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On the dynamic buckling of stochastically imperfect finite cylindrical shells under step loading

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Abstract

The dynamic buckling load of stochastically imperfect finite right circular cylindrical shells subjected to step loading is determined by means of regular perturbation procedures .The imperfection is assumed to be a Gaussian random function of position and consequently is homogeneous. The result obtained is implicit in the load parameter and is asymptotically valid for small magnitude of the random imperfection, which is itself taken as the first term in a Fourier sine expansion.

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1.0 Introduction

There is already in existence an enormous wealth of literature on stochasticity as applied to the analyses of the buckling of elastic structures. A few of the earlier ones include those in the references [1-5] and the references there cited. The present exposition is therefore a contribution to a vast avalanche of knowledge that needs further expansion. Right circular cylindrical shells are elastic structures whose imperfection-sensitivity and static buckling under various time independent loading conditions have been exhaustively analyzed by various researchers including those cited in the references [6-8]. However most of these earlier studies concentrated primarily on analyzing the stability of the structures statically. Perhaps one of the earliest analytical studies to address the problem dynamically was that by Lockhart and Amazigo [9]. In all, there have been by far more investigations on the static stability of the structures than on dynamic stability and our present analysis is therefore aimed at enquiring into an area of relative infrequent investigation.

2.0 Karman-Donnell Equations

A cylindrical shell is characterized by its outward radial displacement W(X,Y,T) and airy stress function F(X,Y,T) where X and Y are the axial and circumferential spatial variables respectively while T is the time variable .The membrane stress resultants N_X ,N_Y, and N_{XY} are given in terms of the airy stress function as (see Figure 1 and Figure 2)

$$N_X=F,_{YY}\ ,N_Y=F,_{XX}\ ,N_{XY}=-F,_{XY}$$

where a subscript following a comma indicates partial differentiation. We shall assume an initial stress-free random imperfection $\overline{W}(X,Y)$which is in fact an initial outward normal displacement .The relevant compatibility equation and equilibrium equation [9] as amended to dynamic setting are respectively given by

$$\frac{1}{Eh}\nabla^4 F-\frac{1}{R}W,_{XX}=-S\left(W,\frac{1}{2}W+\overline{W}\right), \qquad (2.1)$$

and

$$\rho W,_{TT}+D\ \nabla^4 W+\frac{1}{R}F,_{XX}=S\left(W+\overline{W},F\right)-\overline{P}\left(T\right), \qquad (2.2)$$

where E is the Young's modulus, *h* is the shell thickness, R is the radius, ρ is the mass per unit area, D is the bending stiffness given by D= $\frac{Eh^3}{12\left(1-\upsilon^2\right)}$, υ is the Poisson's ratio, $\bar{P}\left(T\right)$ is the time dependent loading history, ∇^4 is the two –dimensional biharmonic operator given by

$$\nabla^4 \equiv \left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)^2 \tag{2.3}$$

while S is the bilinear operator

$$S(P,Q) = P,_{XX}Q,_{YY}+P,_{YY}Q,_{XX}-2P,_{XY}Q,_{XY} \tag{2.4}$$

We now introduce the following nondimensional quantities:

$$x=\frac{\pi X}{L},\ y=\frac{Y}{R}\ ,\in \bar{w}=\frac{\bar{W}}{h},\quad w=\frac{W}{h},$$
$$t=\frac{T\pi^2\left(\frac{D}{\rho}\right)^{\frac{1}{2}}}{L^2}\ ,\ \lambda=\frac{L^2R\,\bar{P}}{\pi^2 D}\ ,A=\frac{L^2\sqrt{12\left(1-\upsilon^2\right)}}{\pi^2 R\ h}, \tag{2.5}$$
$$\xi=\frac{L^2}{\pi^2R^2}\ ,\ K\left(\xi\right)=-\frac{A^2}{\left(1+\xi\right)^2}\ ,\ H=\frac{h}{R},$$

where *L* is the shell length and ∈ is a small deterministic amplitude of the random imperfection function $\bar{w}(x,y)$. We shall consider homogeneous initial displacement and velocity and shall neglect both axial and circumferential inertia terms. We shall assume simply supported boundary conditions and shall neglect the boundary layer effects by assuming that the pre- buckling displacement is constant. Thus as in [9] we let

$$F=-\frac{1}{2}\bar{P}R\left(X^2+\frac{1}{2}\alpha Y^2\right)+\frac{Eh^2L^2}{\pi^2R^2\left(1+\xi\right)^2}f, \tag{2.6a}$$

$$W=\frac{\bar{P}\left(1-\frac{1}{2}\alpha\upsilon\right)}{Eh}+hw. \tag{2.6b}$$

The first terms in (6a,b) are the pre-buckling approximations. The parameter α takes the value α=1 if pressure contributes to axial stress through end plates while at the same time it takes the value α=0 if pressure acts laterally. We shall assume a step loading condition where

$$\bar{P}\left(T\right)=\begin{cases}1, T>0\\ 0, T<0\end{cases}. \tag{2.7}$$

On substituting (2.6a,b) and (2.7) into (2.1) and (2.2), using (2.5) and simplifying we get

$$\bar{\nabla}^4 f-\left(1+\xi\right)^2 w,_{xx}=-H\left(1+\xi\right)^2\bar{s}\left(w,\frac{1}{2}w+\in\bar{w}\right), \tag{2.8}$$

$$\bar{\nabla}^4 w-K\left(\xi\right)f,_{xx}+\lambda\left[\frac{1}{2}\alpha\left(w+\in\bar{w}\right),_{xx}+\xi\left(w+\in\bar{w}\right),_{yy}\right]=-HK\left(\xi\right)\bar{s}\left(w+\in\bar{w},f\right), \tag{2.9}$$

$$0<x<\pi\ ,\ 0<y<2\pi \tag{2.10}$$

$$w=w,_{xx}=f=f,_{xx}=0\ at\ x=0,\pi, \tag{2.11a}$$

$$w=w,_t=0\ at\ t=0, \tag{2.11b}$$

where $\bar{\nabla}^4\equiv\left(\frac{\partial^2}{\partial x^2}+\xi\frac{\partial^2}{\partial y^2}\right)^2$; $\bar{s}\left(P,Q\right)=P,_{xx}Q,_{yy}+P,_{yy}Q,_{xx}-2P,_{xy}Q,_{xy}$.

The procedure to be adopted shall include first determining a uniformly valid asymptotic expression of the normal (radial) displacement *w(x,y,t)* given the Gaussian random stress-free initial displacement $\overline{w}(x,y)$ with its known statistical characterizations. The random nature of $\overline{w}(x,y)$ automatically confers some element of randomness on the displacement *w(x,y,t)* whose statistical characterizations are evaluated once those of $\overline{w}(x,y)$ are known. We next determine the autocorrelation of *w(x,y,t)* and also determine the mean square displacement $\Delta^2(x,y,t)$ as a function of space and time variables. We follow this up by determining the maximum mean square displacement. Lastly we derive an expression for determining the dynamic buckling load λ_D, which is here defined as the maximum load parameter for which the mean square displacement remains bounded for all time $t > 0$.

