

Eigenvalues define conditions of stability in liquid-liquid miscible displacement process

K. I. Idigbe* and B. H. Caudle,**

***Department. of Petroleum Engineering, University of Benin, Benin City, Nigeria.**

****Department of Petroleum and Geosystems Eng., University of Texas at Austin, Austin, Tx, USA.**

Abstract

A miscible displacement process is primarily governed by both convective flow and hydrodynamic dispersion. The proper classification of stability condition in the miscible displacement process is a major requirement for a successful field application of this enhanced crude oil recovery mechanism. This paper derives characteristics functions, the eigenfunctions, which provide guidelines for achieving flow stability in miscible displacement processes. A growth parameter, the eigenvalue, shows the necessary and sufficient conditions for stability in the liquid-liquid miscible displacement process. The critical velocity and wavelength necessary for stability are defined for the miscible displacement process.

Keywords: Convective Flow, Hydrodynamic Dispersion, Stability Condition, Eigenfunctions, Critical Velocity and Wavelength.

pp 167 - 172

1.0 Introduction

The considerations of stability are very important in miscible displacement processes. The miscible fluids such as liquid propane are usually very expensive. Any premature breakup of these in their displacement of in-situ oil must be avoided. Miscibility at the front must be maintained at all time to achieve the desired volumetric sweep efficiencies [1,2].

Instability is a major factor that compromises the miscibility at front and greatly contributes to the low volumetric sweep efficiencies of the liquid-liquid miscible displacement process in the field, especially for the slug process [3,4].

The proper classification of the stability conditions for a miscible process is a necessary condition for its successful implementation in the field. Lower than expected recoveries, early breakthrough of the displacing fluid miscible turned immiscible displacement, are some of the problems reported in the field application of the miscible displacement process [5,6].

To ensure profitability and economics of the process, target recoveries of in-situ oil are desired for any miscible displacement process. Most often, the velocity of injection of the miscible fluid is always greater than the critical velocity required to achieve stability, and thus, this result in less than expected oil recoveries.

Mathematically, the requirements for achieving stability and subsequently high volumetric sweep efficiencies are determined and presented in the following sections.

The stable liquid-liquid miscible displacement process for two incompressible materials, the displacing fluid and the in-situ oil, is primarily governed by both convective flow and hydrodynamic dispersion. Mathematically we have:

$$U_{mix} = -\nabla \frac{K(p - \rho_{mix} g \sin \theta)}{\mu_{mix}} \quad (1.1)$$

$$\nabla \cdot (\phi D \nabla c) - \nabla \cdot (c U_{mix}) = \phi \nabla c \quad (1.2)$$

U_{mix} , K and D are tensors.

Equation (1.1) represents a phase balance – a single phase, and equation (1.2) represents component balance, with two components – the displacing liquid and the in-situ oil are miscible but do not react chemically with each other or the porous medium through which they flow. By virtue of the perturbation and hydrodynamic theories, studies were initiated to investigate the conditions for stability, and the sources of stabilization of miscible displacement processes.

In developing the solution to the investigations, a moving boundary problem in the x , y , z co-ordinate system, in which the in-situ oil is displaced by a miscible material at a constant superfluous velocity, V , in the positive x - direction of infinite extent, is considered. It is further assume that the co-ordinate axes are oriented parallel to

the principal axes of the porous medium. Additionally, there exists the possibility of the x - y plane being inclined at an angle θ to the horizontal.

2.0 Mathematical derivations

For an unperturbed stable state, it is assumed that a vertical interface defined by the y - z surface separates the two fluids; in-situ oil properties on one side and the miscible liquid (solvent) properties on the other side.

Seeking solution to equations (1.1) and (1.2), let:

$$U_{mix} [x, y, z, t] = [V, 0, 0] \quad (2.1)$$

$$C[x, y, z, t] = c_u [x, t] \quad (2.2)$$

$$p = P \quad (2.3)$$

where $c_u [x, t]$, is a function, that satisfies the one-dimensional form of equation (1.2), subject to the appropriate boundary and initial conditions. Equations (2.1) to (2.3) define the state of this unperturbed moving surface as in Figure 1.

