frac{s_{ij}}{n+1}\dot{\varepsilon}_{ij}\left\{1+n\,\omega\left[1+(n+1)\omega\right]\right\} \qquad (3.28)$$

and we have for $\omega = (n+1)f_h$ $\Phi = n\Phi_0\left\{\tfrac{1}{n}+(n+1)f_h\left[1+(n+1)f_h\right]\right\}$ (3.29)

We can then conjecture the result

$$\Phi = \begin{cases} \Phi_0 & \text{for no micro-crack} \\ n(n+1)f_h\Phi_0\left[1+(n+1)f_h\right] & \text{for micro-cracks} \end{cases} \qquad (3.30)$$

Noting (3.23) we can write this result as

$$\Phi = \begin{cases} \dfrac{s_{ij}}{n+1}\dot{\varepsilon}_{ij} & \text{for nomicro} - \text{crack} \\ \dfrac{s_{ij}}{n+1}\dot{\varepsilon}_{ij}\left\{ n\left(n+1\right)f_h\left[1+\left(n+1\right)f_h\right]+1\right\}, & \text{for micro} - \text{crack} \end{cases} \tag{3.31}$$

4.0 **Crack tip fields**

We note as in [23] that under surface diffusion mechanism the void becomes flatter and more crack-like so that the void can be idealized as a penny-shaped crack of constant width and semi-circular tip. Thus for the crack-tip in the power law creep mechanism zone the singularity fields following Hutchinson [12], Rice and Rosengren [26] can be written as

$$\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_0} = \alpha K_\sigma^n r^{-\left(n/_{n+1}\right)}\tilde{\varepsilon}_{ij}\left(\theta,n\right); \qquad \frac{\dot{\sigma}_{ij}}{\dot{\sigma}_0} = \alpha K_\sigma^n r^{-\left(n/_{n+1}\right)}\tilde{\sigma}_{ij}\left(\theta,n\right) \tag{4.1}$$

where $\left(\tilde{\varepsilon}_{ij},\tilde{\sigma}_{ij}\right)$ are non-dimensional field quantities and ε_{ij} is given by (3.21). In this case Rice's J-integral [25] characteristics also apply so that $J = \int_{-\pi}^{\pi}\left(Wn_x - n_j\sigma_{ij}v_{i,x}\right)r\,d\,\theta$ (4.2)

where W is now the strain rate potential and v_i is the velocity of propagation of the void given by

$$v_i - \dot{v}_i = \alpha\varepsilon_0 K_\sigma^n r^{1/_{n+1}}\tilde{v}_i\left(\theta,n\right) \tag{4.3}$$

Also the *HRR* (so called after Hutchinson [12] and Rice and Rosengren [26] results becomes applicable and

$$J = \alpha\,\varepsilon_0\sigma_0 K_\sigma^n I_n \tag{4.4}$$

in which case

$$K_\sigma = \left(\frac{J}{\alpha\,\varepsilon_0\sigma_0 I_n}\right)^{1/_{n+1}} \tag{4.5}$$

which on substituting (3.25) and (3.26) give

$$\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_0} = \left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n}\right)^{n/_{n+1}}\tilde{\varepsilon}_{ij} \quad and \quad \frac{\dot{\sigma}_{ij}}{\sigma_0} = \left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n}\right)^{1/_{n+1}}\tilde{\sigma}_{ij} \tag{4.6}$$

The strain gradient plasticity analysis has been shown [8] more appropriate for the analysis of crack tip when micro-structural theory is being used. As shown in [16] at the time neighbouring micro-voids start to coalesce, the initiation of macro-crack begins. This is buttressed by Kim and Lee [14] who using the earlier results of Lu et al [18] used asymptotic and numerical methods to analyses crack tip phenomenon under creep. They concluded that HRR problem is only valid for the limiting case of no damage thus considering *HRR* case as the behaviour of non-damage material. They also averred that in the small region of crack microstructure the loading parameter based on fracture mechanics derived from *HRR* singular field might be invalid.

Macro-crack propagation is controlled by the distribution of the void-area fraction in the crack-tip region. Thus the analysis of the crack tip field should be based on the SGP theory for a correct analysis.