2.1 **Classical Theory**

The classical buckling load λ_c was obtained in [9] by disregarding the imperfection and solving the associated linear problem assuming

$$(w,f) = (a_{mk},b_{mk}) \sin(ky+\phi_{mk}) \sin mx. \qquad (2.12a)$$

The relevant compatibility and equilibrium equations are respectively given by

$$\overline{\nabla}^4 f-(1+\xi)^2 w,_{xx}=0, \qquad (2.12b)$$

$$\overline{\nabla}^4 w-K(\xi)f,_{xx}+\lambda\left[\frac{\alpha}{2}w,_{xx}+\xi\, w,_{yy}\right]=0. \qquad (2.12c)$$

The result as obtained in [9] is

$$\lambda_c=\left[\frac{(1+\zeta)^2-\dfrac{(1+\xi)^2K(\xi)}{(1+\zeta)^2}}{\dfrac{\alpha}{2}+\zeta}\right], \qquad (2.13a)$$

$$\zeta=n^2\xi. \qquad (2.13b)$$

In the above, *n* taken as an integer, is the critical value of *k*. The associated displacement and Airy stress function are given by

$$(w,f)=\left\{1\ ,\ -\left(\frac{1+\xi}{1+\zeta}\right)^2 a_{1n}\sin(ny+\phi_{1n})\sin x\right\}. \qquad (2.13c)$$

2.2. **Dynamic Theory**

Following the report in [9], we shall assume

$$\overline{w}(x,y)=\overline{a}\sin x\sin ny, \qquad (2.14a)$$

where $\overline{a}$ is a random parameter whose statistical properties are easily evaluated once those of $\overline{w}(x,y)$ are known. We shall now let

$$\tau=\epsilon^2 t\ ;\ 0<\epsilon<<1 \qquad (2.14b)$$

and $w(x,y,t) = V(x,y,t,\tau,\epsilon)$. Thus we have

$$w,_t=V,_t+\epsilon^2V,_\tau\ ;\quad w,_{tt}=V,_{tt}+2\epsilon^2V,_{t\tau}+\epsilon^4V,_{\tau\tau} \qquad (2.14c)$$

We shall now let

$$\begin{pmatrix}V(x,y,t,\tau,\epsilon)\\ f(x,y,t,\tau,\epsilon)\end{pmatrix}=\sum_{i=1}^{\infty}\begin{pmatrix}V^{(i)}(x,y,t,\tau,)\\ f^{(i)}(x,y,t,\tau,)\end{pmatrix}\epsilon^i \qquad (2.15)$$

We now substitute (2.14b) -(2.15) into (2.8) and (2.9) and get the following sequence of equations

$$L^{(1)}(f^{(1)},V^{(1)})=\overline{\nabla}f^{(1)}-(1+\xi)^2V,_{xx}^{(1)}=0, \qquad (2.16a)$$

$$M^{(1)}\left(f^{(1)},V^{(1)}\right)=V,_{tt}^{(1)}+\overline{\nabla}V^{(1)}-K(\xi)\,f,_{xx}^{(1)}+\lambda\left[\frac{\alpha}{2}V,_{xx}^{(1)}+\xi V,_{yy}^{(1)}\right]$$

$$=-\lambda\left[\frac{\alpha}{2}\overline{w},_{xx}+\xi\,\overline{w},_{yy}\right], \qquad (2.16b)$$

$$L^{(1)}\left(f^{(2)},V^{(2)}\right)=-H\,(1+\xi)^2\left\{\frac{1}{2}\overline{s}\left(V^{(1)},V^{(1)}\right)+\overline{s}\left(V^{(1)},\overline{w}\right)\right\}, \qquad (2.17a)$$

$$M^{(1)}\left(f^{(2)},V^{(2)}\right)=-HK(\xi)\left\{\overline{s}\left(V^{(1)},f^{(1)}\right)+\overline{s}\left(f^{(1)},\overline{w}\right)\right\}, \qquad (2.17b)$$

$$L^{(1)}\left(f^{(3)},V^{(3)}\right)=-H\,(1+\xi)^2\left\{\frac{1}{2}\overline{s}\left(V^{(1)},V^{(2)}\right)+\overline{s}\left(V^{(2)},\overline{w}\right)\right\}, \qquad (2.18a)$$

$$M^{(1)}\left(f^{(3)},V^{(3)}\right)=-HK(\xi)\left\{\overline{s}\left(V^{(1)},f^{(2)}\right)+\overline{s}\left(f^{(2)},\overline{w}\right)+\left(V^{(2)},f^{(1)}\right)\right\}-2V,_{t\tau}^{(1)} \qquad (2.18b)$$

$$V^{(i)}=V,_{xx}^{(i)}=f^{(i)}=f,_{xx}^{(i)}=0 \text{ at } x=0,\ \pi, i=1,2,3, \qquad (2.19a)$$

$$V^{(i)}\left(x,y,0,0\right)=0\ ,\ i=1,2,3,\Lambda\ , \qquad V,_t^{(k)}\left(x,y,0,0,\right)=0\ ,\ k=1,2\Lambda \qquad (2.19b)$$

$$V,_t^{(s)}\left(x,y,0,0\right)+V,_\tau^{(s-2)}\left(x,y,0,0\right)=0\ ,\ s=3,4,5,...,\quad 0<x<\pi,\quad 0<y<2\pi \qquad (2.19c)$$

All through the analysis, we shall let

$$\begin{pmatrix} V^{(i)}(x,y,t,\tau) \\ f^{(i)}(x,y,t,\tau) \end{pmatrix}=\sum_{p,q=1}^{\infty}\left\{\begin{pmatrix} V_{1r}^{(i)} \\ f_{1r}^{(i)} \end{pmatrix}\cos q\,y+\begin{pmatrix} V_{2r}^{(i)} \\ f_{2r}^{(i)} \end{pmatrix}\sin q\,y\right\}\sin m\,x \qquad (2.20)$$

The subscript r shall indicate *r*-multiples of *n*-circumferential waves such as for example in the expression; "*sin rny sinmx*" Integration with respect to *x* shall have 0 and π as the lower and upper limits respectively while that

with respect to *y* shall have 0 and 2π as the lower and upper limits respectively.

