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The use of third degree polynomial for accurate conversion of seismic time to depth and vice versa

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Abstract

The seismic sections consist of observed travel times versus corresponding impact to geophone distances. These observed times correspond to some depth below the earth surface. In converting these times to depth, various methods may be used. However, in this paper, the conversion is done by solving the third degree polynomial using data from well survey. The results from this study shows that the third degree polynomial can be conveniently fitted to the set of time depth data from the well as it compared favourable with measured values. The result also reveals that the third degree polynomial is a more accurate means of converting the values of seismic time to depth than the use of velocity information. The standard deviation of the polynomial values calculated from the measured values is 6.25milliseconds compared to 7.08milliseconds using the velocity data.

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1.0 **Introduction**

The seismic method is by far the most important geophysical technique in terms of expenditures and number of geophysicists who are involved in the development [1, 12]. Exploration seismic method involves the generation of sound waves and the recording of the time required for the waves to travel from the sources to a series of geophones usually arranged in line [10]. The raw seismic field data therefore consist of observed travel times versus corresponding impact to geophone distances [6.9].

Seismic sections can be converted from a time scale to a depth scale by using velocity information either from the seismic data or well data. Using the seismic data, the interval velocity is determined from stacking velocities. This is then multiplied with the time to give the depth of such horizon. On the other hand, well data may be most precise. It consists of suspending a geophone or hydrophone in the well by means of a cable and recording the time required for energy to travel from a shot fired near the well down to the geophone [8]. The geophone is usually moved between shots so that the results are a set of travel times from the surface down to various depths. A graph of this may be plotted to give the nature of the distribution.

Results of experimental work in science like the one carried out in this study can be displayed in a number of ways. It may be possible to make definitive statements about the relationship between the dependent and independent variables. On the other hand, we may know nothing of the physical relation between the two variables. Usually, our problems lie between these two extremes. In many cases, we can fit an approximate equation or some mathematical function to the set of data because of its functional relation with the data and with the hope that these will shed some light on the underlying relations. There are several approximating equations available, but the most commonly used is a polynomial expansion [3].

In this study, the method of functional approximation using polynomial has been employed in converting the seismic time to depth. In it, the third degree polynomial is fitted to the set of time-depth data collected from the well survey. It is obvious that the velocity information or equations are usually in first degree. However, using a function of higher degree gives a more accurate result [3]. The choice of the third degree was as a result of the functional relationship between the plot of a third degree polynomial and the time depth plot as shown in Figure 1.

2.0 Theoretical background

Given a continuous function, it is convenient to represent the function by a polynomial whose degree depends on the shape of the function. The data under study, which is plotted in Figure 1, correspond to a portion of a third degree plot [5]. In using the third degree polynomial for the conversion of depth to time and vice versa, we will define a third degree polynomial as [4].

$$Z = A + BT + CT^2 + DT^3 \tag{2.1}$$

where Z is the true depth and T is a variable called time.

In order to achieve a desired degree of accuracy, it is convenient to represent the data by the cubic spline. Such an approximation is continuous with discontinuities at the ends of the given interval.

The final forms of the system are as shown in equations (2.2) and (2.3) [11].

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} N & \sum T & \sum T^2 & \sum T^3 \\ \sum T & \sum T^2 & \sum T^3 & \sum T^4 \\ \sum T^2 & \sum T^3 & \sum T^4 & \sum T^5 \\ \sum T^3 & \sum T^4 & \sum T^5 & \sum T^6 \end{bmatrix} \tag{2.2}$$

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} N & \sum Z & \sum Z^2 & \sum Z^3 \\ \sum Z & \sum Z^2 & \sum Z^3 & \sum Z^4 \\ \sum Z^2 & \sum Z^3 & \sum Z^4 & \sum Z^5 \\ \sum Z^3 & \sum Z^4 & \sum Z^5 & \sum Z^6 \end{bmatrix} \tag{2.3}$$

The above notwithstanding, the conversion of time to depth and vice versa using velocity information was also carried out. The equation for velocity at any given depth is [2, 12].

