

*Collisional effect on lower hybrid waves instability in a dusty plasma*

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**Abstract**

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**The effect of particle collisions on lower hybrid modes in a dusty plasma is studied. The dispersion relation derived from fluid theory is numerically solved for plasma parameters relevant to determine the modification in wave propagation due to collisions. This study is relevant to the earth's lower atmosphere, in particular, the mesosphere, where charged dusts and excitation of low frequency waves have been observed**

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**1.0 Introduction**

A dusty plasma is a normal electron-ion plasma with an additional charged component of micro-sized or sub-micro-sized particulates. This extra component of macro particles increases the complexity of the system even further. Dust grains are massive compared to the sizes of electrons and ions. Dusty plasmas are rather ubiquitous in space. They are found in interstellar clouds, solar system, earth's lower atmosphere, etc. [1]. Dust grains may be metallic, conducting, or made of ice particulates. The presence of charged dusts grains modify the existing low frequency waves found in normal plasma, such as Ion acoustic waves, lower hybrid waves, ion waves, etc [2].

Electron streams flowing in magnetized plasmas are a common occurrence in terrestrial and astronomical plasmas, for example, the earth's ionosphere and magnetosphere. These streams have been explained to be responsible for excitation of lower hybrid waves (LHW) in the vicinity of auroral zone [3]. In the lower atmosphere called the mesosphere which is about 80 to 90 km moving charged dust particles known as 'noctilucent clouds' (NLCs) have been observed where there also occur another phenomenon which is referred to as 'polar mesospheric summer echoes' (PMSE). PMSE is thought to be caused by the presence of NLC dusts, through a two-stream instability process [4]. Our aim is to demonstrate that the influence of dust particle collisions is important when computing waves instabilities in lower regions of earth's atmosphere.

**2.0 Theory**

We consider a three-component plasma consisting of electrons, singly charged ions and massive, negatively charged dust particle. The plasma is embedded in a uniform and constant magnetic field  $B$ , directed along the z-axis. An electric field  $E = \nabla\phi$  is also present in the plasma, with  $\phi$  being the electric potential. The magneto-hydrodynamic (MHD) equation describing the behaviour of the three charged

components are the continuity and momentum equations written as: 
$$\frac{\partial n}{\partial t} + \nabla(n_j v_j) = 0 \quad (j=i, e, d)$$

(2.1)

$$T_e \nabla n_e + en_e E + en_e v_e \cdot xB = -v_e n_e m_e v_e \quad (2.2)$$

$$T_i \nabla n_i - en_i E + en_i v_i \cdot xB = -v_i n_i m_i v_i \quad (2.3)$$

$$n_d m_d \frac{\partial v_d}{\partial t} + n_d m_d v_d \cdot \nabla v_d + e Z_d n_d E + e Z_d n_d v_d \cdot xB = -v_d n_d m_d v_d \quad (2.4)$$

We have assumed that the inertia terms for the ions and electrons are negligible compared to that of the much heavier dust. Here,  $n$ ,  $m$ ,  $v$  and  $\nu$  stand for particle number, mass, velocity and collision frequency, respectively. The subscripts i.e  $d$  indicate ion electron and dust respectively. Isothermal conditions are assumed for the ion and electrons. Charge neutrality at equilibrium requires that

$$n_0 = n_{i0} = n_{e0} + Z_d n_d \quad (2.5)$$

The relationship between species velocities and electric field are:

$$\left. \begin{aligned} eE &= -v_e m_e v_{ez} \\ eE &= v_i m_i v_{iz} \\ eZ_d E &= -v_d m_d v_{dz} \end{aligned} \right\} \quad (2.6)$$

Equations (2.1) – (2.6) are solved by linearization, with first order quantities taken to have space and time

dependence,  $e^{i(k_{\perp} r_{\perp} + k_z Z - \omega t)}$ . The desired dispersion relation for dust lower hybrid mode is found as [5].