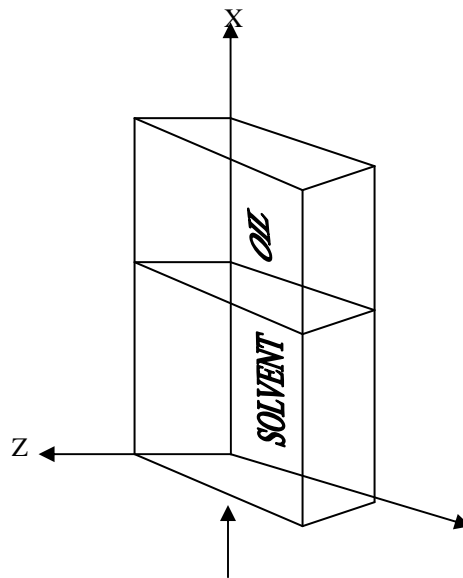


Figure 1: Unperturbed Moving Surface Model [1]

Now suppose a small disturbance or perturbation of an arbitrary form is introduced into the above unperturbed system, and then observed over the course of time. The disturbances could come from the normal variations of properties, such as permeability within the system. When the necessary and sufficient conditions for instability exist, the small disturbances may now grow into channels (viscous fingers, etc.) that result in poor volumetric sweep efficiencies [1, 7, 8].

Let the perturbed system be represented by:

$$\chi = \chi^0 + \varepsilon^1 \chi^1 + \varepsilon^2 \chi^2 + \dots \quad (2.4)$$

where $\chi = F[c, U_x, U_y, U_z, p]$ and $\varepsilon =$ the perturbation parameter; small and dimensionless. The first term in equation (2.4) represents the unperturbed stable displacement while the remainder of the equation, represent the perturbation. Consistent with equation (2.4), the perturbed state is represented by:

$$U_p [x, y, z, t] = [V + W_x, W_y, W_z] \quad (2.5)$$

$$c_p [x, y, z, t] = c_u[x, t] + c^*[y, z, t] \quad (2.6)$$

$$p = P + P^* \quad (2.7)$$

W_x, W_y, W_z, c^* and P^* represent first-order perturbation effects, sufficient to characterize and define the conditions of stability in miscible displacement processes [1]. We define the macroscopic perturbed velocity, W , in terms of the perturbed potential, Φ as:

$$\vec{W} = -\nabla \Phi \quad (2.8)$$

The potential and pressure are define as:

$$\Phi = \frac{K}{\mu} \left[p + \frac{\mu}{K} Vx - \rho g x \sin \theta \right] \quad (2.9)$$

$$p = \frac{\mu}{K} \Phi - \frac{\mu}{K} Vx + \rho g x \sin \theta \quad (2.10)$$

By continuity principle, the divergence of the perturbed velocity must be equal to zero. This defines a Laplace equation, equation (2.11), for this moving boundary problem, associated with the stability of the fluids distribution.

$$-div \vec{W} = \nabla^2 \Phi = 0 \quad -\infty < x < +\infty \quad (2.11)$$

To solve equation (2.11), we choose a Cartesian co-ordinate system with impermeable side boundaries defined by equation (2.12):

$$0 \leq y \leq d, \quad 0 \leq z \leq w \quad (2.12)$$

The normal velocity perturbation components but must vanish at these impermeable boundaries. Thus:

$$W_y [0, z] = W_y [d, z] = W_z [y, 0] = W_z [y, w] = 0 \quad (2.13)$$

Equivalently, the appropriate conditions at these impermeable boundaries are:

$$-\frac{\partial \Phi}{\partial y} [0, z] = 0; \quad 0 \leq z \leq w \quad (2.14)$$

$$-\frac{\partial \Phi}{\partial y} [d, z] = 0; \quad 0 \leq z \leq w \quad (2.15)$$

$$-\frac{\partial \Phi}{\partial z} [y, 0] = 0; \quad 0 \leq y \leq d \quad (2.16)$$

$$-\frac{\partial \Phi}{\partial z} [y, w] = 0; \quad 0 \leq y \leq d \quad (2.17)$$

