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On the effect of temperature dependent thermal conductivity on temperature rise of biologic tissues during microwave heating

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Abstract

We consider the effect of temperature dependent thermal conductivity on temperature rise in biologic tissues during microwave heating. The method of asymptotic expansion is used for finding solution. An appropriate matching procedure was used in our method. Our result reveals the possibility of multiple solutions and it gives insight to avoiding hot-spot in the tissues. Clearly some tissues are heat sensitive while others are heat resistant.

Keywords*:* Thermal conductivity, temperature rise, biologic tissues, microwave heating.

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1.0 Introduction

The subject of heat deposition and consequent rise in temperature in living tissues has been the studies in a number of literatures [2, 3, 7, 9, 12, and 13]. These researches were prompted due to the therapeutic advantage the rise in temperature within a given range had on the cancerous tissues. Diseased cells are destroyed at the rise of temperature within a given time. Increase in number of cells is destroyed after a longer time interval. At a given time and an increase in the rise in temperature greater numbers of cells are destroyed [1, 16]

Heating deposition is not uniform in tissues, destruction of surrounding normal tissues, possibility of hot spot, heat sensitivity or resistance of tissues are some of the questions that are raised in the literature [4, 5, 15, 16].

The physical properties of tissue can lead to formation or hindrance of hot spot as reported by Smyth [15]. Saxene and Arya [14] in their model of temperature distribution in human skin assumed that the rates of blood mass flow, metabolic heat generation and tissue thermal conductivity are different in the three-layer model proposed. The rate of blood flow and metabolic rate was considered a function of position and temperature.

Pal and Pal [13] studied the steady-state temperature distribution in human skin and subcutaneous tissue (SST). Their model accounts for heat conduction, perfusion of the capillary beds and metabolic heat production of the dermis and subcutaneous tissues. Using other simplifying assumptions they obtain their solution in terms of confluent hyper geometric and Airy's function. Very recently Jiang et al [8] discussed the effects of thermal properties and geometrical dimensions in the skin burn injuries. Ng and Chua [12] proposed a comparison of one and two-dimensional programmes for predicting the state of skin burns. Lui and Marchant [11] considered the microwave heating of three-dimensional blocks with a transverse magnetic wave-guide mode in a long rectangular

wave-guide. El-dabe et al [6] in their paper investigated the effects of microwave heating equations in the thermal state of biological tissues. They consequently predicted the effects of the thermal physical properties on the transient temperature of tissues.
In the present work we are studying the effect of temperature dependent thermal conductivity on temperature rise of biologic tissue during microwave heating. In the next section we present the mathematical formulation of the problem, while in section 3 we outline the method of solution. In section 4 we discuss the result.

2.0 Mathematical formulation

Following the works of Kritkos, Foster and Schwaw [10] and Wulff [17] we have our Energy equation as:

$$\frac{\partial}{\partial t}(C_{\rho_t}\,\rho_t T) = \mathrm{Div}(K\rho\,\mathrm{grad}T) - \mathrm{Div}(\rho_b\, C_{\rho_b} T\underline{q}_c) - \rho_b\, C_{\rho_b}\phi T + Hm + Q(x,T,t) \qquad (2.1)$$

We consider the following reasonable simplifying assumptions

(a) We neglect the heat due to metabolic process.

(b) We assume:

(i) Uniform blood flow

(ii) The thermal conductivity

$$K = (1+b_1\theta)^n; \quad \eta \in R \qquad (2.2)$$

(iii) The heating source is taken as

$$\overline{Q}_o = \alpha_h\left(\frac{X}{L}\left(1-\frac{X}{L}\right)\right)^d e^{-nX}u(t) \qquad (2.3)$$

Non-dimensionalising the equation as in Adebile [1] we have

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial\eta}\left\{(1+c_1\theta)^n\frac{\partial\theta}{\partial\eta}\right\} - \alpha_2\frac{\partial\theta}{\partial\eta} - \alpha_o\theta + Q \qquad (2.4)$$

with initial and bounding condition

$$\theta(\eta,0) = \theta_o(\eta),\ \theta(0,\tau) = \chi,\ \theta(1,\tau) = \lambda \qquad (2.5)$$