3.0 **Solution of first order perturbation equations**

Here we shall solve (2.16a,b) by assuming (2.20) in this case where I = 1.Since $\overline{w}$(x,y) is linearly related to $V^{(1)}$and $f^{(1)}$ and since $\overline{w}$(x,y), as given in (2.14a), has no cosine component we are sure that $V^{(1)}$ and $f^{(1)}$ will take the forms

$$\begin{pmatrix} V^{(1)} \\ f^{(1)} \end{pmatrix}=\sum_{p,q=1}^{\infty}\begin{pmatrix} V_{2r}^{(1)} \\ f_{2r}^{(1)} \end{pmatrix}\sin p\,y \sin q\,y. \qquad (3.1)$$

On substituting (3.1) into (2.16a) and simplifying we get

$$f_{2r}^{(1)}=-\frac{(1+\xi)^2 m^2 V_{2r}^{(1)}}{\left(m^2+\xi\,(nr)^2\right)^2}, m=1,2,3,.\Lambda \qquad (3.2a)$$

where (3.2a) is true when $p = m, q = n$ for fixed m and n. Of course when m = 1, r = 1, we have

$$f_{21}^{(1)}=-\frac{(1+\xi)^2 V_{21}^{(1)}}{\left(1+n^2\xi\right)^2}. \qquad (3.2b)$$

We shall have cause to use (3.2b) later. We now substitute (3.2a) into (2.16b) and see that it is only when $m = 1 = r$ that a non-vanishing solution exists. We equally simplify the resultant substitution and get

$$V_{21}^{(1)},_{tt}+\mu^2 V_{21}^{(1)}=\lambda\,\overline{a}\left(\frac{\alpha}{2}+n^2\xi\right), \qquad (3.3a)$$

$$V_{21}^{(1)}(0,0)=V_{21}^{(1)},_t(0,0)=0, \qquad (3.3b)$$

$$\mu^2=\left[\left(1+n^2\xi\right)^2+\left(\frac{A}{1+n^2\xi}\right)^2-\lambda\left(\frac{\alpha}{2}+n^2\xi\right)\right]. \qquad (3.3c)$$

Generally we shall let

$$\left(\mu_r^{(m)}\right)^2 \equiv \mu_r^{(m)^2} = \left[\left(m^2 + (nr)^2 \xi\right)^2 + \left(\frac{Am^2}{m^2 + (nr)^2 \xi}\right)^2 - \lambda\left(\frac{m^2\alpha}{2} + (nr)^2 \xi\right)\right]. \qquad (3.3d)$$

Thus $\mu_1^{(1)^2} = \mu^2$ where $m = r = 1$. It is possible that for some values of the parameters involved, we may have cases where $\mu_r^{(m)^2} \le 0$. We are however not interested in such cases as they may not lead to buckling in the manner suggested in this paper. Thus, we shall assume that for all m and r, $\mu_r^{(m)^2} > 0$. With this assumption in mind we now solve (2.23a-d) and get

$$V_{21}^{(1)}(t, \tau) = \delta_1(\tau)\cos \mu t + \gamma_1(\tau)\sin \mu t + \bar{a}B, \qquad (3.4a)$$

$$\delta_1(0) = 0, \quad \gamma_1(0) = 0, \quad B = \frac{\lambda\left(\frac{\alpha}{2} + n^2 \xi\right)}{\left(1 + n^2 \xi\right)^2 + \left(\frac{A}{1 + n^2 \xi}\right)^2 - \lambda\left(\frac{\alpha}{2} + n^2 \xi\right)} \qquad (3.4b\text{-}d)$$

4.0 Solution of second order perturbation equations

We now substitute the relevant terms on the right sides of (2.17a,b) and simplify to get

$$L^{(1)}\left(V^{(2)}, f^{(2)}\right) = -Hn^2(1+\xi)^2\left\{\frac{1}{2}V_{21}^{(1)^2} + \bar{a}V_{21}^{(1)}\right\}(\cos 2x + \cos 2ny), \qquad (4.1a)$$

$$M^{(1)}\left(V^{(2)}, f^{(2)}\right) = -HKn^2\left\{V_{21}^{(1)} f_{21}^{(1)} + \bar{a}f_{21}^{(1)}\right\}(\cos 2x + \cos 2ny) \qquad (4.1b)$$

We shall now assume (2.20) for $I = 2$ and substitute same into (4.1a) and simplify to get

$$f_{12}^{(2m)} = -\frac{4Hn^2(1+\xi)^2\left(\frac{1}{2}V_{21}^{(1)^2} + \bar{a}V_{21}^{(1)}\right)}{m\pi\left(m^2 + 4n^2\xi\right)^2} - \frac{(1+\xi)^2 m^2 V_{21}^{(2)}}{\left(m^2 + 4n^2\xi\right)^2}, \qquad (4.2a)$$

where

$$f_{12}^{(2)} = \sum_{m=1,3,5,\ldots}^{\infty} f_{12}^{(2m)} \qquad (4.2b)$$

all valid for $r = 2$ and m odd. The above was obtained by multiplying (4.1a) by *cosrnysinmx* and carrying out the necessary simplification. We likewise multiply (4.2a) by *sinrnysinmx* and simplify to get

$$f_{2r}^{(2)} = -\frac{(1+\xi)^2 m^2 V_{2r}^{(2)}}{\left(m^2 + (rn)^2\xi\right)^2} \quad (\forall m, r). \qquad (4.2c)$$

We now multiply (4.1b) by cosrnysinmx and simplify, and get

$$V_{12,tt}^{(2)} + \mu_2^{(m)^2} V_{12}^{(2)} = R_1 V_{21}^{(1)} + R_2 V_{21}^{(1)^2}, \qquad (4.3a)$$

$$V_{12}^{(2)}(0,0) = V_{12,t}^{(2)}(0,0) = 0, \qquad (4.3b)$$

$$R_1 = -\frac{4\bar{a}(nA)^2 H}{m\pi}\left\{\frac{1}{\left(1+n^2\xi\right)^2} + \frac{2m^2}{\left(m^2 + 4n^2\xi\right)^2}\right\}, \qquad (4.3c)$$

$$R_2 = -\frac{4\bar{a}(nA)^2 H}{m\pi}\left\{\frac{1}{\left(1+n^2\xi\right)^2} + \frac{m^2}{\left(m^2 + 4n^2\xi\right)^2}\right\} \qquad (4.3d)$$

We note that (4.3a) is valid for $r = 2$. Similarly we multiply (4.1b) by *sinrnysinmx* and simplify to get

$$V_{2r,tt}^{(2)} + \mu_r^{(m)^2} V_{2r}^{(2)} = 0, \qquad (4.4a)$$

$$V_{2r}^{(2)}(0,0) = V_{2r,t}^{(2)}(0,0) = 0. \qquad (4.4b)$$

The solution of (4.4a) will certainly depend on the homogeneous initial conditions of (4.4b) and so, without solving explicitly for $V_{2r}^{(2)}$ we know that on the final analysis we shall eventually have

$$V_{2r}^{(2)}(t,\tau) = 0. \tag{4.5a}$$

The result of (4.3a-e) is certainly non-vanishing and we expect that the non-vanishing displacement corresponding to this order of perturbation will be of the form

$$V^{(2)}(x, y.t, \tau) = \sum_{m=1,3,5,...}^{\infty} V_{12}^{(2)}(t,\tau)\cos 2ny \sin mx \tag{4.5b}$$

It has been shown [6] that displacements in the shape of the imperfection have dominant effect on the buckling load of the structures. Though this finding was primarily shown to hold for the case of static buckling, it also holds for dynamic buckling. Consequently, since (4.5b) is not in the shape of (2.14a), we shall henceforth neglect it.

