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Non-uniqueness in the interpretation of resistivity sounding -I

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Abstract

Non-uniqueness in the interpretation of resistivity soundings is a known major problem in Vertical Electrical Soundings results because it is extremely difficult to derive a suitable earth model that fits the field curve uniquely even in Idogun and Lonla. The aim of this paper is to confirm this fact about electrical resistivity interpretation in Idogun and Lonla, and to suggest solution, which is being demonstrated in Non-Uniqueness in the interpretation of resistivity soundings-II. The solution to the problem is by conducting four or more VES at 50-100m apart using the same electrode array and electrode spacing factors. The geoelectric layers are correlated across all the VES to identify and confirm their thicknesses and apparent resistivities These layer thicknesses and apparent resistivities are used to compute the curve for each VES.Howover,this is a better alternative method than the drilling of control boreholes to acquire logs which has been the practice.

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1.0 **Introduction**

Despite the operational simplicity of the resistivity method, “the problems of interpretation are among the most difficult in geophysics,” [2]. The quantitative interpretation of vertical electrical sounding curves is hampered by the well-known principle of equivalence, which means that many different layered models may produce practically the same resistivity curve. To select the model that best represents the true conditions of the subsurface, additional hydrogeologic information is needed [8]. The subject of electrical equivalence in the 1-D inversion of resistivity sounding data has received considerable attention in literature [4, 6, 9]. Hence, inversion of the data cannot resolve the true layer parameters for sounding curves, which differ by only a few percent. Conversely, most of the inversion algorithms used in geoelectrical topography have focused largely on the construction of images of the potential distribution and little on the uncertainty, or non-uniqueness, of such images even though it is recognized that the possibility for equivalence exists in 2-D inversion [5].

Many researchers prefer curve matching interpretation techniques in order to evade the mathematical problems of theoretically generating curves that will match their field data curves. In curve matching interpretation method field curves are matched with already existing sets of computer generated curves. Unfortunately however, even when a catalogue of such curves in their thousand is available for isolated VES, it is possible not to get a single theoretical curve that will perfectly match a given field curve but a group of VES in an area can enable compute unique geoelectric section for each curve. The resistivity fieldwork described here was carried out at Idogun and Lonla in Ondo state.

2.0 **Theoretical analysis**

A differential equation, which is the basis of all resistivity prospecting with direct current, is given by

$$\nabla \sigma_{ij} \nabla V = 0 \qquad (2.1)$$

where σ_{ij} is conductivity and V is potential. In the isotropic case the conductivity at the point in the ground is independent of direction, equation (2.1) reduces to Laplace's equation.

$$\nabla^2 V = 0 \qquad (2.2)$$

Solutions to equations (2.1) and (2.2) may be developed for a particular model of the earth by selecting a coordinate system to match the geometry of the model and by imposing appropriate boundary conditions. With a model of horizontal, homogeneous and isotropic layers, it is necessary to find the solution to Laplace's equation as expressed in equation (2.2) for the potential at the surface of the distance, *r*, from the current source.

Ehrenburg and Watson in [1] pursued the optical analogy and developed a solution for any number of layers of fixed thickness *h*. This restriction on thickness ensures that the positions of current images are readily predictable. The surface potential was formulated as:

$$V(r) = \frac{I\rho_1}{2\Pi}\left[\frac{1}{r} + 2\sum_{n=1}^{\infty}\frac{Q_n}{\left(r^2 + 4n^2h^2\right)^{1/2}}\right] \qquad (2.3)$$

By applying separation of variables to Laplace's equation in cylindrical coordinates in [7] were able to arrive at a general solution for the potential at the surface of an n-layered earth having arbitrary resistivities and thickness.

$$V(r) = \frac{l\rho_i}{2\Pi}\left[\frac{1}{r} + 2\int_0^{\infty}\theta_n(\lambda)\, J_0(\lambda r)\, d\lambda\right] \qquad (2.4)$$

where *J*o is the zero- order Bessel function of the first kind and θ_n, called the kernel function, is a function of the thickness and reflection coefficients for an assumed earth model. By differentiating equation (2.4), the Schlumberger apparent resistivity over an n-layer earth becomes:

$$\rho_a(r) = \rho_1\left[1 + 2r^2\int_0^{\infty}\lambda\,\theta_n(\lambda) J_1(\lambda r)\, d\lambda\right] \qquad (2.5)$$

where J_1 is the first-order Bessel function of the first kind.