$V(z)=V_0+Kz,\ V(z)=\frac{dz}{dt}=V_0+Kz$. Hence $dt=\frac{dz}{V_o}+Kz$. Integrating both sides gives

$$t=\frac{1}{K}ln\left(1+\frac{Kz}{V_O}\right)+B_O, \qquad B_O=\frac{lnV_O}{K}+B$$

where B is the constant of integration.
t = 0, corresponds to z = 0, thus B_O = 0
when t = T, z = Z and one obtains equation (2.4)

$$T=\frac{1}{K}ln\left(\frac{KZ}{V_O}+1\right) \tag{2.4}$$

Inverting equation (2.4) gives equation (2.5)

$$Z=\frac{V_O}{K}(Exp(KT)-1) \tag{2.5}$$

3.0 Results and calculations

From the well survey, the time-depth data as shown in Table 1 were recorded. The depth is measured in meters while the time is measured in milliseconds.

Preparing and substituting the values of time and depth (TZ) into equations (2.2) and (2.3) yields equations (3.1) and (3.2).

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}=\begin{bmatrix} 17 & 16.65 & 18.54 & 23.82 \\ 15.65 & 18.54 & 23.32 & 31.91 \\ 18.54 & 23.91 & 31.91 & 43.80 \\ 23.82 & 31.91 & 43.80 & 61.14 \end{bmatrix}^{-1}\begin{bmatrix} 19.40 \\ 23.36 \\ 30.30 \\ 40.83 \end{bmatrix} \tag{3.1}$$

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}=\begin{bmatrix} 17 & 19.40 & 29.50 & 49.25 \\ 19.40 & 29.50 & 49.25 & 86.11 \\ 29.50 & 49.25 & 86.11 & 154.67 \\ 49.25 & 86.11 & 154.67 & 282.73 \end{bmatrix}^{-1}\begin{bmatrix} 15.65 \\ 23.36 \\ 38.60 \\ 67.09 \end{bmatrix} \tag{3.2}$$

Equations (3.1) and (3.3) were solved and substituted into equation (2.1) first for depth and then for time to give equations (3.3) and (3.4).

$$Z = 0.0104 + 0.9786T + 0.1786T^2 + 0.00251T^3 \tag{3.3}$$
$$T = -0.0119 + 1.04062Z - 0.2015Z^2 + 0.0327Z^3 \tag{3.4}$$

where Z is in kilometers and T in seconds. The best linear velocity fit for $V(Z)$ was also obtained using the field data as V(Z) = 6661 + 0.644Z where 6661.8 is the intercept and 0.644 the slope of the straight line plot of the field data. Equation (2.4) and (2.5) becomes

$$T=\frac{2000}{0.644}ln\left[\frac{3.28*0.644Z}{6661.8}+1\right] \tag{3.6}$$

$$Z=\frac{6661.8}{3.28*0.644}\left[exp\left(\frac{0.644T}{2000}\right)-1\right] \tag{3.7}$$

where Z is in meters and T in milliseconds. Using equations (3.6) and (3.7), a comparison of the measured time from the well and the one calculated using both velocity information and the polynomial times were prepared and presented in Figures 2 and 3. The standard deviation (*SD*) of the calculated polynomial time values and that of the velocity are carried out using equation (3.8) [7].

$$SD = \sqrt{\frac{\sum\left(T_P - T_M\right)^2}{N}} \qquad (3.8)$$

where T_p is the polynomial time calculated using equation (3.5) and T_m is the measured time obtained from the well survey. The calculation reveals that the *SD* for the polynomial time is 6.25milliseconds while the *SD* for the velocity time is 7.08milliseconds.