5.0 Solution of Third order perturbation equation

Having neglected $V^{(2)}$, we now substitute for terms on the right hand sides of (2.18a,b), using (4.5c) and simplify to get

$$L^{(1)}\left(V^{(3)}, f^{(3)}\right) = 0, \tag{5.1a}$$

$$\begin{aligned} M^{(1)}\left(V^{(3)}, f^{(3)}\right) = -\frac{HK}{4}\sum_{m=1,3,5,...}^{\infty}\left(V_{21}^{(1)} f_{12}^{(2m)} + \bar{a} f_{12}^{(2m)}\right)\Big[\left(4n^2 + m^2n^2 + 4n^2m\right)\times \\ \{\sin 3ny\cos(m-1)x + \sin ny\cos(m+1)x\} + \left(4n^2m - m^2n^2 - 4n^2\right)\times \\ \{\sin 3ny\cos(m+1)x + \sin ny\cos(m-1)x\}\Big] - 2V_{21,t\tau}^{(1)}\sin ny\sin mx. \end{aligned} \tag{5.1b}$$

We now multiply (5.1a) by *sinrny sinmx* and simplify to get

$$f_{21}^{(3)} = -\frac{(1+\xi)^2 m^2 f_{21}^{(3)}}{(m^2+n^2\xi)^2} \tag{5.2a}$$

valid for $r = 1$, m odd. Similarly when $r = 3$ and m odd we have from the same multiplication above

$$f_{23}^{(3)} = -\frac{(1+\xi)^2 m^2 V_{23}^{(3)}}{(m^2+9n^2\xi)^2}. \tag{5.2b}$$

In the same token we multiply (5.1a) by cosrny *sinmx* and simplify to get

$$f_{1r}^{(3)} = -\frac{(1+\xi)^2 m^2 V_{1r}^{(3)}}{(m^2+(nr)^2\xi)^2} \tag{5.2c}$$

On multiplying (5.1b) by sinrny *sinmx* and simplifying using (31a,b) we get

$$V_{21,tt}^{(3)} + \mu_1^{(m)^2} V_{21}^{(3)} = -r_o\sum_{m=1,3,5,...}^{\infty}\omega_m\left(V_{21}^{(1)} f_{12}^{(2m)} + \bar{a} f_{12}^{(2m)}\right) + 2\mu\left(\delta_1'\sin\mu t - \gamma_1'\cos\mu t\right), \tag{5.3a}$$

$$V_{21}^{(3)}(0,0) = 0, \quad V_{21,t}^{(3)}(0,0) + V_{21,\tau}^{(1)}(0,0) = 0 \tag{5.3b,c}$$

$$\omega_m = \pi\left[\left(4n^2 + m^2n^2 + 4n^2m\right)\left(\frac{1}{2m+1} - 1\right) + \left(4n^2m - m^2n^2 - 4n^2\right)\left(\frac{1}{2m-1} - 1\right)\right] \tag{5.3d}$$

$$r_o = \frac{HK}{2\pi^2} \tag{5.4e}$$

When $r = 3$, we get the following from the same multiplication above

$$V_{23,tt}^{(3)} + \mu_3^{(m)^2} V_{23}^{(3)} = -r_o\sum_{m=1,3,5,...}^{\infty}\theta_m\left(v_{21}^{(1)} f_{12}^{(2m)} + \bar{a} f_{12}^{(2m)}\right), \tag{5.5a}$$

$$V_{23}^{(3)}(0,0) = 0, \quad V_{23,t}^{(3)}(0,0) = 0, \tag{5.5b,c}$$

$$\theta_m = \pi\left[\left(1 + \frac{1}{2m-1}\right)\left(4n^2 + m^2n^2 + 4n^2m\right) + \left(4n^2m - m^2n^2 - 4n^2\right)\left(\frac{1}{2m-1} - 1\right)\right] \tag{5.5d}$$

We now multiply (5.1b) by *cosrny sinmx,using* (5.2c) and observe that because of the ensuing homogeneous initial conditions associated with $V_{1r}^{(3)}(t,\tau)$ we shall eventually have

$$V_{1r}^{(3)}(t,\tau) = 0. \tag{5.5e}$$

We shall now let

$$V_{21}^{(3)} = \sum_{m=1,3,5,...}^{\infty} V_{21}^{(3m)}, \qquad V_{23}^{(3)} = \sum_{m=1,3,5,...}^{\infty} V_{23}^{(3m)}. \tag{5.6}$$

The following simplifications shall be useful later:

$$\left(V_{21}^{(1)}\right)^3 \equiv V_{21}^{(1)^3} = \left(\delta_1 \cos\mu t + \gamma_1 \sin\mu t + \bar{a}B\right)^3 = q_o + q_1 \cos\mu t + q_2 \sin\mu t$$
$$+ q_3 \cos 2\mu t + q_4 \sin 2\mu t + q_5 \cos 3\mu t + q_6 \sin 3\mu t \tag{5.7a}$$

where

$$q_o = \frac{3\gamma_1^2(\bar{a}B)}{2} + (\bar{a}B)^3 + \frac{3\delta_1^2(\bar{a}B)}{2}; \qquad q_o(0) = \frac{5}{2}(\bar{a}B)^3, \tag{5.7b}$$

$$q_1 = \frac{3\delta_1^3}{4} + 3\delta_1\left\{\frac{\gamma_1^2}{4} + (\bar{a}B)^2\right\}; \qquad q_1(0) = -\frac{15(\bar{a}B)^3}{4}, \tag{5.8a}$$

$$q_2 = \frac{3\delta_1^2\gamma_1}{4} + \frac{3\gamma_1^3}{4} + 3\gamma_1(\bar{a}B)^2; \qquad q_2(0) = 0, \tag{5.8b}$$

$$q_3 = \frac{3\delta_1^2(\bar{a}B)}{2} - \frac{3\gamma_1^2(\bar{a}B)}{2}; \quad q_3(0) = \frac{3(\bar{a}B)^3}{2}, \quad q_4 = 3\delta_1\gamma_1(\bar{a}B); \quad q_4(0) = 0, \tag{5.8c}$$

$$q_4 = \frac{\delta_1^3}{4} - \frac{3\delta_1\gamma_1^2}{4}; \quad q_5(0) = -\frac{(\bar{a}B)^3}{4}, \quad q_6 = \frac{3\delta_1^2\gamma_1}{4}; \quad q_6(0) = 0. \tag{5.8d}$$

