Journal of the Nigerian Association of Mathematical Physics,
Volume 8 (*November 2004*).

Lower hybrid waves instability in a velocity–sheared inhomogenous charged dust beam

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Abstract

An electrostatic linear kinetic analysis of velocity-sheared inhomogeneous charged dust streaming parallel to a magnetic field in plasma is presented. Excited mode and the growth rates are derived in the lower hybrid-like mode regime, with collisional effects included. In the case where the drift velocity u is very small the velocity shear ∇u would dominate over density anisotropy ∇n in providing the free energy to drive the relevant instability in space plasmas.

Keywords*:* Lower-hybrid waves; Dusty plasma, Space environment.

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1.0 Introduction

Plasma occurs all over our universe and dusty plasma regime consists, in addition to the usual neutral particles, ions and electrons, of dispersed charged dust particles. Such dusty plasmas are a main constituent of many space and astrophysical environments such as interstellar clouds, circum stellar clouds, asteroid zones, earth's atmosphere, planetary rings, interplanetary dust, nebulae, comet tail, etc. The charged dust particles play a significant role in the dynamics and wave behaviour of many natural systems. Several analyses, treating the dust as a charged particle species of uniform mass and charge, have shown that the presence of charged dust component leads to the appearance of new plasma modes arising from the grain dynamics. de Angelis *et al*. [1] studied the linear properties of ion acoustic waves in the presence of massive, immobile charged dust grains in an unmagnetized plasma. They applied their result in interpreting the low frequency electrostatic noise enhancement associated with Halley's comet. Rao *et al*. [2] considered the dynamics of a tenous dust fluid and assumed Boltzmann distributions for electrons and ions fluids. They predicted the existence of dust acoustic wave in dusty unmagnetized plasmas. D'Angelo [3] investigated low – frequency electrostatic waves in dusty magnetoplasma, and studied the ion acoustic and cyclotron modes. In a study of dust drift waves, it was found that the dust could modify the usual drift waves and also lead to the appearance of a dust drift wave arising from the dynamics of the dust grains [4]

In addition, the two-stream instability has also received some attention. The presence of charged dust in plasma can also affect the behaviour of plasma instabilities. Havnes [5] studied streaming instability between solar wind and commentary dust particles using a kinetic model in which the dust particles were drifting relative to a hot background of hot ions and electrons. The effect of dust particle dynamics on ion-ion two stream instability and two stream instability generated by drifting dust beams have been investigated Bharuthram *et al*. [6]. The existence of dust lower hybrid modes arising due to finite Larmor radius effect was demonstrated by Salimullah [7] while Roseberg *et. al*.[8] showed how the mode can be driven by negatively charged dust and went on to apply the result to radar backscatter from Space Shuttle exhaust in the ionosphere. Velocity-sheared dust induced instability was proposed in [9] as a possible means of interpreting the some observations in helical structures and streamer splitting in commentary tails. The authors, using fluid model, determined the critical dust shear for instability to occur. However, a more complete analysis, using the kinetic theory approach, is presented in this letter. We obtained marginal condition for instability and also the growth rate. Our study would be applicable to wider dusty plasma phenomena found in space with relative particle flows. This is because, most of the times, in space plasmas, gradients of parameters of moving particles can be approximated as weak.

2.0 Theory

We consider three-component plasma, which consists of singly, charged ions, electrons and charged beam of dust particles. The external magnetic field, **B,** and the dust relative velocity, u, are taken along the z-axis while the density in-homogeneity is assumed to be oriented along the x-axis i.e., across the field **B,** (Figure1).

Taking the quasi-neutrality condition for the system to be given as $Z_d n_d + n_e + n_i = 0$. The kinetic dispersion relation for electrostatic modes in a collision-less plasma becomes

$$D(\omega) = 1 + \chi_i + \chi_e + \chi_d = 0 \qquad (2.1)$$

The magnetized ions and electrons having Maxwellian distributions functions have their responses, respectively, as

$$\chi_i = \frac{\omega_i^2}{k^2 v_i^2}[1+\xi_i Z(\xi_i)]\Gamma_0(b_i)\left[1-\frac{i\nu_i}{kv_i}Z(\xi_i)\Gamma_0(b_i)\right]^{-1} \qquad (2.2a)$$

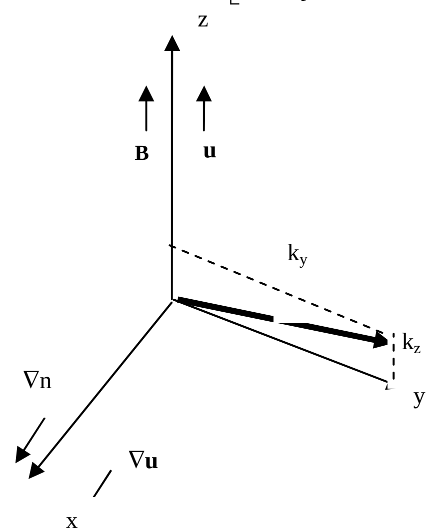

Figure 1: The coordinate geometry for the density gradient velocity-sheared dust, magnetic field B and wave-vector k.