3.0 Method of solution

We solve for the steady state problem in this paper. Hence the equation we shall solve is

$$\frac{\partial}{\partial\eta}\left\{(1+b_1\theta)^n\frac{\partial\theta}{\partial\eta}\right\} - \alpha_2\frac{\partial\theta}{\partial\eta} - \alpha_o\theta + \alpha_h\{\eta(1-\eta)\}^d e^{-m\eta} = 0 \qquad (3.1)$$

$$\theta(0) = \chi,\ \theta(1) = \lambda \qquad (3.2)$$

using the series method $\theta(\eta) = \sum_{r=0}^{\infty} a_r\eta^r,\ \eta \le \frac{1}{2}$ (3.3)

for solution near $\eta = 0$, and $\theta(\eta) = \sum_{r=0}^{\infty} p_r(1-\eta)^r,\ \frac{1}{2} \le \eta \le 1$ (3.4)

for solution near $\eta = 1$, in equation (3.1) and (3.2) we have for a selected $d = 0$

$$a_2 = \frac{\left[\alpha_o a_o - \alpha_h - a_1^2\left\{\eta b_1 + \eta(\eta-1)b_1^2 a_0 + \frac{3\eta(\eta-1)(\eta-2)b_1^3 a_o^2}{6}\right\} + \alpha_2 a_1\right]}{2\left\{1+\eta b_1 a_o + \eta(\eta-1)b_1^2 a_o^2 + \frac{\eta(\eta-1)(\eta-2)b_1^3 a_o^3}{6}\right\}} \tag{3.5}$$

$$a_3 = \left[\alpha_o a_1 - \alpha_h m - 2a_1\left[\eta b_1 a_2 + \frac{(a_1^2 + (a_1^2 + 2a_o a_2)\eta(\eta-1)b_1^2)}{2}\right]\right.$$
$$+\left[\frac{\eta(\eta-1)(\eta-2)}{6}\right](a_o^2 a_2 + 2a_1^2 a_o) + a_o(a_1^2 + 2a_o a_1) \tag{3.6}$$
$$\frac{\left. -4a_2\left\{\eta b_1 a_1 + \eta(\eta-1)b_1^2 a_o a_1 + \frac{\eta(\eta-1)(\eta-2)b_1^3 a_o^2 a_1}{6}\right\} + 2\alpha_2 a_2\right]}{6\left\{1+\eta b_1 a_o + \frac{\eta(\eta-1)b_1^2 a_o^2}{2} + \frac{\eta(\eta-1)(\eta-2)b_1^3 a_o^3}{6}\right\}}$$

$$p_2 = \frac{\left[\alpha_o p_o - \alpha_h \lambda^{-m} - p_1^2\left\{\eta b_1 + \eta(\eta-1)b_1^2 p_o + \frac{3\eta(\eta-1)(\eta-2)b_1^3 p_o^2}{6}\right\} - \alpha_2 p_1\right.}{2\left\{1+\eta b_1 p_o + \eta(\eta-1)b_1^2 p_o^2 + \frac{\eta(\eta-1)(\eta-2)b_1^3 p_o^3}{6}\right\}}$$
$$\frac{\left. -4p_2\left\{n b_1 p_1 + n(n-1)b_1^2 p_o p_1 + \frac{n(n-1)(n-2)b_1^3 p_o^2 p_1}{6}\right\} - 2\alpha_2 p_2\right]}{6\left\{1+\eta b_1 p_o + \eta(\eta-1)b_1^2 p_o^2 + \frac{\eta(\eta-1)(\eta-2)b_1^3 p_o^3}{6}\right\}} \tag{3.7}$$

$$p_3 = \left[\alpha_o p_1 - \alpha_h m e^{-m} - 2p_1\left[\eta b_1 p_2 + \frac{(p_1^2 + 2p_o p_2)n(n-1)b_1^2)}{2}\right] + \right.$$
$$\left[\frac{n(n-1)(n-2)}{6}\right]\left\{p_o^2 p_2 + p_1^2 p_o + p_o(2p_o p_1 - p_1^2\right\} \tag{3.8}$$