Particular solutions to equation (2.11) can easily be found by using the method of ordinary differential equation, by applying the product method. Thus, let [9, 10]:

$$\Phi [x, y, z, t] = X(x) F(y, z) T(t) \quad (2.18)$$

where $X(x)$, $F(y, z)$ and $T(t)$ are eigenfunctions or characteristic functions that give the solution to the Laplace equation (2.11). $F(y, z)$ is expressed as a double Fourier cosine series, defined in the intervals given by equation (2.12), as:

$$F[y, z] = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{ij} \cos 2\alpha_i y \cos 2\beta_j z \quad (2.19)$$

A_{ij} is integration constant, and the eigenvalues α_i and β_j are given as: $\alpha_i = \frac{\pi}{d} i$; $i=1, 2, 3 \dots n$
 $\beta_j = \frac{\pi}{w} j$; $j=1, 2, 3 \dots n$. With the following solution for $X(x)$ [1]:

$$X(x) = C_{ij} \exp[\pm v_{ij} x] \quad (2.20)$$

where C_{ij} is integration constant and v_{ij} is the propagation vector in the y, z plane of magnitude, and expressing $T(t)$ as an exponential function, we obtain the new expression $\Phi[x, y, z, t]$ that satisfies equation (13) and the boundary conditions:

$$\Phi[x, y, z, t] = E(\cos 2\alpha y \cos 2\beta z) \exp(\pm vx + nt) \quad (2.21)$$

In equation (2.21), the subscripts ij have been dropped, and E is a product of the integration constants. The eigenvalue, n , can take negative, zero, or positive values. The simple exponential function has the property of rapid growth for positive values of n and rapid negative values of n . To ensure uniform flow far from the moving interface, we require that the x -component of the perturbed velocity vanish at $x = \pm \infty$, respectively. At the perturbed interface both kinematic and hydrodynamic conditions must be satisfied. Defining and separating the macroscopic perturbed into its unperturbed stable component and the unstable one with first-order effects [8, 12], and satisfying kinematic conditions at the interface, we define equation (2.22):

$$\eta[c_p, y, z, t] = \frac{v}{n} E_o(\cos 2\alpha y \cos 2\beta z) \exp[nt] \quad (2.22)$$

To apply the hydrodynamic condition at the perturbed interface, the equation of continuity for the displacing miscible fluid (the solvent) as defined by equation (1.2) is invoked. The perturbed form of this equation is:

$$\nabla(\phi D \nabla c_p) - \nabla(c_p W_{mix}) = \phi \left(\frac{\partial c_p}{\partial t} \right) \quad (2.23)$$

where c_p = the perturbed solvent concentration, D = the effective dispersion tensor, (l^2/t)

$$\phi \left(\frac{\partial c_p}{\partial t} \right) + W_{mix} \cdot \nabla c_p = \nabla(\phi D \nabla c_p) \quad (2.24)$$

The assumption of incompressible fluids further reduces equation (2.23) to the left hand side of the equation (2.24) defines the so-called substantive time derivative following the motion of fluid. Equivalently, equation (2.24) can be written in terms of the first-order perturbed interface, η , where it is linear function of the perturbed solvent concentration, c_p as shown in equation (2.25).

$$\frac{\partial \eta}{\partial t} - \nabla(\phi D \nabla \eta) = 0 \quad (2.25)$$

Equation (2.25) has the units of velocity. The first term describes a displacement velocity attained by any moving iso-concentration surface following the motion of fluid under the action of a unit dispersive force [11, 12]. With the assumption of constant dispersion coefficients which considering a particular velocity, equation (2.25) further reduces to

$$U_D = \phi D_i \nabla^2 \eta[c_p, y, z, t] = 0 \quad (2.26)$$

Equation (2.26) allows for the subsequent spreading of the interface due to the effects of molecular diffusion and convective dispersion [13]. Due to the effects of the molecular diffusion and convective dispersion, a transition zone is created between the two miscible liquids—the pure oil and the pure solvent. By solving equation (2.26) with the expression for η from equation (2.22), results are obtained for the investigations.