Experimental results of Elcsner et al [5] show that plastic strain gradients appear either because of the geometry of loading or because of inhomogenous deformation in the material. As noted also by Tolle and Kassner [28] large strain plasticity is required for cavity growth, and it would appear that the cavity wall must be a dislocation source. Thus the common limitations lie with the continuum models. We are quite aware of the works of Nguyen et al [21], O'Dowd and Shih [22] based upon the work of Li and Wang [17] which considered an asymptotic series for the crack tip and conjectured that additional terms in the series beside the dominant takes care of the micro-structural aspect of void growth. We feel that the geometric aspect of the loading should be part of the analysis. This dictates our preference for the SGP analysis.

The basic theory of strain gradient analysis and all its invariants are available in the literature [6, 8, 10, 11, 7]. Our interest in this section is the crack-tip field analysis. As noted in Wei and Hutchinson [29] stresses ahead of a crack-tip in elastic-plastic solids with strain gradient effects could be more than 2.5 times their counterparts in materials without strain gradient fields - the *HRR* field.

The crack tip fields of HRR derived above will be shown below as limits of the crack tip fields derived through the SGP analysis. The strain gradient plasticity analysis recognizes the essence of a length parameter for the correct analysis of crack tip fields under a micro-structural consideration which experimental evidences [5, 20, 29] confirm. We now proceed to give an SGP analysis of the crack tip fields.

The following generalization of Rice's path independent J integral [25] exists for the deformation theory solid [30, 31].

$$J = \int_C \left(Wn_1 - T_i u_{i,1} - q_i \theta_{i,1}\right) ds \qquad (4.7)$$

where C is any contour circling the crack tip originating on the lower crack surface and ending on the upper crack surface with s as the distance along the contour. W is taken to be the energy density for the deformation solid given by

$$W(E, \varepsilon_m) = w\ (\ E\ ) + \tfrac{1}{2}\kappa\varepsilon_m^2 \qquad (4.8)$$

where the elastic part of the strain energy due to deviatoric strains is included in w. κ is the bulk modulus of the solid. T_j is the traction vector acting on an element with unit normal n_i, that is

$$T_j = \left(\sigma_{ij} + \tau_{ij}\right) n_i \qquad (4.9)$$

q_j is the couple stress traction vector and θ_i is the linearized rotation vector. E is the effective strain quantity defined by

$$E^2 \equiv \varepsilon_e^2 + l^2 \chi_e^2 \qquad (4.10)$$

where l is a material length quantity which becomes the sole additional constitutive parameter in the theory; ε_e is regarded as a measure of the density of statistically stored dislocations and χ_e is a measure of the density of geometrically necessary dislocations produced by the strain gradient. The basic relationships applicable to this analysis are given as

$$\theta_i \equiv \tfrac{1}{2} e_{ijk} u_{k,j}\,;\ w(E) = \frac{n}{n+1}\sigma_0 \varepsilon_0 \left(\frac{E}{\varepsilon_0}\right) \text{ such that } \frac{\Sigma}{\sigma_0} = \left(\frac{E}{\varepsilon_0}\right) \qquad (4.11)$$

where σ_0 is a measure of the tensile yield stress and ε_0 is the associated elastic tensile strain at that stress, Σ is the effective stress, e_{ijk} is the Levi-Civita symbol, u_k the displacement vector. This generalization suggests that $W \to \frac{f\ (\theta\ )}{r}$ as $r \to 0$ which the case is indeed. In general, the solution in plane strain can be written as [23]

$$\left[\varepsilon_{ij}, l\chi_{ij}\right] = \varepsilon_0 \left(\frac{J}{\sigma_o \varepsilon_0 I_n r}\right)^{n/n+1} \left[\hat{\varepsilon}_{ij}\left(\theta, \tfrac{r}{l}, n\right), \hat{\chi}_{ij}\left(\theta, \tfrac{r}{l}, n\right)\right],$$

$$\left[\sigma_{ij}, l^{-1} m_{ij}, \tau_{ij}\right] = \sigma_0 \left(\frac{J}{\sigma_o \varepsilon_0 I_n r}\right)^{n/n+1} \left[\hat{\sigma}_{ij}\left(\theta, \tfrac{r}{l}, n\right), \hat{m}_{ij}\left(\theta, \tfrac{r}{l}, n\right), \hat{\tau}_{ij}\left(\theta, \tfrac{r}{l}, n\right)\right] \qquad (4.12, 4.13)$$

where $\chi_{ij} = \theta_{i,j}$ is the linearised curvature tensor so that $\chi_{ij} = e_{ikl}\varepsilon_{jl,k}$. m_{ij} is the unsymmetric deviatoric couple stress tensor given by

$$m_{ij} = \frac{\partial W}{\partial \chi_{ij}} = \frac{\partial w}{\partial \chi_{ij}} \qquad (4.14)$$