4.0 Discussion of results

A comparison of the measured values of time and depth and that calculated from polynomial and velocity information was carried out as shown in Figures 2 and 3. It reveals that the third degree polynomial gives a more accurate means of estimating the values of seismic time and depth better than using the velocity information. From equations (3.4) and (3.5), it is very convenient to interpolate within the range of the data given and also to extrapolate beyond given data. The calculation of the standard deviation reveals that the deviation of the polynomial values from the measured value is 6.25milliseconds, which is less compared to that of the velocity value that is 7.08milliseconds.

5.0 **Conclusions**

Based on the findings from this study, the polynomial method of Time-Depth conversion is recommended. The present work has shown that the method can be used quite successfully. Also, the method yielded fairly reasonable result when compared with results obtained from well (actual) measurement. It is known that one millisecond of error in terms of conversion to depth could mean about five to ten feet hence the polynomial method will be most useful because of the closeness of its calculated values to the measured values. The above not withstanding, it is necessary to state the range of the reliability when using the polynomial method.

Table 1: Time depth data from the seismic survey

Number	Depth (meters)	Time (*msec*)
1	0.00	0.00
2	162.03	150.40
3	323.58	301.40
4	405.87	371.60
5	558.27	503.80
6	771.63	680.00
7	893.55	782.30
8	1045.95	898.20
9	1198.35	1010.20
10	1381.23	1140.40
11	1533.63	1230.40
12	1655.55	1302.40
13	1777.47	1378.40
14	1853.67	1428.40
15	1887.20	1458.40
16	1929.87	1480.40
17	2021.31	1536.40

Table 2: Comparison of measured and calculated values

No.	Actual Depth (m)	Actual Depth (km)	Measured Time Tm($msec$)	Polynomia Time Tp (msec)	Tp – Tm (msec)	Velocity Time Tv (msec)	Tv – Tm (msec)
1	0.00	0.00	0.00	-11.90	-11.90	0.00	0.00
2	162.03	0.162	150.40	151.50	1.10	155.62	5.22
3	323.58	0.3236	301.40	304.83	3.43	303.36	1.96
4	405.87	0.4059	371.60	379.40	7.80	376.00	4.40
5	558.27	0.5583	503.80	511.89	8.09	506.18	2.38
6	771.63	0.7716	680.00	686.12	6.12	679.69	-0.31
7	893.55	0.8936	782.20	780.35	-1.85	774.65	-7.55
8	1045.95	1.0460	898.20	893.52	-4.68	889.39	-8.81
9	1198.35	1.1984	1010.20	1002.09	-8.11	1000.03	-10.17
10	1381.23	1.3812	1140.40	1127.15	-13.25	1127.79	-12.61
11	1533.63	1.5336	1230.40	1228.02	-2.38	1230.37	-0.03
12	1655.55	1.6556	1302.40	1306.97	4.57	1310.06	7.66
13	1777.47	1.7775	1378.40	1381.52	3.12	1387.74	9.34
14	1853.67	1.8537	1428.40	1432.94	4.54	1435.33	6.93
15	1887.20	1.8872	1458.40	1454.06	-4.34	1456.04	-2.36
16	1929.87	1.9299	1480.40	1480.89	0.49	1482.19	1.79
17	2021.31	2.0213	1536.40	1538.26	1.86	1537.51	1.11