and

$$\chi_e = \frac{\omega_e^2}{k^2 v_e^2}[1+\xi_e Z(\xi_e)]\Gamma_0(b_e)\left[1-\frac{i\nu_e}{k_z v_e}Z(\xi_e)\Gamma_0(b_e)\right]^{-1} \qquad (2.2b)$$

A weak inhomogenous charged dust beam moving with sheared velocity can also be represented by Maxwellian distribution as follows: $F(\lambda, x) = \frac{n_d(x)}{(2\pi v_d^2)^{3/2}} exp[-\lambda(x)]$, where $\lambda(x) = \frac{v_x^2 + v_y^2 + (v_z^2 - u(x))^2}{v_d^2}$. The response is obtained, following Alexandrov *et. al.* [10], from the operation: $\left(1-\frac{k_y v_d^2}{\omega\Omega_d}\frac{\partial}{\partial x}\right)\lambda(x)[\varepsilon_d - 1]$ where ε_d takes the form of χ above. The result is

$$\chi_d = \frac{\omega_d^2}{k^2 v_d^2}\left\{1+\left[\left(\xi_d - \frac{k_y v_d^2}{k_z \Omega_d}\frac{\partial \ln n_d}{\partial x}\right)Z(\xi_d) - \frac{k_y v_d^2}{k_z \Omega_d}\frac{\partial u}{\partial x}\frac{\partial Z(\xi_d)}{\partial \xi_d}\right]\Gamma_0(b_d)\right\}\times \quad (2.2c)$$

$$\left[1 - \frac{i\nu_i}{k_z v_d}Z(\xi_d)\Gamma_0(b_d)\right]^{-1}$$

The plasma, cyclotron and collision frequencies are $\omega_\alpha = 4\pi ne^2/m_\alpha$ $\Omega_\alpha = qB/m_\alpha$, and υ_α, respectively and; $\xi_i = (\omega+i\upsilon_i)/\sqrt{2}\, kv_i$, $\xi_e = (\omega+i\upsilon_e)/\sqrt{2}\, k_z v_e$, $\xi_d = (\omega-k_y u+i\upsilon_d)/\sqrt{2}\, k_z v_d$,. Z_d, e, $-e$ and m_d, m_i, m_e are the charges and masses of the charged dusts, ions and the electrons, respectively. $\Gamma_0(b) = exp(-b)I_0(b)$, $b_\alpha = k_y^2 r_{L\alpha}^2/2$, $r_{L\alpha} = v_\alpha/\Omega_\alpha$. I_0 is the modified Bessel function of order 0, $Z(\xi)$ is the plasma dispersion function r_L is Lamor radius and v_α is thermal velocity. In the lower hybrid case where $\Omega_d^2 << |\omega|^2 << \Omega_i^2 << \Omega_e^2$, $\xi_\alpha >> 1$ [11], $n_d << n_e, n_i$, $k_z << k_y$, $k_y^2 r_{Ld}^2 << 1$ and $\nu_e, \nu_i, \nu_d < \omega$, equation (1) becomes

$$D(\omega) \approx 1 - \frac{\omega_i^2}{a} + \frac{1}{b}\left(\frac{k_y^2}{k^2}\frac{\omega_e^2}{\Omega_e^2} - \frac{k_z^2}{k^2}\frac{\omega_e^2}{\omega^2}\right) + \frac{1}{c}\left(\frac{k_y}{k}\frac{k_z}{k}u'\frac{\omega_d^2}{\Omega_d\omega^2} + \frac{\sqrt{2}k_y}{k^2 L}\frac{\omega_d^2}{\Omega_d\omega}\right) -$$

$$i\frac{1}{c}\left(\frac{\sqrt{\pi}k_y\omega_d^2}{k^2 v_d k_z L\Omega_d}exp(-\xi_d^2) + \frac{\sqrt{\pi}\xi_d u' k_y \omega_d^2}{k_z\Omega_d k^2 v_d^2}exp(-\xi_d^2)\right) \quad (2.3)$$

where $a \approx \omega(\omega+i\nu_i) - k^2 v_i^2 \dfrac{i\nu_i}{\omega+i\nu_i}$, $b \approx 1 - \dfrac{k_z}{k}\dfrac{i\nu_e}{\omega+i\nu_e}\left(1-\dfrac{k^2 v_e^2}{\Omega_e^2}\right)$, $c \approx 1 - \dfrac{k_z}{k}\dfrac{i\nu_d}{\omega - k_y u + i\nu_d}$

