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The critical role associated with beach slope and its width in evolution of swell near the shoreline

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Abstract

Using perturbation method, the shallow water wave equation is investigated. We are, however, interested in the case in which the incident wave train propagate in the radial direction towards the shoreline. This is rather more general than the case in which the trains of progressive waves propagate strictly in x–direction. The essential part of this study is the determination of the critical role associated with the width of the shelf and the beach gradient in relation to the transformation of the beach waves. There from, it is deduced that the wave energy is an increasing function of the beach bottom gradient and the shelf width. The later should, however, be finite.

Further, the nonlinear interactions between the wave trains and the resulting excitations of the seabed are also discussed.

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1.0 Introduction

This study concerns the evolutions of a train of the nearly uniform progressive waves moving towards the shoreline obliquely. Incidentally, the shoreline makes an angle with mean direction of this approaching wave train. The crests of the reflected waves will not, therefore, be parallel to those of the incident wave trains. The development is therefore in accordance with the behaviour of diffracted waves in which the angles of incidence and reflection are generally equal. Traditionally, the appropriate framework for the formulation of the above problem is the cylindrical polar coordinate systems. The use of the polar coordinate system in describing the evolutions of the long crested shallow water waves with curved profile is quite universal in the field of dynamical oceanography. For example, this approach provides a realistic method of modeling the problems of wave actions on structures that are cylindrical in shape. The cases in which the structures were vertical surface – piercing circular cylinders were considered by Hunt and Williams [1], Zohu and Liu [2]). In these considerations, wave forces and turning moments on the structure were efficiently calculated.

However, what concerns us in this study is the intense wave activities near the shoreline which are associated with the long crested wave trains approaching the shoreline obliquely. The roles associated with the sea bottom gradient, α, and width of the shallow water, *d*, will be analysed in this consideration. The width d in this context is the distance measured seaward from the shoreline over which an incoming swell is associated with appreciable bottom pressure. Appreciable, in that the bottom pressure induced by the wave should be active enough to excite micro-seismic disturbances detectable even in the far field, Asor [3].

We shall confine our discussion to cases relating to swell with mean wavelength of about 450m propagating in the beach with average depth of 4m. Thus, the ratio 4/450 is quite small and the shallow water theory can realistically describe the evolutions of this swell in the shallow water zone. In these considerations, previous calculations made by Okeke [4], Okeke and Asor [5] were quite close to the observed data. Thus, the use of full Euler's equations of hydrodynamics will not improve the calculation quite significantly.

The nonlinear shallow water wave equation had been intensively used in the description of a number of beach phenomena. Because, most of the previous models concerned waves that were approaching the shorelines at right angles, it is the objective of this study to generalize previous studies [6, 7], which concern the transformations of swell on a beach; and thus, incorporate the more realistic case of oblique incidence in the model.

2.0 Specifications

In the polar coordinate system (R, Z, θ), the origin is on the shoreline, R is a measure of radial distance seaward but from the origin, Z–axis points negatively towards the body of the water with θ as the angle measured from the normal direction to the shoreline. $z = \eta(R,\theta,t)$ and $Z = -h(R)$ are the equations of the surface wave profile and the sea bottom topography respectively. ρ and g are the constant water density and gravity acceleration respectively.

3.0 The basic equations governing the evolution of the long-crested beach waves

q_R and q_θ are fluid particle velocity components in radial and transverse directions respectively for the motion associated with long-crested beach waves. Thus, the usual related equations of hydrodynamics in the polar coordinate system are

$$\frac{\partial q_R}{\partial t} + q_R \frac{\partial q_R}{\partial R} + \frac{q_\theta}{R}\frac{\partial q_R}{\partial \theta} = -g\frac{\partial \eta}{\partial R} \quad \textbf{(3.1)}$$

$$\frac{\partial q_\theta}{\partial \theta} + q_R \frac{\partial q_\theta}{\partial R} + \frac{q_\theta}{R}\frac{\partial q_\theta}{\partial \theta} = -{}^{g}\!/\!_{R}\frac{\partial \eta}{\partial \theta} \quad \textbf{(3.2)}$$

$$\frac{\partial}{\partial R}\left[R\, q_R(\eta + h)\right] + \frac{\partial}{\partial \theta}\left[q_\theta(\eta + h)\right] = R\frac{\partial \eta}{\partial t} \quad \textbf{(3.3)}$$