The following relationships also apply: $m_{ij} = \frac{2}{3} l^2 \frac{\Sigma}{E}\chi_{ji}, \Sigma^2 = \sigma_e^2 + l^{-2} m_e^2$, where

$$\sigma_e = \sqrt{\tfrac{3}{2} s_{ij} s_{ij}}\,,\ m_e = \sqrt{\tfrac{3}{2} m_{ij} m_{ij}}\ \ so\ that\ \ s_{ij} = \tfrac{2}{3}\frac{\Sigma}{E}\varepsilon'_{ij} \qquad (4.15)$$

and $\varepsilon'_{ij} = \varepsilon_{ij} - \varepsilon_m \delta_{ij}$. In their finite element analysis Xia and Hutchinson [30] found that elastic compressibility does not affect the most singular fields. Assuming the displacement generating the strains as the tip is approached are not irrotational they found that the dominant fields at the tip are irrotational in such a way that the strain and not curvatures are dominant at the crack tip so that by (4.10) and (4.11) the conclusion is reached that as in the HRR field.

$$W \to r^{-1}, \quad s = \frac{n}{n+1}, \quad \varepsilon_e \to r^{-n/(n+1)}, \quad \sigma_e \to r^{-1/(n+1)} \qquad (4.15a)$$

It becomes clear that the dominantly singular crack tip fields are such that $\hat{\varepsilon}_{ij}, \hat{\sigma}_{ij}, and\, \hat{\tau}_{ij}$ are all finite as $r/l \to 0$, while $\hat{\chi}_{ij}$ *and* $\hat{m}_{ij}$ approach zero. In the outer field where the HRR solution is approached, $\hat{\varepsilon}_{ij}, and\ \hat{\sigma}_{ij}$, respectively approach $\tilde{\varepsilon}_{ij}, and\ \tilde{\sigma}_{ij}$. As $r/l \to 0$, the dominant singular fields are given by

$$(\sigma_m, \tau_{r\theta}) = \sigma_0 \left(\frac{J}{\sigma_0 \varepsilon_0 I_n r} \right)^{1/(n+1)} (\hat{\sigma}_m, \hat{\tau}_{r\theta}) \ with\ (\hat{\sigma}_m, \hat{\tau}_{r\theta}) = \frac{1}{\sqrt{3}} \left(\frac{2A}{\sqrt{3}} \right)^{1/n} (f(\theta), g(\theta)). \qquad (4.16)$$

where

$$f(\theta) = \frac{2n}{n+1} cos\left(\frac{1}{n+1}\theta \right) + \frac{n-1}{n+1} cos\left(\frac{n+2}{n+1}\theta \right),$$
$$g(\theta) = -\frac{2n}{n+1} sin\left(\frac{1}{n+1}\theta \right) - \frac{n-1}{n+1} sin\left(\frac{n+2}{n+1}\theta \right) \qquad (4.17)$$

with

$$A = \frac{\sqrt{3}}{2} \left(\frac{n+1}{2n\pi} I_n \right)^{n/(n+1)} \qquad (4.18)$$

It becomes obvious that we will be having the result that for $\frac{r}{l} \to 0, i.e. \frac{r}{l} << 1$ the SGP theory applies while for $\frac{r}{l}$ large the HRR result applies (see Figure 4). This translates to the very obvious conclusion: f_h large we have the HRR results while for f_h small f_h<<1 the SGP results is the more appropriate.

5.0 Conclusion

We have considered the constitutive behaviour and crack tip fields for materials undergoing coupled creep constrained cavitations. A series like relationship was derived for the coupled mechanism and used in the crack-tip analysis. This relationship reveals that at high stresses the power law mechanism is more appropriate than any other mechanism. We also show that the Kachanov-Rabotnov results are special cases of our results and constitute an upper bound for the analysis. We also show that in creep constrained cavitations the power law can be used to deduce diffusion mechanism results. We show through the strain gradient plasticity theory that the HRR results for crack tip fields hold only when the continuum theory holds but fails under micro-structural analysis.

