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Higher order MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction

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Abstract

We present a higher order correction for the temperature field. It is shown that the correction has a maximum and that the maximum θ_{1max} depends on the Prandtl number P_r and the heat absorption coefficient ϕ_*. Moreover $\theta_{1max} \to \infty$ as $\phi \to \phi_*$

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1.0 Introduction

The study of flow and heat and mass transfer in the boundary layer induced by a surface moving with a uniform or non-uniform velocity in a quiescent ambient fluid is important in several manufacturing processes in industry which include the boundary layer along material handling conveyers, the extension of plastic sheets, the cooling of an infinite metallic plate in a bath [1]. Such a problem also arises in glass blowing, continuous casting and spinning of fibers [1]. Sakiadis [10] studied the flow induced by a surface moving with a constant velocity in an ambient fluid.

Vajravelu and Hadjinicolau [8] and [9] investigated a convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream and they showed that heat generation or absorption effects in moving fluid is important.

Pioneer worker on heat and mass transfer included Chen and Yuh [2] while Gupta and Gupta [6] investigated heat and mass transfer on a stretching sheet with suction or blowing. Crane [3] studied the flow induced by a surface moving with constant velocity in an ambient fluid while the heat transfer of the same problem was studied by Erickson [4]. Recently Muthucumaraswamy [7] studied the effects heat generation/absorption and magnetic effects while Chamkha [1] studied a generalization of the problem investigated in [7].

In this paper we study an MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation / absorption which results from a quadratic reaction. Thus the present problem is a generalization of [1].

2.0 Governing equations

Following [1] and modifying for an Arhenins reaction, we obtain

$$\frac{\partial v}{\partial y} = 0 \qquad (2.1)$$

$$v\frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\,\beta_T\,(T - T\infty) + g\,\beta_c\,(c - c_\infty) - \frac{\sigma B_0^2}{p}u \qquad (2.2)$$

$$\rho c_p v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + (c - c_\infty)^n\,\{Q_0\,(T - T_\infty) + Q_1\,(T - T_\infty)^2\} \qquad (2.3)$$

$$v\frac{\partial C}{\partial y} = D\frac{\partial^2 c}{\partial y^2} - (c - c_\infty)^n\,\{y_0\,(T - T_\infty)^2 + y_1(T - T_\infty)^2\} \qquad (2.4)$$

where y is the horizontal or transverse coordinate, it is the axial velocity, v is the transverse velocity, T is the fluid temperature, c is the concentration, $T\infty$ is the ambient temperature, $c\infty$ is the ambient concentration, and p, g, B_T, Bc v σ, β_c Q_0, D, γ_{0_1} γ_1 and n are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, kinematics viscosity, fluid electrical conductivity, magnetic induction,

heat generation/coefficient and the chemical reaction parameter and real number respectively. The physical boundary conditions for the problem are $U(0) = U_w$, $V(o) = - v_w$, $T(0) = T_w$, $c(0) = c_w$

$$Y \to \infty,\ u \to 0,\ T \to T_\infty,\ C \to C_\infty\ , \qquad (2.5)$$

where u_w, $v_w > 0$, T_w and c_w are surface velocity, suction velocity, surface temperature and concentration respectively. Assuming V = constant, $n = 0$

$$y^1 = yv_w / \upsilon, \qquad u^1 = \frac{u}{u_w}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}\ ,\ c^1 = \frac{c - c_\infty}{c_w - c_\infty} \qquad (2.6)$$

We obtain, dropping $^{||}{}^{\prime}{}^{||}$

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + G_{rT}\,\theta + G_{rc}\,c - M^2\,u = 0 \qquad (2.7)$$

$$\frac{d^2\vartheta}{dy^2} + P_r\frac{d\vartheta}{dy} + \phi\,\theta + \in\phi_1\theta^2 = 0 \qquad (2.8)$$

$$\frac{d^2c}{dy^2} + s_c\frac{dc}{dy} - \left(\delta_0\,\theta + \delta_1\,\theta^2\right) = 0 \qquad (2.9)$$

where

$$G_{rT} = g\frac{B_T\,v\left(T_w - T_0\right)}{U_w\,V_w^2}, G_{rc} = g\frac{B_c\,v(Cw - C_\infty)}{U_w V_w^2},\quad P_r = \frac{\mu cp}{k},\ S_c = \frac{V_w}{D},\ \delta_0 = \frac{\gamma_0}{D}\,(T_w - T_\infty),\ \delta_1 = \gamma_1\frac{\left(T_w - T_\infty\right)}{D}$$

$$\frac{v\,Q_0}{\mu\,c\,p\,v_w^2} = \frac{\upsilon\,Q_0}{\rho\,c_p v_w^2}.\frac{\upsilon\,\rho\,c_p}{k} = \phi P\,r\ ,\frac{vQ_1}{\mu\,c\,p\,v_w^2}\frac{R\,T_0^2}{E} = \varepsilon\ \phi_1\ Pr \qquad (2.10)$$