Using the relevant boundary condition we have $\theta(0) = \chi = a_o$ (3.9)

$$\theta(1) = \lambda = p_0 \tag{3.9a}$$

We now match the inner solution (θ^i) and outer solution (θ^o) using the matching condition

$$\left.\begin{aligned} \theta^i(\eta) &= \theta^o(1-\eta) \\ \frac{\partial^i \theta}{\partial \eta} &= \frac{\partial \theta^o}{\partial \eta}(1-\eta) \end{aligned}\right\} at\, \eta = ½ \tag{3.10}$$

After much algebra we obtain

$$\frac{\left(a_1+\frac{(a+a^2c_1)}{2a^2B_1}\right)^2}{\frac{\$}{a^2B_1}}-\frac{\left(p_1+\frac{(b+b^2c_2)}{2b^2B_2}\right)^2}{\frac{\$}{b^2B_2}}=1 \qquad (3.11)$$

and
$$\frac{\left(a_1+\frac{(1+2ac_1)}{4aB_1}\right)^2}{\frac{N}{2aB_1}}+\frac{\left(p_1+\frac{(1+2bc_2)}{4bB_2}\right)^2}{\frac{N}{2bB_2}}=1 \qquad (3.12)$$

where
$$\$=-\{(a_o-p_o)+(a^2A_1-b^2A_2)\}+\left(\frac{a+a^2c_1}{2a^2B_1}\right)^2-\left(\frac{b+b^2c_2}{2b^2B_2}\right)^2 \qquad (3.13)$$

$$\mathrm{N}=\left(\frac{1+2ac_1}{4aB_1}\right)^2-\left(\frac{1+2bc_2}{4aB_2}\right)^2-(aA_1-bA_2) \qquad (3.14)$$

$$a=b=\tfrac{1}{2}$$
$$a_2=A_1+B_1a_1^2+c_1a_1,\; p_2=A_2+B_2p_1^2+c_2p_1 \qquad (3.15)$$

The solution to the problem becomes

$$\theta^i(\eta)=a_o+a_1\eta+a_2\eta^2+\Lambda\ h.o.t,\; 0<\eta\le\tfrac{1}{2}$$
$$\theta^o(\eta)=p_o+p_1(1-\eta)+p_2(1-\eta)^2+\Lambda\ h.o.t,\; \tfrac{1}{2}\le\eta\le 1 \qquad (3.16)$$

Solving the equations in (3.11) and (3.14), we obtain a_1 and p_1 for specific variables. Other results such as a_2 and p_2 follows. For the purpose of our graph, we denote $\alpha_0\equiv a_{10}, \alpha_h\equiv a_{1h}, \alpha_2\equiv a_{12}, \theta\equiv \text{Thetha}$ (where S1 means solution 1 and S2 solution 2), all other variables were used as they appear in the text on the graph.

4.0 **Result and discussion**

Our result revealed the existence of more than one solution at any given location within the tissue volume. This observation is clear from Figures 1-8. We can hereby state that solution to the temperature rise is unstable when the thermal conductivity is dependent on temperature. Great care must be exercised so that normal tissues are not destroyed since the part of solution at any given stage cannot be predicted.

In Figure 1 it is clear that boundary condition affects the possible rise in temperature at a given tissue location. Adjusting the boundary condition will surely adjust the location of a desired maximum temperature rise.

Figure 2 reflects that the highest temperature rise can be located at about the mid-point of the tissue. This knowledge can be used effectively for centrally located tumours.

In Figure 3 and 4, the rise in temperature exhibited different profiles for different thermal conductivities. We can see that hot spot formation can be hindered with the aid of the thermal conductivity. Higher rise in temperature is possible for lower thermal conductivity. Similarly the behaviour of thermal conductivity can promote cold spot as seen in Figure 4. This will be useful in Hyperthemia.