3.0 Results

In the transition zone, one permeability, the absolute permeability will exit. Writing the appropriate terms for the pressures and potential in oil and solvent zones, equation (2.22), defines the eigenvalue n , the growth or decay parameter [1].

$$n = \frac{v[L3 - (\rho_o - \rho_s)g \sin \theta - 4M_s v^2]}{L4} \quad (3.1)$$

where

$$L3 = V(\mu_o - \mu_s)/K \quad (3.2)$$

$$L4 = (\mu_o - \mu_s)/K \quad (3.3)$$

The least stable term of the decomposed Fourier components will have the highest growth rate, yet stability requires that no growth occur [7, 8, 11, 12]. This means that the growth or stability parameter, the eigenvalue, n , must be zero or negative. The necessary and sufficient condition for growth or instability is that n , the growth parameter, be positive. This will always be satisfied whenever [1]:

$$\frac{(\mu_o - \mu_s)V - (\rho_o - \rho_s)g \sin \theta}{K} > 4M_s v^2 \quad (3.4)$$

$$M_s = \frac{\phi D_t \mu_{mix} L_c}{K}; \quad (3.5)$$

L_c = Length of the transition or mixing zone for a stable displacement. The first and second terms on the left side of the equation (3.4), give the effect of the viscous and gravity forces. A critical velocity, V_c , is defined as that at which there is a balance between the viscous and gravity forces. Negating the effect of mixing, this can be define from equation (3.4), as:

$$V_c = \frac{K(\rho_o - \rho_s)g \sin \theta}{\mu_o - \mu_s} \quad (3.6)$$

Equation (3.6) is the well-known expression in the Industry [14] for critical velocity for an initially sharp liquid-liquid interface in a homogenous porous medium. Subtracting equation (3.6) from equation (3.4), and putting the expression for v , the propagation vector, one obtains

$$\frac{(\mu_o - \mu_s)(V - V_c)}{K} > 4M_s \left[\frac{(i\pi)^2}{d^2} + \frac{(j\pi)^2}{w^2} \right] \quad (3.7)$$

Rearranging equation (3.7), putting the expression for M_s , and defining H_e , the heterogeneity factor, the

following condition for instability, is obtained

$$\frac{(\mu_o - \mu_s)(V - V_c)d^2 w^2}{\mu_{mix} \phi D_t H_e L_c (i^2 w^2 + j^2 d^2)} > 4\pi^2 \quad (3.8)$$

As always, the least stable term will determine the onset of instability, and this will occur at the minimum values

of the eigenvalue, v , the propagation vector, at $i = j = 1$. From equation (3.8), satisfied simultaneously, the necessary and sufficient conditions for instability are given by equation (3.9) to (3.11):

$$\mu_o > \mu_s \quad \text{or} \quad \frac{\mu_o}{\mu_s} > 1 \quad (3.9)$$

$$V > V_c \quad (3.10)$$

$$\lambda > \lambda_c \quad (3.11)$$

where

$$\lambda_c = 4\pi \left[\frac{\phi D_t H_e \mu_{mix} L_c}{(\mu_o - \mu_s)(V - V_c)} \right]^{0.5} \quad (3.12)$$

The critical wavelength is defined by equation (3.12), derived from equation (3.4), with $\lambda = 2\pi/\gamma$, defining the wavelengths of the individual Fourier components.

4.0 Discussion

From the investigations, it is seen that the conditions of stability in liquid-liquid miscible displacement processes are defined by eigenvalues. The expression on the left hand side equation (3.8) must be equal to or less than $4\pi^2$ for stability. There are two types of hydrodynamic stability in miscible displacements, namely [1, 3, 4]: unconditional stability, and conditional stability. When the expression on the left side of equation (3.1) is separated into various distinct dimensionless scaling groups as shown in equation (4.1), the forces that affect the performances of miscible displacement processes are clearly seen. This is discussed extensively in Refs. [1] and [4].

$$N_s = N_v [1 - N_g] N_p N_d N_f \quad (4.1)$$

N_s = the expression on the left hand side of equation (3.8). It is then seen that the stability of a displacement in a given system (model or field), is determined by the interplay of forces such as that the behavior might change with different displacement and system parameters.