The Dimensionless boundary conditions are $U(0) = 1$, $\theta(0) = 1$, $C(0) = 1$

$$Y \to \infty\quad u \to 0,\ \theta \to 0,\ C \to 0 \qquad (2.11)$$

3.0 Method of solution

In this paper we consider only

$$\frac{d^2\theta}{dy^2} + P_r\frac{d\theta}{dy} + \phi\,\theta + \in\phi_1\,\theta^2 = 0 \qquad (3.1)$$

$$\theta(0) = 1, \theta \to 0 \text{ as} y \to \infty \qquad (3.2)$$

We seek asymptotic solution in the limit $\in \to 0$. Thus

$$\theta = \theta_0 + \in\theta_1 + \in^2\theta_2 + \qquad (3.3)$$

We obtain

$$\frac{d^2\theta_0}{dy^2} + P_r\frac{d\theta_0}{dy} + \phi\theta_0 = 0 \qquad (3.4)$$

$$\theta_0\,(0) = 1,\ \theta_0\,(\infty) = 0 \qquad (3.5)$$

$$\frac{d^2\theta_1}{dy^2} + P_r \frac{d\theta}{dy} + \phi\theta_1 + \phi_1 \theta_0^2 = 0 \qquad (3.6)$$

$$\theta_1(0) = \theta_1(\infty) = 0 \qquad (3.7)$$

The solution to (3.4) – (3.7) are

$$\theta_0 = \exp(-my) \qquad (3.8)$$

$$\theta_1 = \frac{Pr\,\phi_1}{4m^2 + 2P_r\, m + Pr\,\phi}\left[exp\,(-my) - exp - (2my)\right] \qquad (3.9)$$

where

$$m = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - 4P_r\,\phi}\right] \qquad (3.10)$$

We display in Figure 1 the graph of ϕ against y. In Figure 2 we show how maximum θ_1 depends on ϕ.

4.0 Conclusion

In this paper we provide a higher order correction to previously obtained temperature field. It shows maximum correction, and indeed previously obtained steady temperature field do not exist when the scaled heat generation/absorption coefficient is greater than one quarter of the prandtl number.