In Figure 5, comparism is made of the rise in temperature with different thermal conductivity. This figure actually supports the discussion in Figure 2-4.

On the other hand, figure 6 reveals the possible rise due to a given heating device. This will guide the medical expert regulate the parameters used in design of the heating devices to achieve the maximum/desired temperature rise. The size of engine can also lead to a greater or smaller rise in temperature.

Figure 8 gives a clear picture of the effect of different thermal conductivity. Our result is a significant one for use by medical expert to promote effective microwave hyperthermia. Our result agrees with experimental findings of instability of temperature rise due to non-uniform deposition of heat in tissue. The effect of the thermophysical property will throw more light on the need to know precisely tissue properties before hyperthermia commences. These result no doubt improve medical technology and gives guides to engineer in the design of machine.

Fig. 1

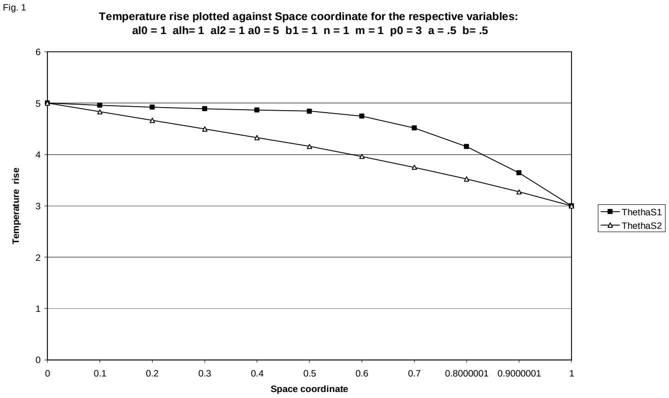


Fig. 2

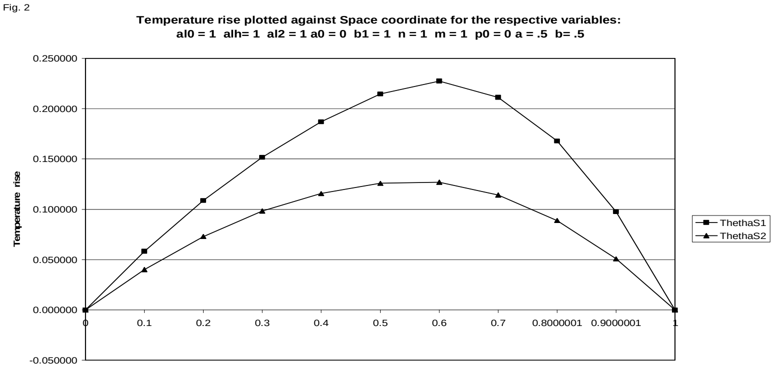


Fig. 3

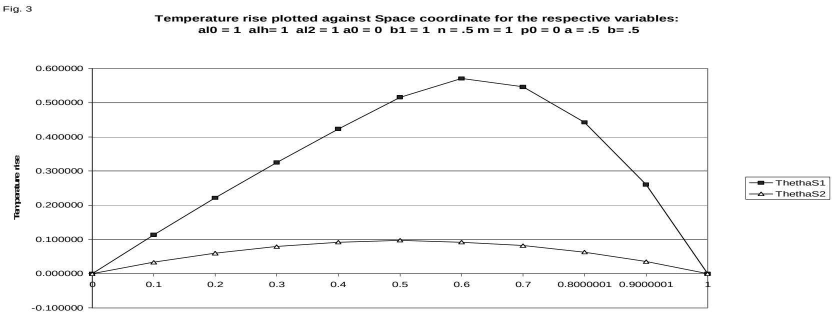


Fig. 4

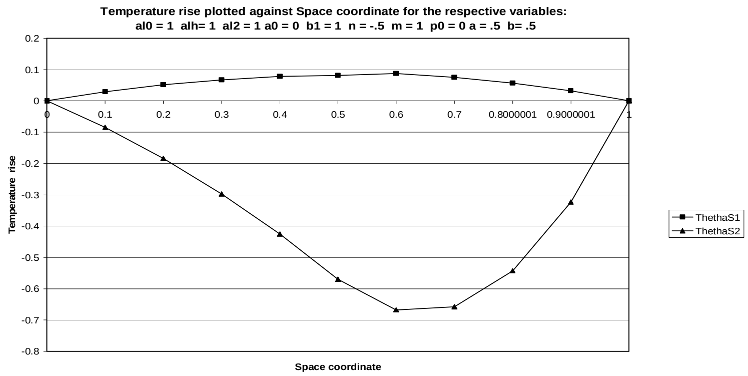


Fig. 5

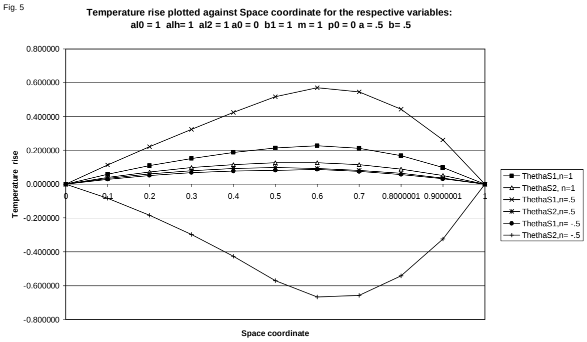


Fig. 6

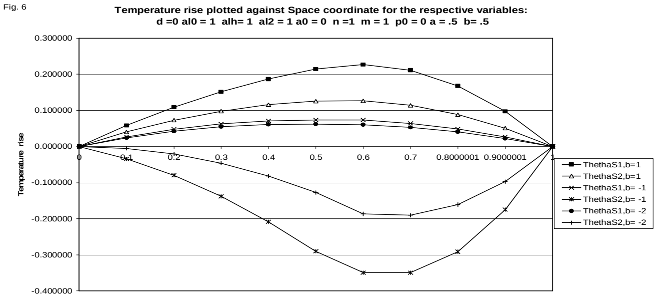


Fig. 7

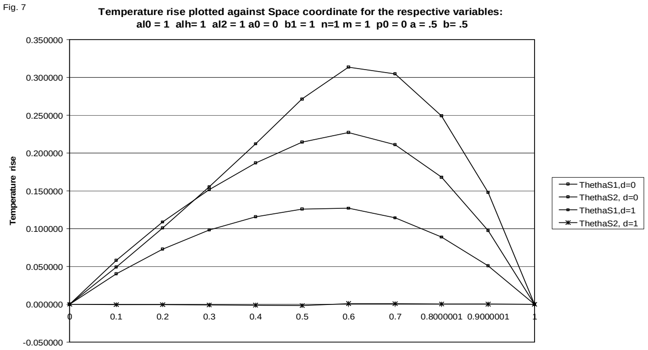
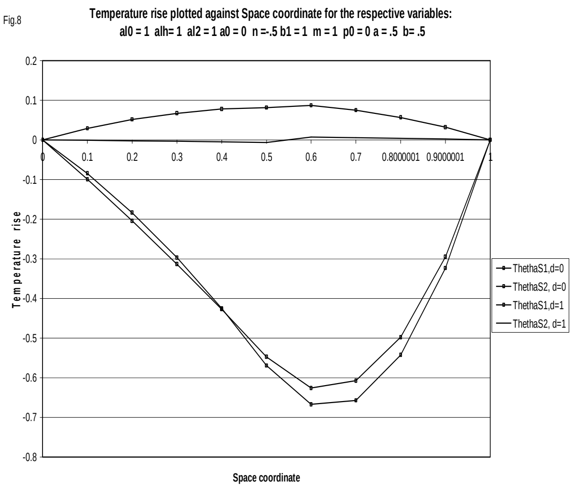


Fig.8



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