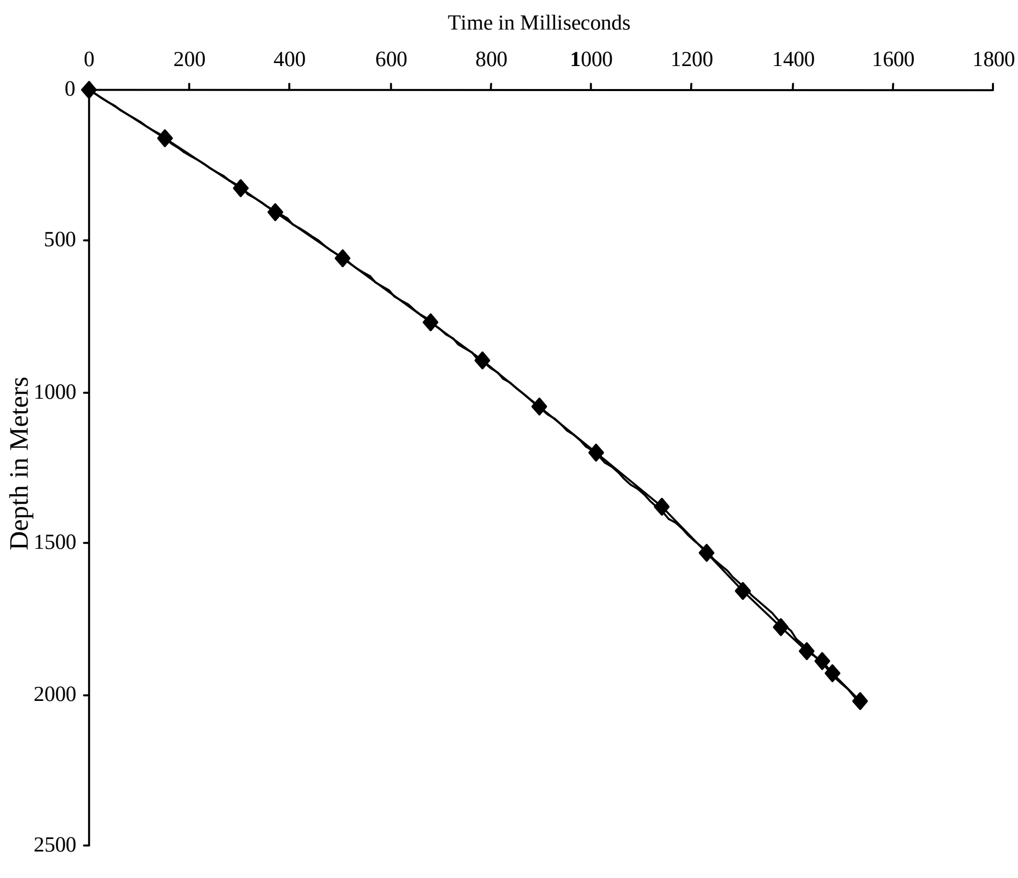

Figure 1: A plot of depth versus time

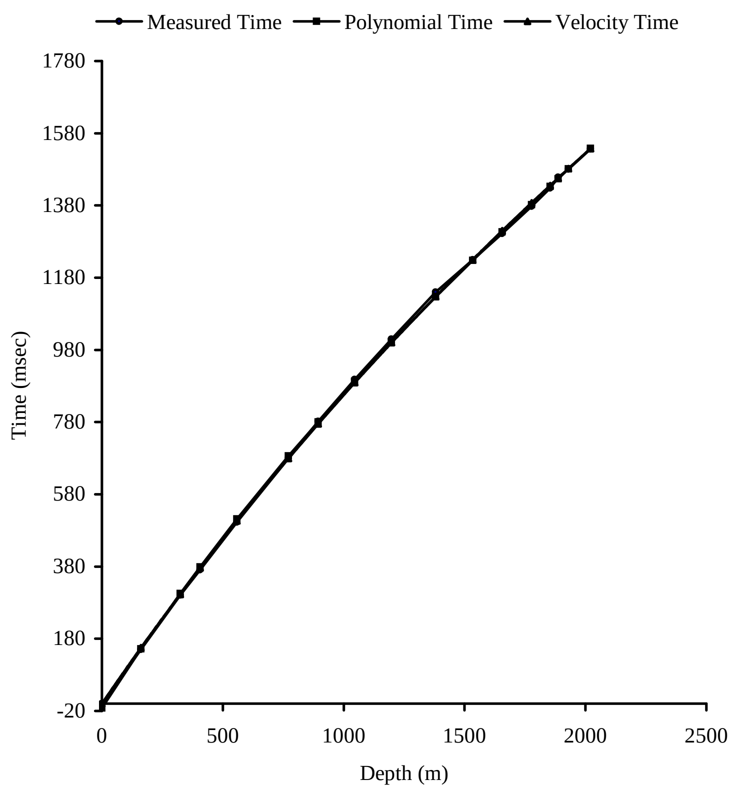

Figure 2a: Comparison of measured and calculated values.