Similarly we have the following simplification

$$\left(V_{21}^{(1)}\right)^2 \equiv V_{21}^{(1)^2} = q_7 + q_8 \cos\mu t + q_9 \sin\mu t + q_{10} \sin 2\mu t + q_{11} \cos 2\mu t. \tag{5.9a}$$

where

$$q_7 = \left\{\frac{\delta_1^2}{2} + \frac{\gamma_1^2}{2} + (\bar{a}B)^2\right\}; \quad q_7(0) = \frac{3(\bar{a}B)^2}{2}, \quad q_8 = 2\delta_1(\bar{a}B); \quad q_8(0) = -2(\bar{a}B)^2. \tag{5.9b}$$

$$q_9 = 2\gamma_1(\bar{a}B); \quad q_9(0) = 0, \quad q_{10} = \delta_1\gamma_1; \quad q_{10}(0) = 0. \tag{5.9c}$$

$$q_{11} = \frac{1}{2}\left(\delta_1^2 - \gamma_1^2\right); \quad q_{11}(0) = -\frac{(\bar{a}B)^2}{2} \tag{5.9d}$$

If we substitute in (5.3a)-(5.5d), using (3.4a), (4.2a,b) and (5.6), we simplify to get

$$V_{21,tt}^{(3m)} + \mu_1^{(m)^2} V_{21}^{(3m)} = -\varphi_m\left\{\frac{1}{2}V_{21}^{(1)^3} + \frac{3}{2}\bar{a}V_{21}^{(1)^2} + \bar{a}^2 V_{21}^{(1)}\right\} + 2\mu\left(\delta_1' \sin\mu t - \gamma_1' \cos\mu t\right) \tag{5.10a}$$

$$V_{21}^{(3m)}(0,0) = 0 \ (\forall m \text{ odd}), \quad V_{21,t}^{(31)}(0,0) + V_{21,\tau}^{(1)}(0,0) = 0 \ (\text{for } m = 1), \tag{5.10b}$$

$$V_{21,t}^{(3m)}(0,0) = 0 \quad (m = 3,5,7,\Lambda), \tag{5.10c}$$

$$\varphi_m = r_o \omega_m l_m, \quad l_m = -\frac{4(1+\xi)^2 n^2 H}{m\pi\left(m^2 + 4n^2\xi\right)^2}. \tag{5.10d}$$

$$V_{23,tt}^{(3m)} + \mu_3^{(m)^2} V_{23}^{(3m)} = -\Lambda_m\left\{\frac{1}{2}V_{21}^{(1)^3} + \frac{3}{2}\bar{a}V_{21}^{(1)^2} + \bar{a}^2 V_{21}^{(1)}\right\} \tag{5.11a}$$

$$V_{23}^{(3m)}(0,0) = 0, \quad V_{23,t}^{(3m)}(0,0) = 0 \ (\forall m \text{ odd}), \ \Lambda_m = r_o \theta_m l_m. \tag{5.11b}$$

Yet some other simplifications of (5.10a) and (5.11b) are necessary thus:

$$V_{21,tt}^{(3m)} + \mu_1^{(m)^2} V_{21}^{(3m)} = -\varphi_m\left[q_{12} + q_{13}\cos\mu t + q_{14}\sin\mu t + q_{15}\cos 2\mu t + q_{16}\sin 2\mu t\right.$$
$$\left. q_{17}\cos 3\mu t + q_{18}\sin 3\mu t\right] + 2\mu\left(\delta_1' \sin\mu t - \gamma_1' \cos\mu t\right), \tag{5.12a}$$

and

$$V_{23,tt}^{(3m)} + \mu_3^{(m)2} V_{23}^{(3m)} = -\Lambda_m \left[q_{12} + q_{13}\cos\mu t + q_{14}\sin\mu t + q_{15}\cos 2\mu t + q_{16}\sin 2\mu t \right. \qquad (5.12b)$$
$$\left. q_{17}\cos 3\mu t + q_{18}\sin 3\mu t \right] + 2\mu\left(\delta_1'\sin\mu t - \gamma_1'\cos\mu t\right),$$

where $$q_{12} = \frac{1}{2}q_o + \frac{3\,\bar{a}}{2}q_7 + \bar{a}^3 B; \quad q_{12}(0) = \frac{5\left(\bar{a}B\right)^3}{4} + \frac{9\bar{a}^{\;3}B}{4},, \qquad (5.12c)$$

$$q_{13} = \frac{q_1}{2} + \frac{3\bar{a}q_8}{2} + \bar{a}^2\delta\ ;\quad q_{13}(0) = -\frac{15\bar{a}^3B^3}{8} - 3\bar{a}^3B^2 - \bar{a}^3B \qquad (5.12d)$$

$$q_{14} = \frac{1}{2}q_2 + \frac{3\,\bar{a}\,q_9}{2} + \bar{a}^2\gamma_1\,;\quad q_{14}(0) = 0, \qquad (5.12e)$$

$$q_{15} = \frac{1}{2}q_3 + \frac{3\,\bar{a}\,q_{11}}{2};\quad q_{15}(0) = 0, \qquad (5.12f)$$

$$q_{16} = \frac{1}{2}q_4 + \frac{3\,\bar{a}\,q_{10}}{2};\quad q_{16}(0) = 0, \qquad (5.12g)$$

$$q_{17} = \frac{1}{2}q_5\,;\quad q_{17}(0) = -\frac{\left(\bar{a}\,B\right)^3}{8}, \qquad (5.12h)$$

$$q_{18} = \frac{1}{2}q_6\,;\quad q_{18}(0) = 0. \qquad (5.12i)$$

When $m = 1$, we ensure a uniformly valid solution in (5.12a) by setting to zero the coefficients of $\cos\mu t$ and $\sin\mu t$ and getting

$$\gamma_1' - \frac{\varphi_1 q_{13}}{2\mu} = 0;\quad \delta_1' - \frac{\varphi_1 q_{14}}{2\mu} = 0 \qquad (5.13)$$

where φ_1 is the value of φ_m at $m = 1$. We have no intention of solving for $\gamma_1(\tau)$ and $\delta_1(\tau)$ in full from (5.13) because we can always extract every necessary information needed from these two functions direct from (5.13). For example, by determining (41) at $\tau = 0$, we have

$$\gamma_1'(0) = \frac{\varphi_1 q_{13}(0)}{2\mu} = \bar{a}^3 Q_o;\quad Q_o = \frac{\varphi_1}{2\mu}\left(\frac{15B^3}{8} + 3B^2 + B\right),\quad \delta_1'(0) = \frac{\varphi_1 q_{14}(0)}{2\mu} = 0. \qquad (5.14a,b,c)$$