$L = \dfrac{\partial \ln n_d}{\partial x}$ is scale length of dust inhomogeneity and ∇u or $u' = \dfrac{\partial u}{\partial x}$ is the dust velocity shear. Equation (2.3. is expanded to obtain

$$1 - \frac{\omega_i^2}{\omega^2} - \frac{k_z^2}{k^2}\frac{\omega_e^2}{\omega^2} + \frac{k_y^2}{k^2}\frac{\omega_e^2}{\Omega_e^2} + \frac{k_y}{k}\frac{k_z}{k}u'\frac{\omega_d^2}{\Omega_d\omega^2} + \frac{\sqrt{2}k_y}{k^2 L}\frac{\omega_d^2}{\Omega_d\omega} +$$

$$i\left(\frac{\nu_i\omega_i^2}{\omega^3} - \frac{k_y^2}{k^2}\frac{k_z}{k}\frac{\nu_e\omega_e^2}{\omega\Omega_e^2} - \frac{\sqrt{2}k_y k_z}{kLk^2}\frac{\nu_d\omega_d^2}{\Omega_d\omega^2} + \right.$$

$$\left.\frac{\sqrt{\pi}k_y\omega_d^2}{k_z L\Omega_d k^2 v_d}exp(-\xi_d^2) + \frac{\sqrt{\pi}k_y u'\omega_d^2}{k_z\Omega_d k^2 v_d^2}\xi_d\, exp(-\xi_d^2)\right) = 0 \quad (2.4)$$

having ignored the wave-particle resonance of the background plasma. We define $\omega = \omega_r + i\gamma$ and $D = D_r + iD_i$ where real $\omega \equiv \omega_r$; imaginary part of $\omega \equiv \gamma$ and real $D \equiv D_r$; imaginary part of $D \equiv D_i$ and assume $|\omega| >> |\gamma|$; $|D_r| >> |D_i|$. Therefore, the wave mode is obtained from $D_r(k,\omega_r) = 0$. However, in the limit where $L \to \infty$, the excited mode becomes

$$\omega_r^2 \approx (1+\mu)^{-1}\left[\omega_i^2 + \frac{k_z^2}{k^2}\omega_e^2 - \frac{k_y}{k}\frac{k_z}{k}u'\omega_d^2\right] \quad (2.5)$$

while the growth rate under the same condition is

$$\gamma = -\omega_r^3\left[\omega_i^2 + \frac{k_z^2}{k^2}\omega_e^2 - \frac{k_y}{k}\frac{k_z}{k}u'\omega_d^2\right]^{-1}\left(\frac{\nu_i\omega_i^2}{\omega_r^3} - \frac{k_y^2}{k^2}\frac{k_z}{k}\frac{\nu_e\omega_e^2}{\omega_r\Omega_e^2} - \frac{\sqrt{2}k_y k_z}{kLk^2}\frac{\nu_d\omega_d^2}{\Omega_d\omega_r^2} + \right.$$

$$\frac{\sqrt{\pi}k_y\omega_d^2}{k_z L\Omega_d k^2 v_d} exp(-\xi_d^2) + \frac{\sqrt{\pi}k_y u'\omega_d^2}{k_z\Omega_d k^2 v_d^2}\xi_d\, exp(-\xi_d^2)\Bigg) \qquad (2.6)$$

where $\mu = \left[\frac{k_y^2}{k^2}\frac{\omega_e^2}{\Omega_e} - \frac{k_y}{k_z}\frac{u'\omega_d^2}{\Omega_d k^2 v_d^2}\right]$

In the cold plasma approximation and negligible collision frequencies, (i.e. υ_i, υ_e, $\upsilon_d = 0$), the excited frequency mode becomes

$$\omega_r^2 = \frac{1}{(1+\mu^2)}\left[\omega_i^2 + \left(\frac{k_z}{k}\right)^2\omega_e^2 + \left(\frac{k_z}{k}\right)\left(\frac{k_y}{k}\right)u'\omega_d^2\right] \qquad (2.7)$$

The condition for instability in this case is approximately

$$\frac{1}{\Omega_d}\frac{\partial u}{\partial x} > \frac{1}{Z_d^2}\left(\frac{k^2\omega_i^2}{k_y k_z\omega_d^2} + \frac{k_z\omega_e^2}{k_y\omega_d^2}\right) \qquad (2.8)$$

3.0 Conclusion

We have found a resonant instability for velocity-sheared charged dust streaming in plasma condition. The wave, which is near the lower hybrid frequency, is excited by the velocity gradient. The threshold for the onset of the instability in cold plasma limit was determined. The theory presented here should be applicable in wide ranges of phenomena in space plasmas occurring due to charged dust with low but sheared relative velocity, and weak density gradient.

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