This model will describe the transformations of the old and smooth streamlined swell on a beach. Thus, the parameter, $\in$, which is the ratio of the vertical extent of the fluid to the related wavelength will be significantly small. Using the Stokes expansion as applied to variables in (3.1) to (3.3) in terms of the parameter $\in$, then,

$$q_R = \in q_R^{(1)} + \in^2 q_R^{(2)} + 0(\in^3) \tag{3.4}$$

$$q_\theta = \in q_\theta^{(1)} + \in^2 q_\theta^{(2)} + 0(\in^3) \tag{3.5}$$

$$\eta = \in \eta^{(1)} + \in^2 \eta^{(2)} + 0(\in^3) \tag{3.6}$$

Thus, equations of order $\in$ are

$$\frac{\partial}{\partial R}\left[R\,h\frac{\partial}{\partial R}\eta^{(1)}\right] + \frac{\partial}{\partial\theta}\left[\frac{h}{R}\frac{\partial\eta^{(2)}}{\partial\theta}\right] = -\frac{h}{R}\frac{\partial^2\eta^{(1)}}{\partial t^2} \tag{3.7}$$

$$\frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial}{\partial R}(R\,h\,q_R^{(1)})\right] + \frac{\partial}{\partial\theta}\left[\frac{1}{R}\frac{\partial}{\partial R}(h\,q_\theta^{(1)})\right] = -\frac{1}{g}\frac{\partial^2 q_R^{(1)}}{\partial t^2} \tag{3.8}$$

Following Okeke (1972), (3.7) has general solution

$$\eta_o^{(1)}(R) = \left({}^d\!/_R\right)^{1/2} J_o(\beta R^{1/2}) \tag{3.9}$$

$$\beta = 2\,\omega_o(\alpha\,g)^{-1/2} \tag{3.10}$$

$J_o(\beta R^{1/2})$ **is the zero order Bessel function of the first kind. ω_o is the frequency of the dominant wave train; hence**

$$\eta^{(1)} = Re\left\{\eta_o^{(1)}(R)\,cos({}^\theta\!/_2)\,exp\,(i\omega_o t)\right\} \tag{3.11}$$

We now introduce the scalar potential $\phi(R,\theta,t)$ expressed by

$$q_R = -\frac{\partial\phi^{(1)}}{\partial R},\ \ q_\theta = -\frac{1}{R}\frac{\partial\phi^{(1)}}{\partial\theta} \tag{3.12}$$

Insert (3.12) into (3.8) and integrate, then

$$\frac{\partial}{\partial R}\left(R\,h\frac{\partial\phi^{(1)}}{\partial R}\right) + \frac{h}{R}\frac{\partial^2\phi^{(1)}}{\partial\theta^2} = -\frac{h}{g}\frac{\partial^2\phi^{(1)}}{\partial t^2} \tag{3.13}$$

Introducing $h = \alpha R$ in (3.13)

$$\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\,\phi^{(1)}}{\partial R}\right)+\frac{\partial^2\phi^{(1)}}{\partial\theta^2}=-{}^{R}\!/_{\alpha g}\,\frac{\partial^2\phi^{(1)}}{\partial\,t^2} \tag{3.14}$$

Let $$\phi^{(1)}(R,\theta,t)=Re\left[\phi_o^{(1)}(R)\sin\left(\frac{\theta}{2}\right)e^{i\omega_0 t}\right] \text{ following from Equation (3.9)} \tag{3.15}$$

Using (3.14) with (3.15), then,

$$R^2\,\frac{d^2\phi_o^{(1)}}{dR^2}+2\,R\,\frac{d\phi_o^{(1)}}{dR^2}+\left(\frac{R\omega_o^2}{\alpha\,g}-m^2\right)\phi_o^{(1)}=0 \tag{3.16}$$

This gives ***m*** **= ½ and consequently, the same equation for** $\eta_o^{(1)}(R)$**; thus if c is the wave speed, the solution of (3.16) gives**

$$\phi_o^{(1)}(R)=\frac{g}{c}\left(\frac{d}{R}\right)^{1/2}J_o(\beta\,R^{1/2}) \tag{3.17}$$

$$\phi^{(1)}(R,\theta,t)=Re\left\{\phi_o^{(1)}(R)\;e^{i(\alpha t+m\theta)}\right\} \tag{3.18}$$