The solution of the remaining equation in (5.13a) subject to (5.10b,c) is

$$V_{21}^{(3m)}(t,\tau) = \delta_3^{(m)}\cos\mu_1^{(m)}t + \gamma_3^{(m)}\sin\mu_1^{(m)}t - \varphi_m\left[\frac{q_{12}}{\mu_1^{(m)2}} + \frac{C\left(q_3\cos\mu t + q_4\sin\mu t\right)}{\left(\mu_1^{(m)2} - \mu^2\right)}\right. \qquad (5.15a)$$
$$\left. + \frac{\left(q_{15}\cos 2\mu t + q_{16}\sin 2\mu t\right)}{\left(\mu_1^{(m)2} - 4\mu^2\right)} + \frac{\left(q_{17}\cos 3\mu t + q_{18}\sin 3\mu t\right)}{\left(\mu_1^{(m)2} - 9\mu^2\right)}\right],$$

where $C = 1$ if $m \neq 1$, $C = 0$ if $m = 1$ and

$$\delta_3^{(m)}(0) = \frac{\varphi_m\bar{a}^3 Q_{1m}(\lambda)}{\mu_j^{(m)2}},\quad Q_{1m}(\lambda) = \left[\left(\frac{5B^3}{4} + \frac{9B}{4}\right) - \mu_j^{(m)2}\left\{\frac{C\left(\frac{15B^3}{8} + 3B^2 + B\right)}{\left(\mu_j^{(m)2} - \mu^2\right)}\right.\right.$$

$$\left.\left. - \frac{3\left(B^3 - B\right)}{4\left(\mu_j^{(m)2} - 4\mu^2\right)} + \frac{B^3}{8\left(\mu_j^{(m)2} - 9\mu^2\right)}\right\}\right],\quad \gamma_3^{(m)}(0) = 0 \quad (\forall m \text{ odd}). \qquad (5.15b\text{-}d)$$

Similarly, the solution of (5.12b-d) subject to (5.11b,c) is

$$V_{23}^{(3m)}(t,\tau)=\delta_4^{(m)}\cos\mu_3^{(m)}t+\gamma_4^{(m)}\sin\mu_3^{(m)}t-\Lambda_m\left[\frac{q_{12}}{\mu_3^{(m)2}}+\frac{(q_3\cos\mu t+q_4\sin\mu t)}{(\mu_3^{(m)2}-\mu^2)}\right.$$
$$\left.+\frac{(q_{15}\cos 2\mu t+q_{16}\sin 2\mu t)}{(\mu_3^{(m)2}-4\mu^2)}+\frac{(q_{17}\cos 3\mu t+q_{18}\sin 3\mu t)}{(\mu_3^{(m)2}-9\mu^2)}\right], \quad (5.16a)$$

where

$$\delta_4^{(m)}(0)=\frac{\Lambda_m\bar{a}^3Q_{3m}(\lambda)}{\mu_3^{(m)2}},\quad Q_{3m}(\lambda)=\left[\left(\frac{5B^3}{4}+\frac{9B}{4}\right)-\mu_3^{(m)2}\left\{\frac{\left(\frac{15B^3}{8}+3B^2+B\right)}{(\mu_3^{(m)2}-\mu^2)}\right.\right. \quad (5.16b\text{-}d)$$
$$\left.\left.-\frac{3(B^3-B)}{4(\mu_3^{(m)2}-4\mu^2)}+\frac{B^3}{8(\mu_3^{(m)2}-9\mu^2)}\right\}\right],\quad \gamma_4^{(m)}(0)=0\quad(\forall m \text{ odd}).$$

So far we summarize the asymptotic expression for normal (radial) displacement as

$$V(x,y,t,\tau,\in)=\in V_{21}^{(1)}\sin ny\sin x+\in^3\sum_{m=1,3,5,\ldots}^{\infty}\left(V_{21}^{(3)}\sin ny+V_{23}^{(3)}\sin 3ny\right)\sin mx+0\left(\in^4\right). \quad (5.17)$$

We note that (5.17) is uniformly valid. For the purpose of further analysis we now 'extract' the random imperfection amplitude $\bar{a}$ by performing a Taylor series expansion of each function of τ about $\tau = 0$ in (5.17), using (2.14b), and retaining an adequate number of terms necessary for the next line of analysis. Thus we get

$$V=B\bar{a}\,(1-\cos\mu t)\sin ny\sin x+\bar{a}^3\,\varepsilon^3\left[t\,\gamma_1'(0)\,\sin\mu t\sin ny\sin x+\right.$$
$$\left.\sum_{m=1,3,5,\ldots}^{\infty}\left\{\frac{\varphi_mQ_2(t)\sin ny}{\mu_1^{(m)^2}}+\frac{\Lambda_mQ_3(t)\sin 3ny}{\mu_3^{(m)^2}}\right\}\sin mx\right]+0\left(\varepsilon^4\right) \quad (5.18a)$$

where

$$Q_2(t)=\left[\left(\frac{5B^3}{4}+\frac{9B}{4}\right)(\cos\mu t-1)+\mu_1^{(m)^2}\left\{\frac{C\left(\frac{15B^3}{8}+3B^2+B\right)(\cos\mu t-\cos\mu_1^{(m)}t)}{\mu_1^{(m)^2}-\mu^2}\right.\right.$$
$$\left.\left.-\frac{3(B^3-B)(\cos 2\mu t-\cos\mu_1^{(m)}t)}{4(\mu_3^{(m)^2}-4\mu^2)}+\frac{B^3(\cos 3\mu t-\cos\mu_1^{(m)}t)}{8(\mu_1^{(m)^2}-9\mu^2)}\right\}\right], \quad (5.18b)$$

$$Q_3(t)=\left[\left(\frac{5B^3}{4}+\frac{9B}{4}\right)(\cos\mu_3^{(m)}t-1)+\mu_3^{(m)}\left\{\frac{\left(\frac{15B^3}{8}+3B^2+B\right)(\cos\mu t-\cos\mu_3^{(m)}t)}{(\mu_3^{(m)^2}-\mu^2)}\right.\right.$$
$$\left.\left.-\frac{3(B^3-B)(\cos 2\mu t-\cos\mu_3^{(m)}t)}{4(\mu_3^{(m)^2}-4\mu^2)}+\frac{B^3(\cos 3\mu t-\cos\mu_3^{(m)}t)}{8(\mu_3^{(m)^2}-9\mu^2)}\right\}\right]. \quad (5.18c)$$

Now $\bar{w}(x,y)$ is a random function whose mean $<\bar{w}(x,y)>$ is considered given and this can be written as

$$<\bar{w}(x,y)>=\;<\bar{a}>\sin ny\,\sin x \quad (5.18d)$$

where

$$<\bar{a}>\;=\frac{2}{\pi^2}\int_0^{\pi}\int_0^{2\pi}<\bar{w}(x,y)>\sin ny\sin x\,dydx \quad (5.18e)$$

and where the angular bracket <…> denotes the Mathematical expectation. Again the autocorrelation $R_{\bar{w}}$ of the imperfection function [5,10-12] is given by

$$R_{\bar{w}}\left(\zeta,\eta\right)=<\left\{\bar{w}\left(x,y\right)-<\bar{w}\left(x,y\right)>\right\}\left\{\bar{w}\left(x+\zeta,y+\eta\right)-<\bar{w}\left(x+\zeta,y+\eta\right)>\right\}> \quad (5.19a)$$

We shall however consider a zero-mean Gaussian statistic for $\bar{w}(x,y)$ and so we have

$$<\bar{w}(x,y)>=<\bar{a}>=0 \quad (5.19b)$$

so that

$$R_{\bar{w}}\left(\zeta,\eta\right)=<\bar{w}\left(x,y\right)\bar{w}\left(x+\zeta,y+\eta\right)> \quad (5.19c)$$