Ghosh in [3] introduced a novel approach to the problem of computing sounding curves for stratified models by starting with the integral formula of [7], equation (2.5), and expressed it as

$$\rho_a(r) = r^2\int_0^{\infty}\lambda T(\lambda) J(\lambda r) d\lambda \qquad (2.6)$$

where $T(\lambda) = \rho_1[1 + 2\theta_n(\lambda)]$. The function $T(\lambda)$ is called the resistivity transform function because it is defined by a Hankel transformation

$$T(\lambda) = \int_0^{\infty} r^{-1}\rho_a(r)\, J(\lambda r)\, dr \qquad (2.7)$$

Equation (2.6) is a convolution integral. Therefore, it is possible to determine a linear digital filter $\{b_i\}$, which converts resistivity transform samples into apparent resistivity values for theoretical models:

$$\rho_a(i) = \sum_i b_i T_{m-i} \qquad (2.8)$$

The method is accurate, fast, and simple in operation and has small computer storage requirements. In addition, depths are no longer restricted to integral multiples and may take any arbitrary values.

3.0 **Experimental work**

The fieldwork was carried out in two towns (Idogun and Lonla) in Ondo state. A total of eight Schlumberger Vertical Electrical Soundings stations were conducted using the ABEM SAS 300C Terrameter and the SAS 2000 Booster with four VES in Idogun and four VES in Lonla.Current Electrode saparation(AB/2) varied from 1m to 147m in Idogun and from 1m to 215m in Lonla. The direction of expansion of the electrodes was constrained by topography in Idogun though it is desirable that array should be expanded parallel to probable strike so as to minimize the effect of non-horizontal beddings.

The end result of the field measurement is the computation of an apparent resistivity.The resulting data were plotted as curves of apparent resistivity (ℓ) in Ohm-m against electrode separation (AB/2) in metre using log-log sheet. This constitutes the field curve which was interpreted qualitatively and quantitatively.The quantitative interpretation was curve matching and computation techniques.

4.0 Results and discussion

We employed the principle of equivalence to interpret the curves for two sets of layers parameter which are more conservative and the interpreted depth to the basements were computed for each layer parameter. The results obtained are shown in Tables 1-8 and Figures 1and 2.

Table 1: The two-geoelectric sections for Idogun VES 1

Layer	Model		Model 2	
	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	270.83	1.03	378.40	1.03
2	57.33	8.81	57.33	7.28
3	2440.87	∞	107.00	3.40
4	-	-	3348.25	∞
Depth to the weathered basement (m)	9.84		11.71	
RMS Errors (%)	2.90		2.93	

Table 2: The two-geoelectric sections for Idogun, VES 2.

Layer	Model 1		Model 2	
	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	602.00	1.07	602.00	1.08
2	102.00	1.60	102.00	1.60
3	43.00	4.15	43.00	2.60
4	990.84	∞	102.00	3.40
5	-	-	1004.68	∞
Depth to the weathered basement (m)	6.82		8.68	
RMS Error (%)	1.38		1.22	