4.0 **Second Order wave profile** $\eta^{(2)}(R,\theta,t)$

Collecting Term in $\in^2$ from (3.1) to (3.3) and using the expansions (3.4) to (3.6), then

$$\frac{\partial}{\partial\,t}q_R^{(2)}+q_R^{(1)}\,\frac{\partial\,q_R}{\partial R}+\frac{q_\theta^{(1)}}{R}\,\frac{\partial}{\partial\,\theta}q_R^{(1)}=-g\,\frac{\partial\eta^{(2)}}{\partial R} \tag{4.1}$$

$$\frac{\partial}{\partial\,t}q_\theta^{(2)}+q_R^{(1)}\,\frac{\partial}{\partial R}q_\theta^{(1)}+\frac{q_\theta^{(1)}}{R}\,\frac{\partial}{\partial\theta}q_\theta^{(1)}=-{}^{g}\!/_{R}\,\frac{\partial\eta^{(2)}}{\partial\theta} \tag{4.2}$$

$$\frac{\partial}{\partial R}\left[R\left\{q_R^{(1)}\eta^{(1)}+q_R^{(2)}h\right\}\right]+\frac{\partial}{\partial\theta}\left[q_\theta^{(1)}\eta^{(1)}+h\,q_\theta^{(2)}\right]=R\,\frac{\partial}{\partial\,t}\eta^{(2)} \tag{4.3}$$

Use (4.1) and (4.2) to eliminate $q_R^{(2)}$ and $q_\theta^{(2)}$ from (4.3), then

$$\frac{\partial}{\partial R}\left[Rh\left(g\frac{\partial\eta^{(2)}}{\partial R}+q_R^{(1)}\frac{\partial}{\partial R}q_R^{(1)}+\frac{q_\theta^{(1)}}{R}\frac{\partial q_R^{(1)}}{\partial\theta}\right)\right]+$$

$$\frac{\partial}{\partial\theta}\left[h\left(\frac{g}{R}\frac{\partial}{\partial\theta}\eta^{(2)}+q_R^{(1)}\frac{\partial q_\theta^{(1)}}{\partial R}+\frac{q_\theta^{(1)}}{R}\frac{\partial q_\theta^{(1)}}{\partial\theta}\right)\right]-\frac{\partial^2}{\partial t\partial R}\left(R\eta^{(1)}q_R^{(1)}\right)-\frac{\partial^2}{\partial t\partial\theta}\left(\eta^{(1)}q_\theta^{(1)}\right) \qquad (4.4)$$

$$=-R\frac{\partial^2\eta^{(2)}}{\partial t^2}$$

Re-arranging, and introduce $h(R) = \alpha R$,

$$\frac{\partial}{\partial R}\left(R^2\frac{\partial\eta^{(2)}}{\partial R}\right)+\frac{\partial^2}{\partial\theta^2}\eta^{(2)}+\frac{R}{\alpha g}\frac{\partial^2\eta^{(2)}}{\partial t^2}=-\frac{1}{g}\left[\frac{\partial}{\partial R}\left(R^2q_R^{(1)}\frac{\partial q_R^{(1)}}{\partial R}\right)+\right.$$

$$\left.\frac{\partial}{\partial R}\left(Rq_\theta^{(1)}\frac{\partial q_\theta^{(1)}}{\partial\theta}\right)+\frac{\partial}{\partial\theta}\left(Rq_R^{(1)}\frac{\partial q_\theta^{(1)}}{\partial R}\right)+\frac{\partial}{\partial\theta}\left(q_\theta^{(1)}\frac{\partial}{\partial\theta}q_\theta^{(1)}\right)\right]+$$

$$\frac{1}{\alpha}\left[\frac{\partial^2}{\partial t\partial R}\left(R\eta^{(1)}q_R^{(1)}\right)+\frac{\partial^2}{\partial t\partial\theta}\left(\eta^{(1)}q_\theta^{(1)}\right)\right]=-\frac{1}{g}\left[\left\{\frac{\partial}{\partial R}\left(R^2\frac{\partial\phi^{(1)}}{\partial R}\frac{\partial^2\phi^{(1)}}{\partial R^2}\right)\right.\right. \qquad (4.5)$$

$$\left.+\frac{\partial}{\partial R}\left(\frac{\partial\phi^{(1)}}{\partial\theta}\frac{\partial^2\phi^{(1)}}{\partial R\partial\theta^2}\right)+\frac{\partial}{\partial\theta}\left(R\frac{\partial\phi^{(1)}}{\partial R}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\phi^{(1)}}{\partial\theta}\right)+\frac{\partial}{\partial\vartheta}\left(\frac{1}{R^2}\frac{\partial\phi^{(1)}}{\partial\theta}\frac{\partial^2\phi^{(1)}}{\partial\theta^2}\right)\right)\right]$$