The mean square imperfection is $R_{\bar{w}}(0)$ is given by

$$R_{\bar{w}}=<(\bar{w}(x,y))^2>=<\bar{a}^2>\sin^2 ny\sin^2 x \quad (519d)$$

This gives

$$<\bar{a}^2>=\frac{32}{9\pi^2}\int_0^{\pi}\int_0^{2\pi}R_{\bar{w}}\left(0\right)\sin^2 ny\sin^2 x\,dydx \quad (5.19e)$$

We shall, for simplicity of further analysis, assume

$$R_{\bar{w}}=A_{1n}\sin x\sin^2 ny \quad (5.20a)$$

and note that within the range 0 < x < π, we have $R_{\bar{w}}>0$. Here A_{in} are positive constants. Thus we have

$$\pi\,\bar{a}^2\,\phi=\frac{32A_{in}}{9\pi} \quad (5.20b)$$

In the same token [5,8,10-13], the autocorrelation R_w of the normal displacement is

$$R_w\left(\zeta,\eta,t_1,t_2\right)=<w\left(x,y,t_1\right)w\left(x+\zeta,y+\eta,t_2\right)> \quad . \quad (5.21a)$$

The mean square displacement $\Delta^2(x,y,t)$ is obtained by setting $\zeta=\eta=0;\ \ t_1=t_2=t$ and getting

$$\Delta^2\left(x,y,t\right)=R_w\left(0,t\right)=<\left(w(x,y,t)\right)^2> \quad (5.21b)$$

On substituting (5.18a) into (5.21b) and simplifying we get

$$\Delta^2\left(x,y,t\right)=B^2\epsilon^2<\bar{a}^2>\left(1-\cos\mu t\right)^2\sin^2 ny\sin^2 x$$
$$+2B\epsilon^4<\bar{a}^4>\left[t\gamma_1'\left(0\right)\sin\mu t\left(1-\cos\mu t\right)\sin^2 ny\sin^2 x \quad (5.22)\right.$$
$$\left.+\sum_{m=1,3,5,...}^{\infty}\left\{\frac{\varphi_m Q_2(t)\sin ny}{\mu_1^{(m)^2}}+\frac{\Lambda_m Q_3(t)\sin 3ny}{\mu_3^{(m)^2}}\right\}\left(1-\cos\mu t\right)\sin ny\ \sin x\ \sin mx\right]+\Lambda$$

As noted in [8,13], the two terms in powers of ϵ^2 and ϵ^4 in (5.22) shall be adequate. In establishing the initial post dynamic buckling phenomenon associated with these structures. We shall now determine the maximum mean square displacement $\Delta_a^2=\Delta^2\left(x_a,y_a,t_a\right)$ where xa, y_a ,t_a are the critical values of the associated variables at maximum mean square displacement. The conditions for maximum mean square displacement are

$$\Delta^2_{,x}=\Delta^2_{,y}=0. \quad (5.23a,b)$$

The other condition, to order ϵ^2, is

$$\Delta^2_{,t}=0. \quad (5.24)$$

We shall let x_a, y_a, t_a and τ_a be the critical values of the associated variables at maximum displacement and now let

$$t_a=t_o+\epsilon^2 t_2+\ldots \quad (5.25a)$$

From (5.23a,b), we get

$$x_a=\frac{\pi}{2},\qquad y_a=\frac{\pi}{2n}\ . \quad (5.25b,c)$$

From (5.24), we get,

$$t_0=\frac{\pi}{\mu}\ . \quad (5.25d)$$

We know [8,13,14] that

$$<\bar{a}^4>=3<\bar{a}^2>^2=\frac{1024A_{1n}^2}{27\pi^2}. \quad (5.25e)$$

The maximum normal displacement V_a from (5.17) is

$$V_a = 2\,\bar{a}\,B\in - \in^3 \bar{a}^3 \sum_{m=1,3,5\Lambda}^{\infty} \Big[\left(BQ_{21} + B^2Q_{22} + B^3 Q_{23}\right) - \frac{\varphi_1}{2\mu}\left(15B^3 + 3B^2 + B\right)$$

$$- \left(BQ_{31} + B^2Q_{32} + B^3Q_{33}\right) \Big] \sin\frac{m\pi}{2} + 0\left(\in^4\right), \tag{5.26a}$$

where

$$Q_{21} = \frac{9}{2} + \frac{\mu_1^{(m)^2} c\left(1+\cos\mu_1^{(m)}t_0\right)}{\mu_3^{(m)^2} - \mu^2} - \frac{3\left(1-\cos\mu_1^{(m)}t_0\right)}{4(\mu_1^{(m)^2} - 4\mu^2)};\; Q_{22} = \frac{3\mu_1^{(m)^2} c\left(1+\cos\mu_1^{(m)}t_0\right)}{\mu_1^{(m)^2} - \mu^2} \text{(54b)} \tag{5.26b}$$

$$Q_{23} = \frac{5}{2} + \mu_1^{(m)^2}\left\{\frac{15c\left(1+\cos\mu_1^{(m)}t_0\right)}{8\left(\mu_1^{(m)^2} - \mu^2\right)} + \frac{3\left(1-\cos\mu_1^{(m)}t_0\right)}{4\left(\mu_1^{(m)^2} - 4\mu^2\right)} + \frac{\left(1+\cos\mu_1^{(m)^1}t_0\right)}{8\left(\mu_1^{(m)^2} - 9\mu^2\right)}\right\} \tag{5.26c}$$

$$Q_{31} = \frac{9}{4}\left(\cos\mu_3^{(m)}t_0 - 1\right) - \frac{\mu_3^{(m)^2}\left(1+\cos\mu_3^{(m)}t_0\right)}{\mu_3^{(m)^2} - \mu^2} - \frac{3\left(1-\cos\mu_3^{(m)}t_0\right)}{4(\mu_3^{(m)^2} - \mu^2)} \tag{5.26d}$$

$$Q_{32} = \frac{3\mu_3^{(m)^2}\left(1+\cos\mu_3^{(m)^2}t_0\right)}{\mu_3^{(m)^2} - \mu^2} \tag{5.26e}$$

$$Q_{33} = \left[\frac{5}{4}\left(\cos\mu_3^{(m)}t_0 - 1\right) - \mu_3^{(m)^2}\left\{\frac{15\left(1+\cos\mu_3^{(m)}t_0\right)}{8\left(\mu_3^{(m)^2} - \mu^2\right)} + \frac{3\left(1-\cos\mu_3^{(m)}t_0\right)}{4\left(\mu_3^{(m)^2} - 4\mu^2\right)} + \frac{\left(1+\cos\mu_3^{(m)}t_0\right)}{8\left(\mu_3^{(m)^2} - 9\mu^2\right)}\right\}\right]. \tag{5.26f}$$