5.0 Conclusion

- In this paper, by virtue of the perturbation and hydrodynamic theories, investigations have shown:
- i. The necessary and sufficient conditions for the stability of miscible displacement processes, which must be satisfied simultaneously,
 - ii. Unconditional stability, where the scaling number, N_s , must be less than
 - iii. Neutral stability, where N_s will be equal to $4\pi^2$, and
 - iv. The major forces that manifest the phenomena that affect the performances of miscible displacement processes.

Nomenclature

- d = Width of Rectangular System (l)
 w = Height of Rectangular system (l)
 D_t = Effective Transverse Dispersion Coefficient (l^2/t)
 H_e = Heterogeneity Factor (dimensionless)
 L_c = Length of Transition Zone (l)
 M = Mobility Ratio
 V = Superfluous Injection Velocity (l/t)
 V_c = Critical injection Velocity (l/t)
 μ_0 = Viscosity of the Crude Oil ($M/l-t$)
 μ_s = Viscosity of the Solvent ($M/l-t$)
 μ_{mix} = Fluid Viscosity in the Transition Zone ($M/l-t$)
 $\phi = \phi_i (1 - S_w - S_g)$ (Fraction)
 ϕ_i = Effective porosity (fraction)
 S_w = Initial Water Saturation (fraction)
 S_g = Initial Gas Saturation (fraction)
 K = the absolute permeability tensor (l^2)
 U_{mix} = average flux (velocity) of mixture in the transition zone (l/t)
 C = solvent volumetric concentration (dimensionless)
 t = time (t)
 P = pressure

References

- [1] Idigbe, K.I. (1984): Application of Stability Theory and Finite Element Simulation to Characterize Miscible Displacements, Ph.D. Thesis, University. Of Texas at Austin, Texas.
- [2] Koch, H.A. Jr. and Slobod, R.L. (1957): Miscible Slug Process, Trans., AIME, Vol. 210
- [3] Idigbe, K.I. (2001): Calculating Optimum Sizes of Miscible Slugs Consistent with Stability and Miscibility, Journal of Civil and Environmental Systems Engineering, University of Benin, Benin City, Vol. 2, No. 1.
- [4] Idigbe, K.I. (2003): An Appropriate Dimensionless Number for the Scaling and Modeling of Miscible Displacement Processes, Journal Of Engineering for Development, University of Benin, Benin City, Vol. 5, pp. 1-8
- [5] Habemann, B. (1075): The Efficiency of Miscible Displacement as a Function of Mobility Ratio, SPE Reprint Series, No. 8: pp. 205-213.
- [6] Greenkorn, R.A., Johnson, C.R. and Haring, R.E. (1965): Miscible Displacement in a Controlled Natural System, Trans., AIME, Vol.234.
- [7] Peters, E.J. (1979): Stability Theory and Viscous Fingering in Porous Media, Ph.D. Dissertation, University of Alberta, Alberta, Canada.
- [8] Chuoke, R.L., Van Meurs, P. and van der Poel, C. (1959): The Instability of Slow, Immiscible, Viscous Liquid-Liquid Displacement in permeable Media, Trans., AIME, Vol216, pp. 188-194.
- [9] Cramer, H. (1958): mathematical Method of Statistics, Princeton University Press
- [10] Ames, W.F. (1971): Numerical Methods for Partial Differential Equations, Barnes & Nobles, Inc., New York.
- [11] Outmans, H.D. (1962): Nonlinear Theory for Frontal Stability and Viscous Fingering in Porous Media Soc. Pet. Eng. Journal, pp. 165-176.
- [12] Heller, J.P. (1966): Onset of Instability Patterns between Miscible Fluid in porous Media, Journal of Applied Physics, Vol.37, No. 4
- [13] Bird, R.B., Stewart, W.E. and Lightfoot, E > 1. (1960): Transport Phenomena, John Wiley & Sons, Inc., New York.
- [14] Blackwell, R.J., Rayne, J.R. and Terry, W.M. (1959): Factors Influencing the Efficiency of Miscible Displacements, Trans., AIME, Vol. 216.