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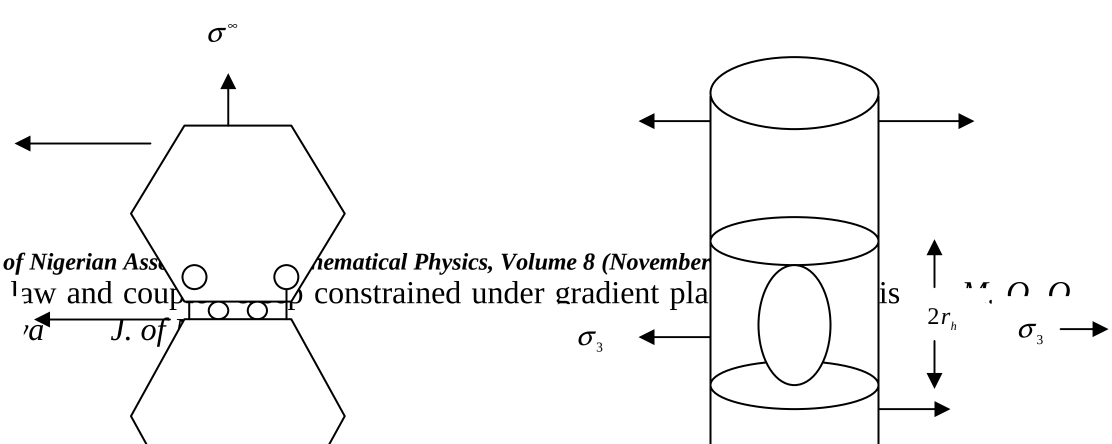

Figure 1b: Void growth on a grain boundary showing the unit of structure.