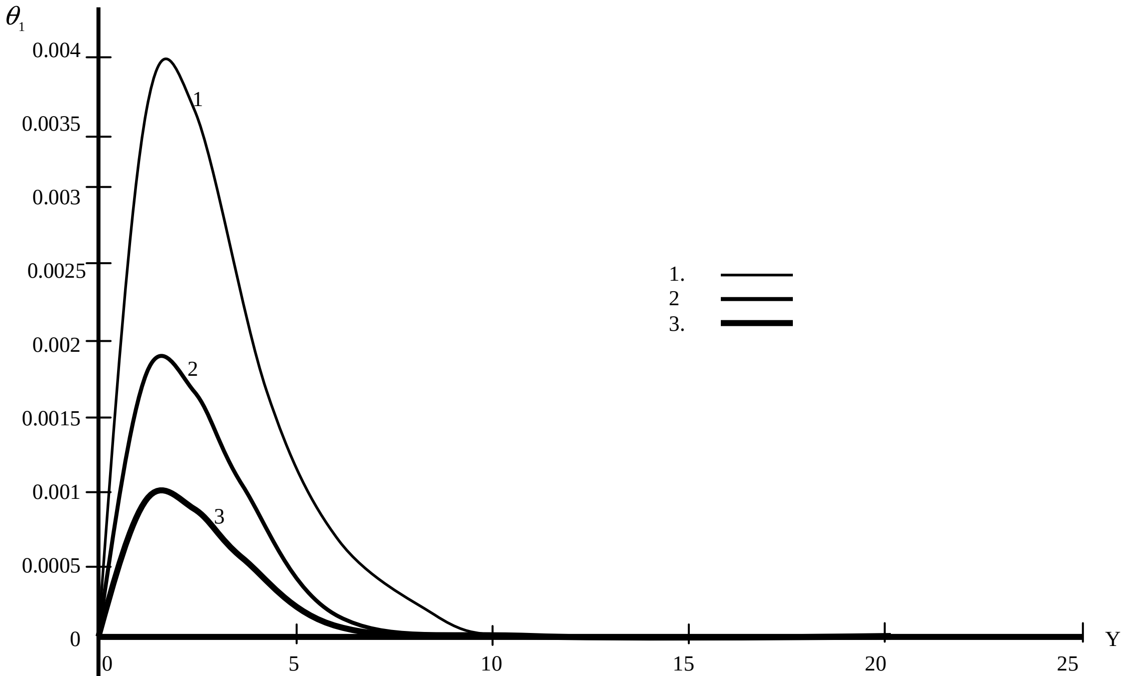

Figure 1: Graph of velocity θ_1 against position Y

1. $Pr = 0\cdot 71,\ \phi = 0\cdot 1,\ \phi_1 = 0\cdot 01,\ M = 0\cdot 5,\ Grt = 1,\ Grc = 1,\ k = 0\cdot 15,\ Sc = 0\cdot 6$
2. $Pr = 0\cdot 71,\ \phi = 0\cdot 1,\ \phi_1 = 0\cdot 01,\ M = 0\cdot 5,\ Grt = 1,\ Grc = 1,\ k = 0\cdot 15,\ Sc = 0\cdot 6$
3. $Pr = 0\cdot 71,\ \phi = 0\cdot 1,\ \phi_1 = 0\cdot 01,\ M = 0\cdot 5,\ Grt = 1,\ Grc = 1,\ k = 0\cdot 15,\ Sc = 0\cdot 6$

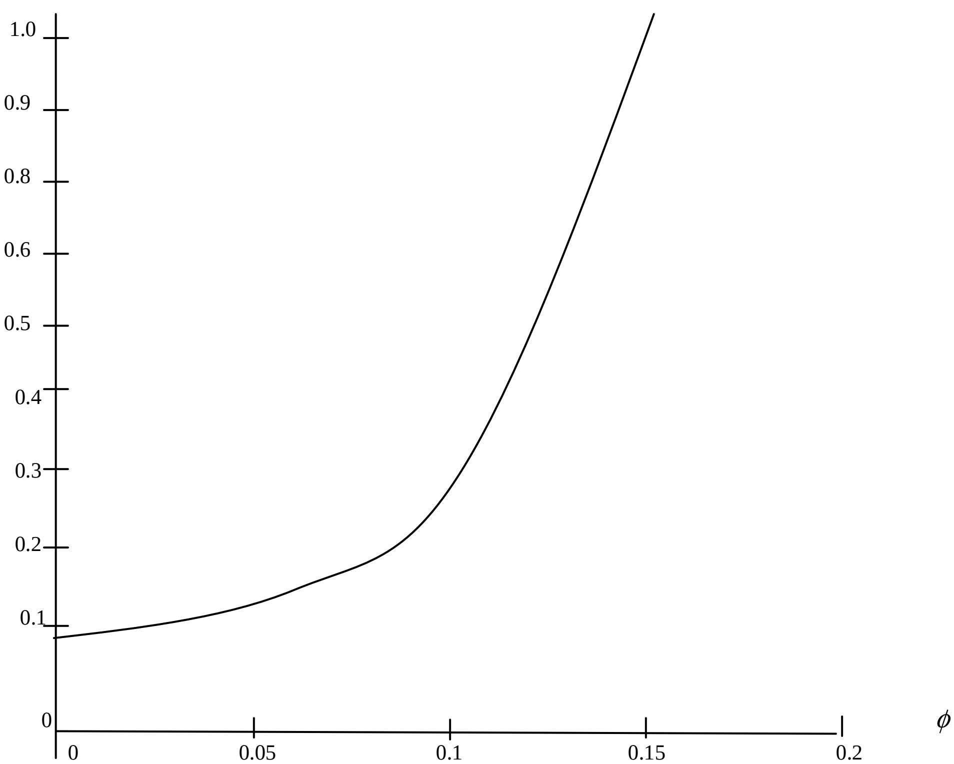

Figure 2: Graph of θ_{1max} against ϕ

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