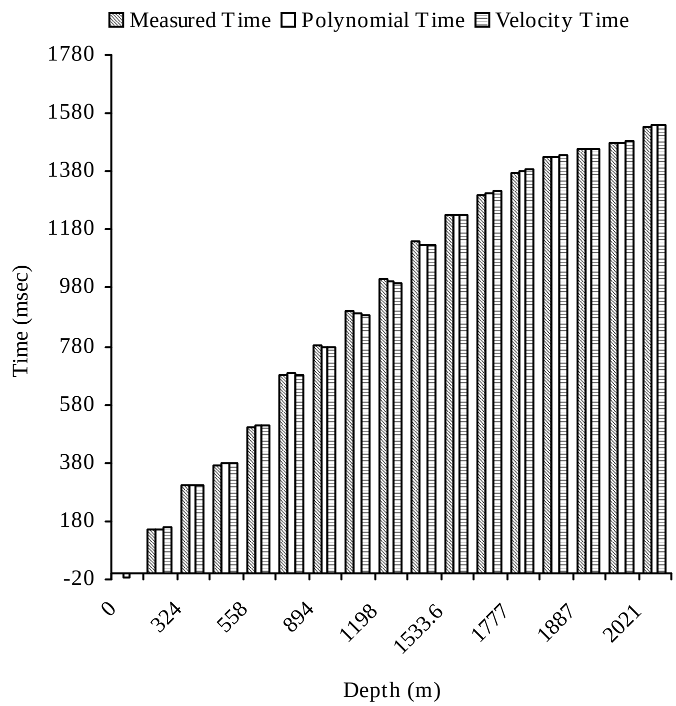

Figure 2b: Bar chart for the comparison of measured and calculated values.

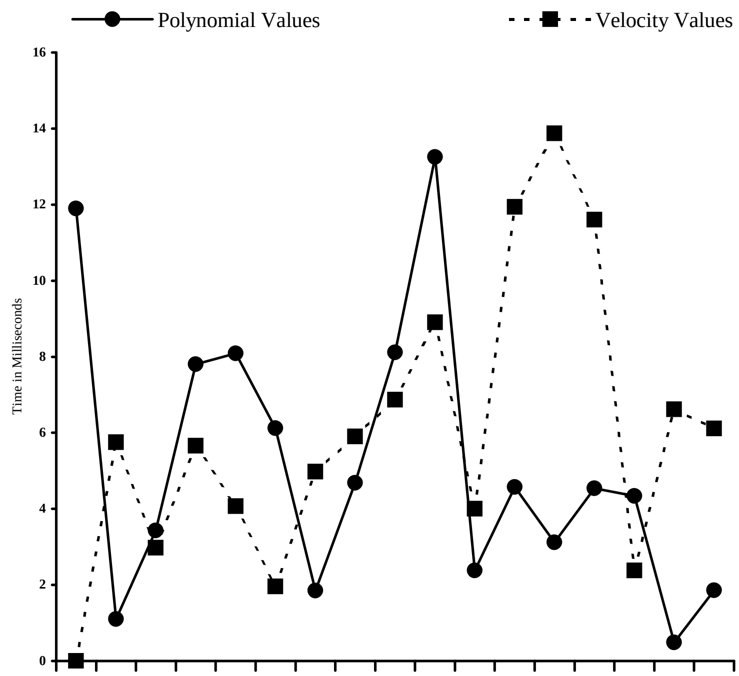

Figure 3: Comparison of difference in absolute residuals between polynomial and velocity values

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