$$+\frac{1}{\alpha}\left[\frac{\partial^2}{\partial t\partial R}\left(R\eta^{(1)}\frac{\partial\phi^{(1)}}{\partial R}\right)+\frac{\partial^2}{\partial t\partial\theta}\left(\frac{\eta^{(1)}}{R}\frac{\partial\phi^{(1)}}{\partial\theta}\right)\right]=E$$

Using (3.12), let $$\eta^{(2)}(R,\theta,t)=Re\left\{\eta_{o(R)}^{(2)}\lambda^{2i(\omega_0 t+\frac{\theta}{4})}\right\} \qquad (4.6)$$

Again, using (3.5) and (4.6), we obtain

$$R^2\frac{\partial\eta_o^{(2)}}{\partial R^2}+2R\frac{\partial\eta_o^{(2)}}{\partial R}-4(m^2+\frac{\omega^2R}{\alpha g})\eta_o^{(2)}=E \qquad (4.7)$$

m = ½ as before; and

$$E=-\frac{1}{g}\left\{R^2\frac{\partial\ \phi_o^{(1)}}{\partial R}\ \frac{\partial^3\phi}{\partial R^3}+R^2\left(\frac{\partial^2\phi_o^{(1)}}{\partial R^2}\right)^2+2R\frac{\partial\phi_o^{(1)}}{\partial R}\frac{\partial^2\phi_o^{(1)}}{\partial R^2}-m^2\left(\phi_o^{(1)}\frac{\partial^2\phi_o^{(1)}}{\partial R^2}+\frac{\phi_o^{(1)}}{R}\left(\frac{\partial\phi_o^{(1)}}{\partial R}\right)+\left(\frac{\phi^{(1)}}{R}\right)^2\right\}$$

$$-\frac{1\omega}{\alpha}\left[\eta_o^{(1)}\frac{\partial\phi_o^{(1)}}{\partial R}+R\frac{\partial\eta_o^{(1)}}{\partial R}\frac{\partial\phi}{\partial R}+R\eta_o^{(1)}\frac{\partial^2\phi_o^{(1)}}{\partial R^2}-m^2\left(\eta_o^{(1)}\frac{\phi^{(1)}}{R}\right)\right] \qquad (4.8)$$

Linear approximation gives $\phi_o^{(1)}=\frac{gk}{\omega_o}\eta_o^{(1)}$ as in (3.18) where k is the wave number and consequently,

$$Re\left\{E(\eta_o^{(1)})\right\}=-\frac{R^2gk^2}{\omega_o{}^2}\left[\frac{3}{16}\frac{\eta_o^{(1)2}}{R^4}-\frac{15}{4R^3}\eta_o^{(1)}\eta_{(o)}^{(1)}+\frac{\eta_0^{(1)}\eta_0^{(1)\prime}}{R^2}-\frac{1}{2}\frac{\eta_0^{(1)}\eta_0^{(1)\prime\prime}}{R}-\frac{3}{2R}\eta_0^{(1)\prime}\eta_0^{(1)\prime\prime}+\eta_0^{(1)\prime}\eta_0^{(1)\prime\prime}\left(\frac{\eta_0^{(1)\prime}}{R}\right)^2\right]$$

(4.9)

where primes, here, indicate derivatives with respect to R. From (3.9), take

$$\eta_o^{(2)}(R)=\eta_{oo}^{(2)}(R)\left(\frac{d}{R}\right)^{\frac{1}{2}} \qquad (4.10)$$

The simplification of (4.9) using (4.10) will involve the derivatives of $J_o(BR^{\frac{1}{2}})$. We shall use the following to eliminate these derivatives