$$i \left[ (1 - \delta_d Z_d) \frac{A}{m_e (\omega - K_z v_{ez} + i C_e^2 A)} + \frac{B}{m_i (\omega - k_z v_{iz} + i C_i^2 B)} \right] - \frac{\delta_d Z_d^2 F}{m_d (\omega - k_z v_{dz})} = 0 \quad (2.7)$$

where  $\delta = n_d / n_i$ ,  $\omega_c = eB / m$  is the cyclotron frequency,  $C = (T/m^2)^{1/2}$ , the thermal speed, and:

$$\begin{aligned} A &= \frac{(k_{\perp}^2 + k_z^2) v_e^2 + k_z^2 \omega_{ce}^2}{(v_e^2 + \omega_{ce}^2) v_e} \\ B &= \frac{(k_{\perp}^2 + k_z^2) (\omega - k_z v_{iz} + i v_i)^2 - k_z^2 \omega_{ci}^2}{((\omega - k_z v_{iz} + i v_i)^2 - \omega_{ci}^2) (\omega - k_z v_{iz} + i v_i)} \\ F &= \frac{(k_{\perp}^2 + k_z^2) (\omega - k_z v_{dz} + i v_d)^2 - k_z^2 \omega_{cd}^2}{((\omega - k_z v_{dz} + i v_d)^2 - \omega_{cd}^2) (\omega - k_z v_{dz} + i v_d)} \end{aligned} \quad (2.8)$$

$\omega$  is the mode frequency which has a real part,  $\omega_r$ , and an imaginary growth rate,  $\gamma$ .

### 3.0 Numerical results

The dispersion relation (2.7) is numerically solved for parameters corresponding to the earth's mesosphere. The mesosphere is the region where excitation of waves in the form of PMSE and presence of charged dust called the NLC have been observed. The region is between 70 - 90km from the earth and has the following average plasma parameters for  $O^-$  [6]:  $T_e = T_i = .1eV$ ,  $n_e = 10^3 cm^{-3}$ ,  $B = .5G$ ,  $m_d = 10^{12} m_i$ ,  $\delta = 10^{-4}$ ,  $v_e = 2 \times 10^5 \omega_e$ ,  $v_i = 7 \times 10^3 \omega_i$ ,  $Z_d = 7 \times 10^4$ ,  $\lambda_z = 200m$ , and  $\lambda_{\perp} = 11m$ .

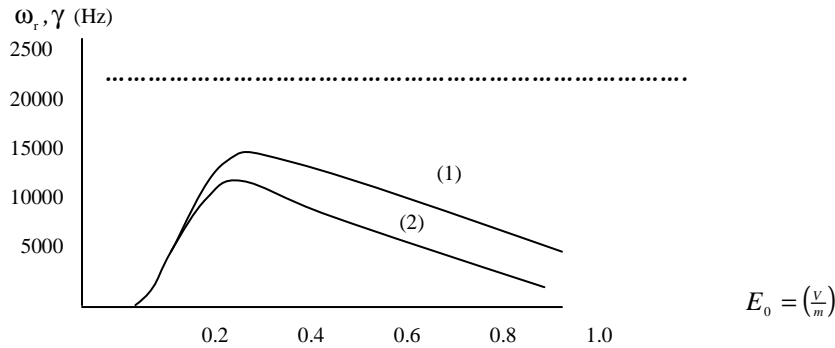


Figure 1: Real frequency,  $\omega_r$  (dotted lines) and growth rates,  $\gamma$ , (solid lines) as functions of electric field: (1) growth rate when  $v_d = 0$  and (2) growth rate when  $v_d \sim 10^{-4} \omega_{ci}$

Figure 1 consists the graphs of the excited frequency mode and the growth rates. Graph (1) is the case where dust collisions have been neglected, ( $v_d = 0$ ), in the computation of the growth rate. In Graph (2), the ion and electron collisions are the same as in Graph (1) while in addition the collision frequency of the dusts particle ( $v_d = 10^{-4} \omega_{ci}$ ) is included. It is seen that the growth rate of the excited mode has reduced as a result of dust collisions. The dotted line is the frequency mode, which is the dust lower hybrid frequency.

### 4.0 Conclusion

In this paper we have investigated the effect of dust-dust collisions on the growth rates of dust lower hybrid wave. The three plasma components, which are electron, ion and negatively charged dust, interact to excite a dust lower hybrid wave. It is shown that inclusion of collision

frequency of charged dust that is usually neglected can affect the calculated growth rate. The data used for the earthly mesosphere shows that the growth is reduced, that is, dust collision has a stabilizing effect.

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