In the same token the maximum mean square normal displacement $\Delta_a^2 \equiv \Delta^2\left(x_a, y_a, t_a, \tau_a\right)$ is, using (5.22) and (5.25b-d),

$$\Delta_a^2 = \frac{128B^2A_{1n}\in^2}{9\pi} + \frac{2048A_{1n}^2\in^4 B}{27\pi^2}\sum_{m=1,3,5\ldots}^{\infty}\left(B^3Q_4^{(m)} + B^2Q_5^{(m)} + BQ_6^{(m)}\right)\sin\frac{m\pi}{2} + \Lambda \tag{5.27a}$$

where

$$Q_4^{(m)} = \frac{15\varphi_1 t_0 \delta_{1m}}{16\mu} - \frac{\varphi_m Q_{23}}{\mu_1^{(m)^2}} + \frac{\Lambda_m Q_{33}}{\mu_3^{(m)^2}} \tag{5.27b}$$

$$Q_5^{(m)} = \frac{3\varphi_1 t_0 \delta_{1m}}{2\mu} - \frac{\varphi_m Q_{22}}{\mu_1^{(m)^2}} + \frac{\Lambda_m Q_{32}}{\mu_3^{(m)^2}} \tag{5.27c}$$

$$Q_6^{(m)} = \frac{\varphi_1 t_0 \delta_{1m}}{2\mu} - \frac{\varphi_m Q_4}{\mu_1^{(m)^2}} + \frac{\Lambda_m Q_{32}}{\mu_3^{(m)^2}} \tag{5.27d}$$

where, δ_{1m} is the Dirac-delta function. We note that each of B, $Q_4^{(m)}, Q_5^{(m)}$ and $Q_6^{(m)}$ depends on the load parameter λ. We shall however rewrite (5.27a) as

$$\Delta_a^2 = \in^2 c_1 + \in^4 c_3 + \Lambda \tag{5.28}.$$

Where c_1 and c_3 are the obvious respective coefficients of $\in^2$ and $\in^4$ as in (5.27a) and are thus dependent on λ through $Q_4^{(m)}, Q_5^{(m)}, Q_6^{(m)}$ and B. The dynamic buckling load λ_D [8,13] is usually obtained from the maximization

$$\frac{d\lambda}{d\Delta_a^2} = 0 \tag{5.29}$$

which is determined at $\lambda = \lambda_D$. However, before invoking the maximization in (5.29), we first [8,13] have to reverse the series (5.27a, 5.28) by expressing the least order of $\in$ in the following series

$$\in^2 = \Delta_a^2 d_1 + \left(\Delta_a^2\right)^2 d_3 + \Lambda \tag{5.30a}$$

where

$$d_1 = \frac{1}{c_1}, \quad d_2 = -\frac{c_3}{c_1^3}. \tag{5.30b}$$

Each d_i and c_i (i =1,3) is a function of λ_D. The maximization (5.29) is now easily executed through (5.30a) to yield, after some simplification

$$\in^2 = \frac{c_1}{4c_3} \qquad (5.31)$$

which is evaluated at $\lambda=\lambda_D$. On further simplifying (5.31), we get

$$\frac{64A_{in}}{3\pi}\in^2 \sum_{m=1,3,5,...}^{\infty}\left(B^2Q_4^{(m)} + BQ_5^{(m)} + Q_6^{(m)}\right)\sin\frac{m\pi}{2} = 1. \qquad (5.32)$$

Equation (5.32) is understood to be evaluated at $\lambda=\lambda_D$

6.0 Analysis of the result

The result (5.32) is asymptotic in nature and is valid for small values of $\in$. Each of the terms in (5.32) is dependent on the load parameter λ_D and the specific value of λ_D that satisfies (5.32) for each value of $\in$ is the dynamic buckling load. We expect the dominant term to come from the case where m = 1 and for this we have

$$\frac{64A_{in}}{3\pi}\varepsilon^2\left(B^2Q_4^{(1)} + BQ_5^{(1)} + Q_6^{(1)}\right) = 1 \qquad (6.1)$$

where $Q_4^{(1)}, Q_5^{(1)}$ and $Q_6^{(1)}$ are the values of $Q_4^{(m)}, Q_5^{(m)}$ and $Q_6^{(m)}$ respectively at m = 1. On simplifying (6.1) further we get

$$\frac{64A_{1n}}{3\pi}\in^2\left[\left(\frac{\lambda_D\left\{\frac{\alpha}{2}+n^2\xi\right\}}{(\mu(\lambda_D))^2}\right)^2 Q_4^{(1)}(\lambda_D)+\left(\frac{\lambda_D\left\{\frac{\alpha}{2}+n^2\xi\right\}}{(\mu(\lambda_D))^2}\right)Q_5^{(1)}(\lambda_D)+Q_6^{(1)}(\lambda_D)\right] = 1 \qquad (6.2)$$

as the equation to be satisfied by λ_D. For convenience we may set A_{1n} = 1. Since Donnell-type of equations are used, it is necessary that n > 5.

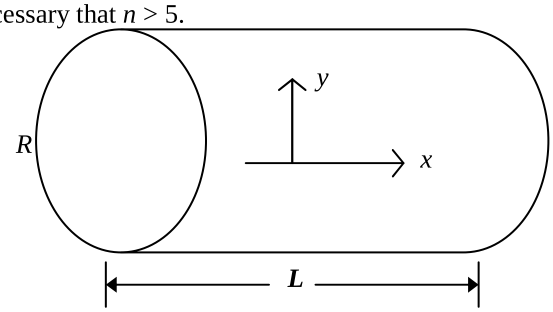
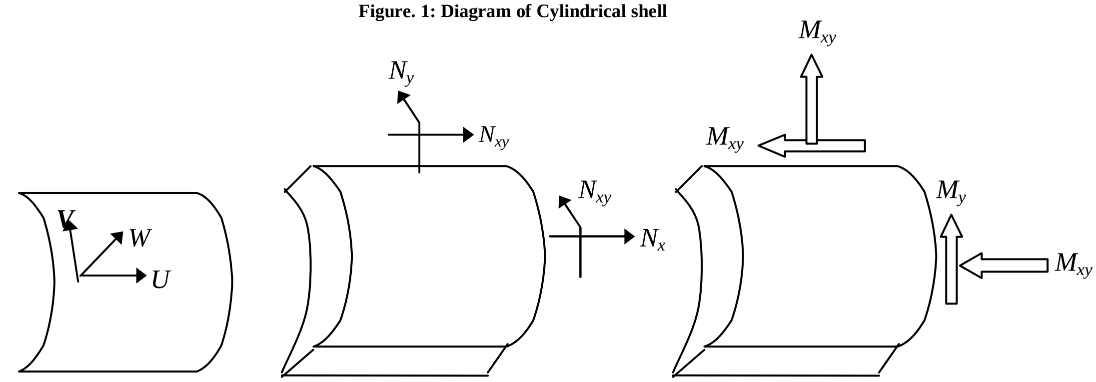


Figure. 1: Diagram of Cylindrical shell



(a) Displacements (b) Stress Resultant (c) Stress Couples

Figure 2: Cylindrical shells showing components of (a) Displacements (b) Stress Resultants and (c) Stress couples

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