$J_o'(x) = -J_1(x),\ J_o''(x) = \frac{J_1(x)}{x} - J_o(x),\ J_o'''(x) = \frac{6}{x^2} J_1(x) - (\tfrac{1}{4} + \tfrac{3}{x}) J_o(x))$ where primes indicate derivatives with respect to *x*. Thus, using solution (3.9), then,

$$Re\{E(\eta_o^{(1)}\} = Re\{E(R)\}$$

$$= -\frac{R\,g\,d}{\omega^2}\left[A_1(R)J_o^2(\beta R^{1/2}) + A_2(R)J_1(\beta R^{1/2})J_o(\beta R^{1/2}) + A_3(R)J_1^2(\beta R^{1/2})\right] \qquad (4.11)$$

$$A_1(R) = \frac{3}{16}R^{-4} - \frac{\beta^2 R^{-3}}{16} + \frac{R^{-5/2}\beta^3}{32},\quad A_2(R) \equiv 2BR^{-7/2} + \frac{9}{16}\beta^3 R^{-5/2} + \frac{\beta^4 R^{-2}}{32},\quad A_3(R) = \frac{\beta^2 R^{-3}}{4}$$

Using (4.10), (4.7) takes the final form when m = ½

$$R^2\eta_o^{(2)''} + R\eta_o^{(2)'} + \frac{4\omega^2}{\alpha g}\eta_o^{(2)} = Re\{E(R)\} \qquad (4.12)$$

(4.12) can be integrated using the method of the variation of parameters. Thus, the general solution is given by

$$\eta_{oo}^{(2)} = A_o J_o(2\beta R^{1/2}) + B_o Y_o(2\beta R^{1/2}) + Y_o(2\beta R^{1/2})\int^R \frac{J_o(2\beta\sigma^{1/2})Re\{E(\sigma)\}}{w(\sigma)}d\sigma$$
$$- J_o(2\beta R^{1/2})\int^R \frac{Y_o(2\beta\sigma^{1/2})Re\{E(\sigma)\}}{w(\sigma)}d\sigma \qquad (4.13)$$

$$W(R) = W[J_o(2\beta R^{1/2}),\ Y_o(\beta R^{1/2})] = \frac{2}{\Pi\,\beta R^{1/2}} \qquad (4.14)$$

A_o and B_o are arbitrary constants, W stands for Wronskian,determinant. Finally,

$$\eta_o^{(2)}(R) = \sqrt{\frac{d}{R}}\eta_{oo}^{(2)}(R) = \sqrt{\frac{d}{R}}\left[A_o J_o(2\beta R^{1/2}) + B_o Y_o(2\beta R^{1/2})\right] + \frac{\Pi\,\beta\sqrt{d}}{2}\left[Y_o(2\beta R^{1/2})C_1 + C_2 J_o(2\beta R^{1/2})\right]$$

$$C_1(R) = \int^R \frac{J_o(2\beta\sigma^{1/2})Re\{E(\sigma)\}}{\sigma^2}d\sigma,\ C_2(R) = \int^R \frac{Y_o(2\beta\sigma^{1/2})Re\{E(\sigma)\}}{\sigma^2}d\sigma \qquad (4.15)$$

In the subsequent numerical calculations, we utilize the following relation, which may be derived using any textbook on Bessel functions.

$$J_o^2(2\beta R^{1/2}) = \sum_{n=o}^{\infty}\frac{(-1)^n(2n)!\,(\beta)^{2n}R^n}{(n!)^4},\quad J_1^2(2\beta R^{1/2}) = \sum_{n=o}^{\infty}\frac{(-1)^n(2n+2)!\,(\beta)^{2n+2}R^{n+1}}{[(n+1)!]^2(n+2)!\,n!}$$

$$J_1(2\beta R^{1/2})\ J_o(2\beta R^{1/2}) = \sum_{n=o}^{\infty}\frac{(-1)^n(2n+1)!\ \beta^{2n+1}R^{\frac{n+1}{2}}}{[(n+1)!]^2(n!)^2}$$

5.0 Discussion

It has been mentioned earlier that the motivation for this analysis is to investigate the critical role associated with beach slope and its width in evolution of swell near the shoreline. In normal beach zone, swell activities before breaking are generally governed by the shallow water equations if the weather is not stormy in the beach locality. Consequently, we shall utilize the wave periods associated with shallow water swell in the subsequent numerical calculation. The width *d* of the beach can be estimated as a function of wavelength using

the relationship, $d = \frac{0.002\pi}{\Delta K_o}$ where ΔK_o is the change in the mode wave numbers within the wave train. Thus, following [5] (2000), $\Delta K = \frac{0.051}{L_o}$ with $L_o = 2\pi/K_o$, K_o being the mean wave number corresponding to wave length, L_o, for the swell. The interesting feature indicated in solutions (3.9) and (4.15) is that both first and second order wave height vary directly as the square root of d. Consequently, these equations seem to have confirmed that the effective wave evolutions on a beach demand that the width of the beach shall be finite. In (4.15), the first two terms form the complimentary solution and are of first order and so, the particular solution will account for the second solution required. From these, Tables I and II are calculated. The main feature of the calculations is the dominance of the wave component with period of 9 seconds for all the realistic values of *d*.