Figure 1a: Void growth on grain boundary

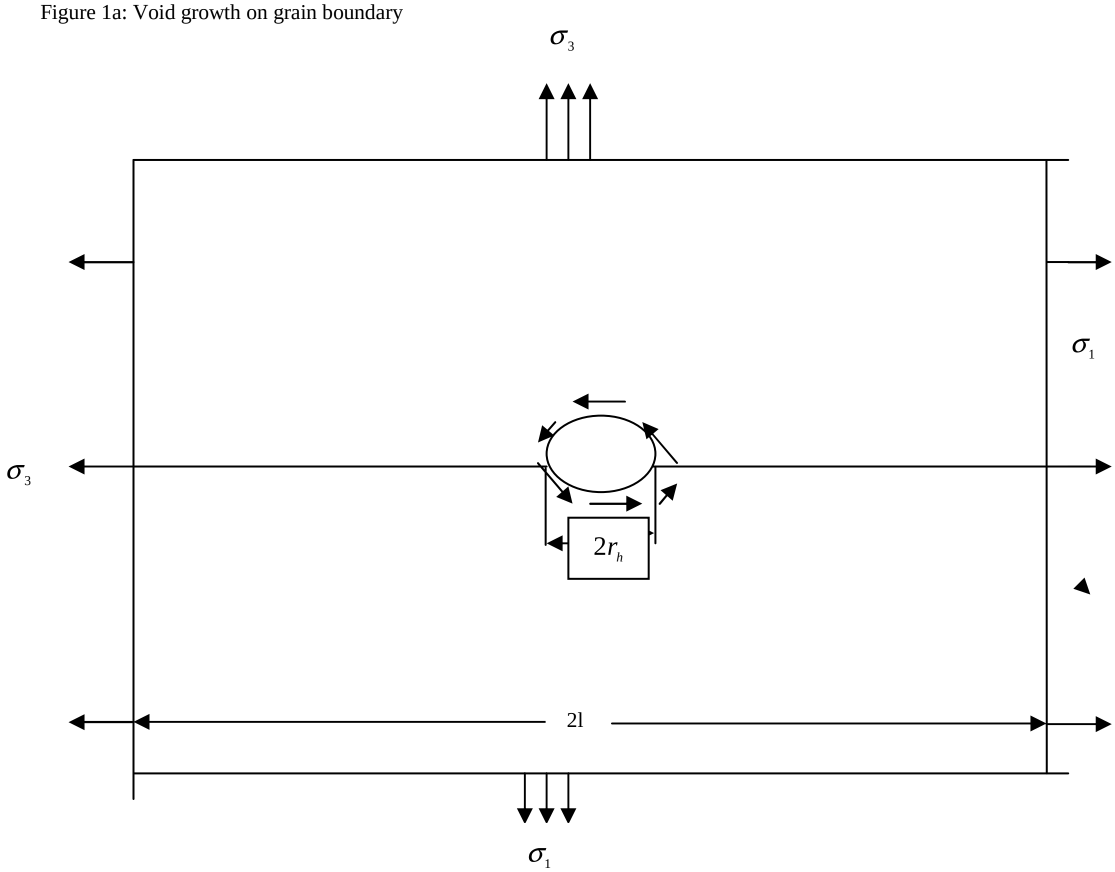

Figure 2a: Void growth by Surface Diffusion Mechanism

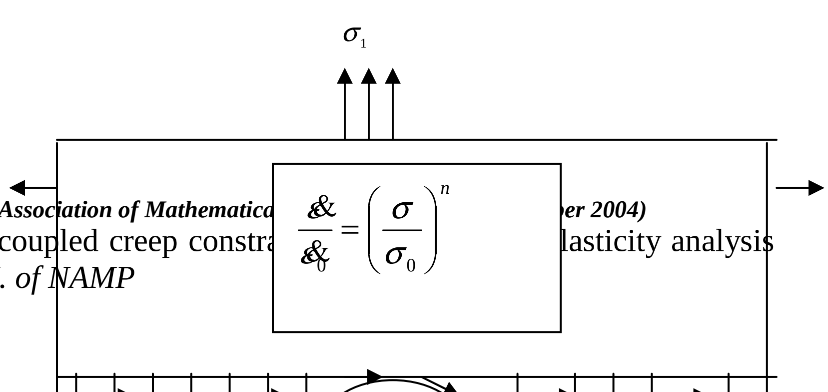

Figure 2b: Void growth by power law

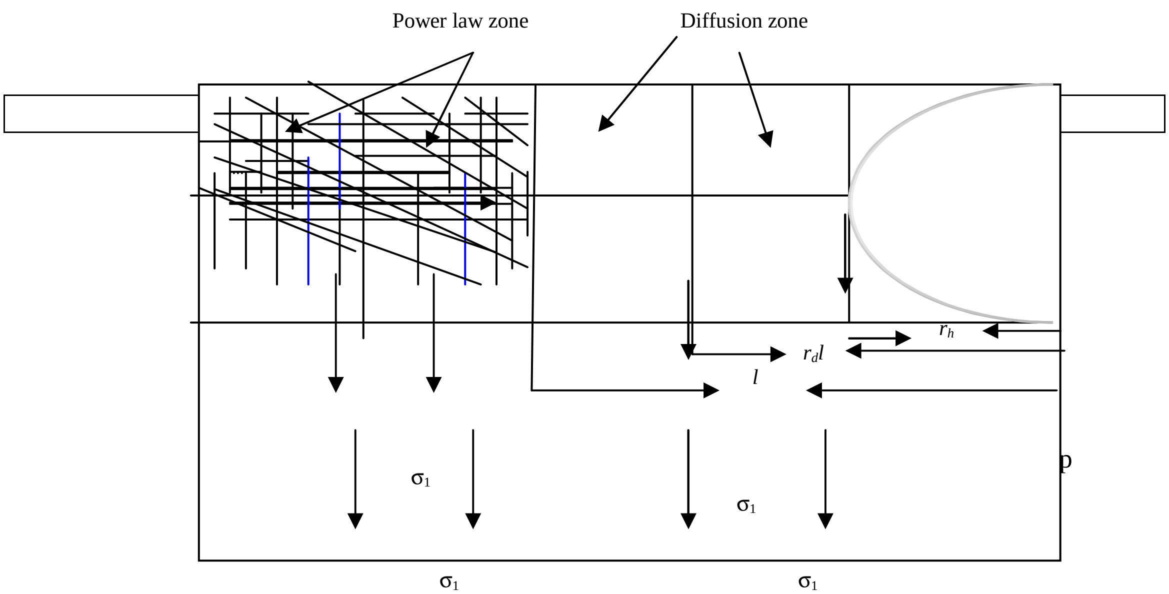

Figure 2c: Void growth by coupled surface diffusion and power law creep

D f_h