Figure 1 is interesting in that, it illustrates that the wave height is an increasing function of the beach bottom gradient. Further, wave energy is proportional to the square of its height. Thus, the wave energy density will increase with increasing beach gradient. But wave energy density is closely related to the microseismic energy density. This appears to suggest that the present theory is in agreement with the observation regarding intense microseismic activities prevalent in the neighbourhood of a steep coast. The observation made by [8] concerning the intense microseismic events is in support of this conclusion, considering the topography of the Irish sea coastal waters with its steep coast from which considerable microseismic signals can radiate.

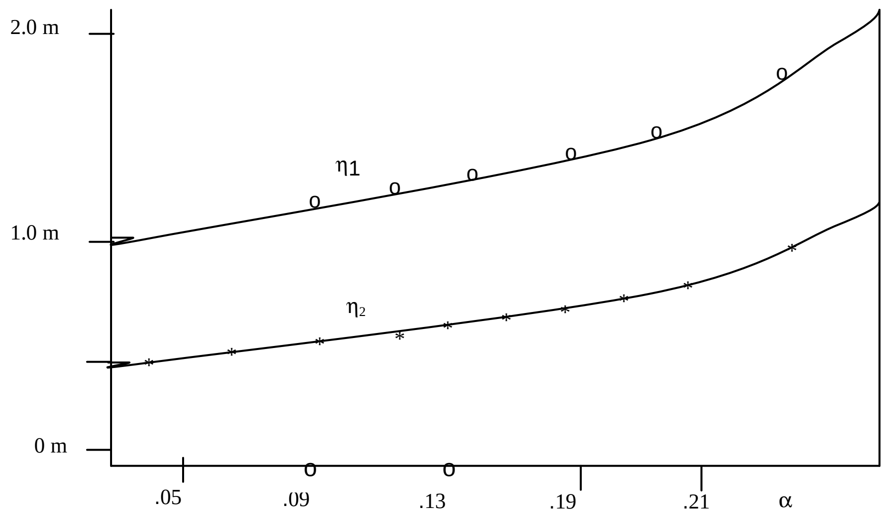

Figure I: Variation of η_1 and η_2 as functions of beach gradient α for period T = 10sec

Table I Computation for C_1

T(*s*) *d* (km)	6	7	8	9	10	11	12
1.5	.227	.25	.271	.310	.280	.261	.241
2.5	.234	.265	.280	.321	.286	.269	.250
3.5	.263	.271	.285	.331	.291	.271	.256
4.5	.271	.281	.291	.341	.298	.278	.261
5.5	.278	.290	.298	.352	.301	.290	.296

Table II: Computation for C_2

T(s) *d* (km)	6	7	8	9	10	11	12
1.5	.13	.15	.1611	.210	.158	.148	.132
2.5	.138	.159	.168	.224	.161	.150	.140
3.5	.141	.161	.169	.238	.173	.161	.148
4.5	.150	.168	.142	.241	.178	.168	.152
5.5	.158	.170	.176	.249	.181	.170	.162

Table III: Computation for $\eta^{(1)}(R, \pi/6, t)$ in metres

T(*s*) / *d* (km)	6	7	8	9	10	11	12
1.5	2.5	2.66	2.63	3.10	2.71	2.61	2.44
2.5	2.61	2.63	2.68	3.22	2.80	2.68	2.48
3.5	2.65	2.69	2.71	3.45	2.85	2.70	2.52
4.5	2.68	2.70	2.80	3.50	2.92	2.75	2.58
5.5	270	2.80	2.95	3.67	3.01	2.81	2.62

Table IV: Computation for $\eta^{(2)}$ $(R, \pi/6, t)$ in metres

T (*s*) / *d* (km)	6	7	8	9	10	11	12
1.5	0.51	0.60	.72	.91	.72	.65	.41
2.5	0.63	.67	.78	.98	.81	.68	.48
3.5	0.68	.71	.81	1.21	.89	.71	.54
4.5	0.69	.79	.86	1.30	.91	.79	.56
5.5	0.73	.80	.91	1.37	.92	.82	.63

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