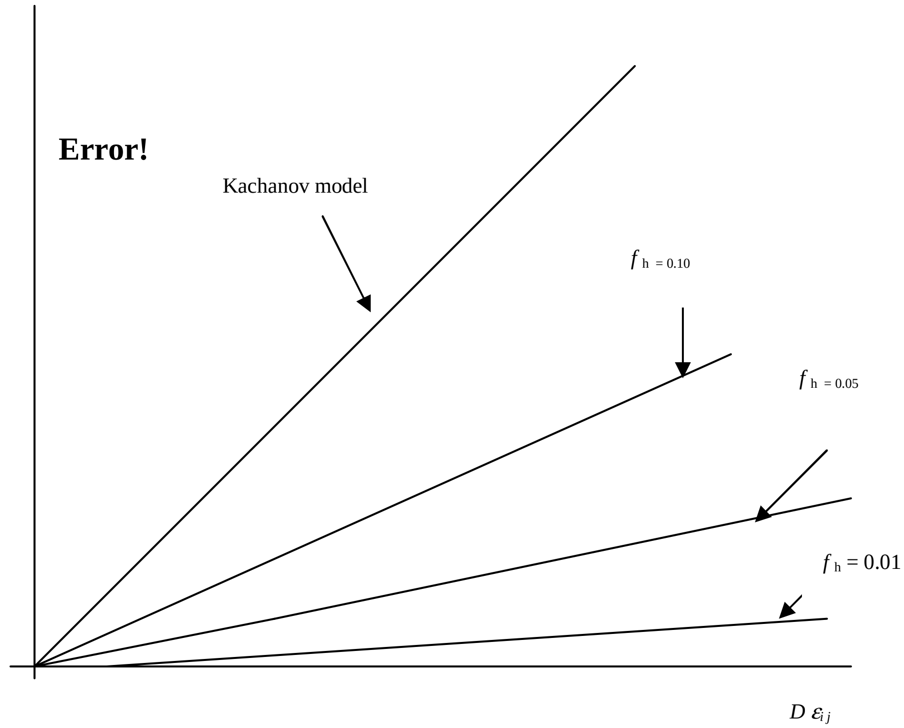

Figure 3: *f* h small model and Kachanov mode

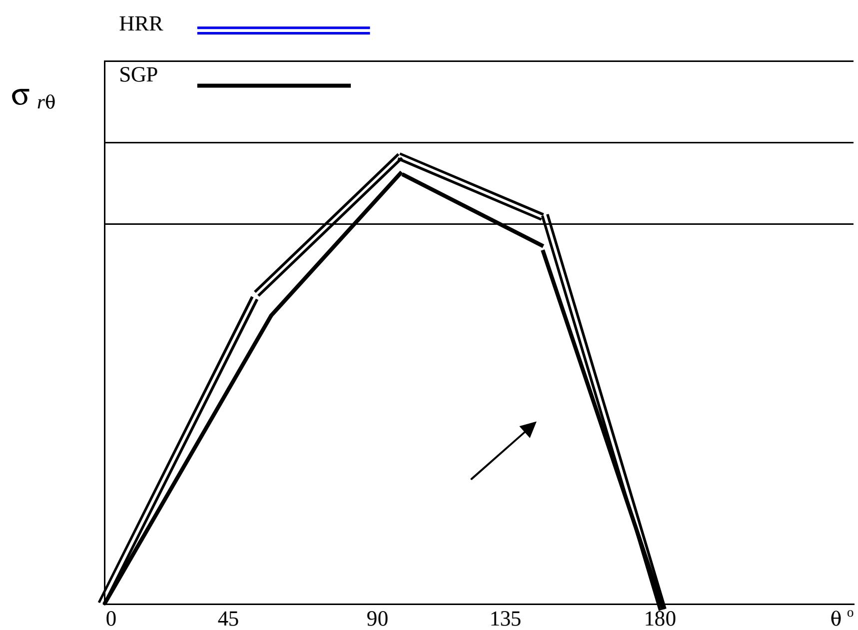

Figure 4a: Distribution of normalised stress components defined in equation (4.12)

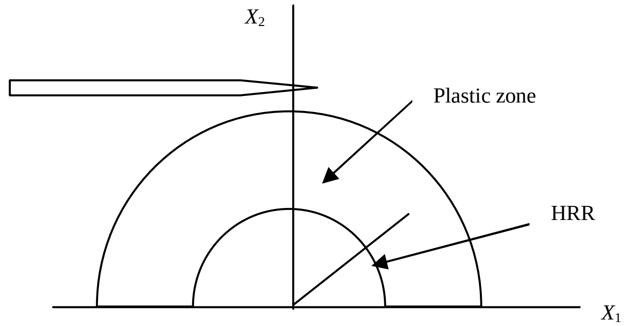

Figure 4b: Asymptotic crack tip geometry

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