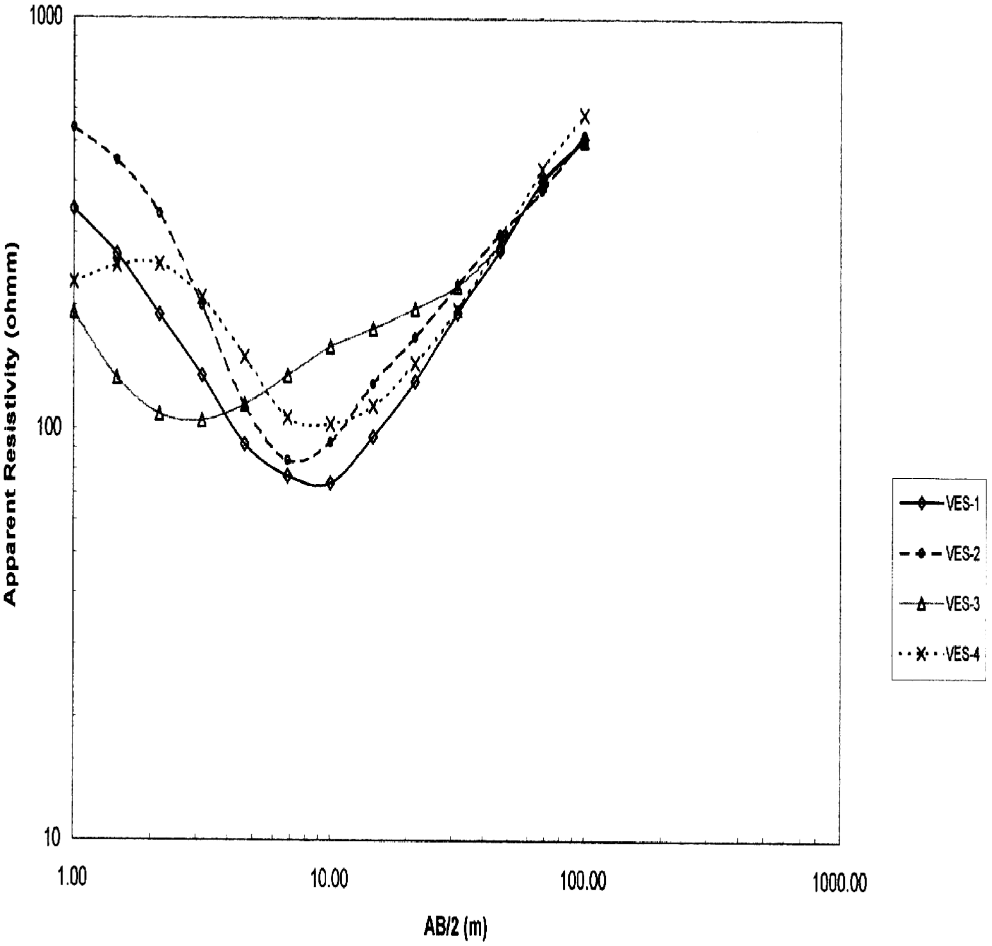

Figure 1: VES curves for Idogun Town.

Table 3: The two-geoelectric sections for Idogun, VES 3.

Layer	Model 1		Model 2	
	Resistivity (Ω*m*)	Thickness (*m*)	Resistivity (Ω*m*)	Thickness (*m*)
1	224.00	0.73	224.00	0.73
2	36.64	0.84	36.64	0.80
3	306.00	4.97	306.00	4.97
4	110.44	12.30	110.44	8.40
5	1838.74	∞	374.00	13.15
6	-	-	1838.74	∞
Depth to the basement (*m*)	18.84		28.05	
RMS Error (%)	1.87		1.85	

Table 4: The two-geoelectric sections for Idogun, VES 4.

Layer	Model 1		Model 2	
	Resistivity (Ω*m*)	Thickness (*m*)	Resistivity (Ω*m*)	Thickness (*m*)
1	221.00	0.73	221.00	0.73

2	395.91	0.82	395.91	0.82
3	96.44	1.90	91.85	1.90
4	26.47	2.84	24.01	1.60
5	1858.22	∞	184.00	9.26
6	-	-	2549.00	∞
Depth to the basement (m)	6.29		14.31	
RMS Error (%)	2.74		2.22	

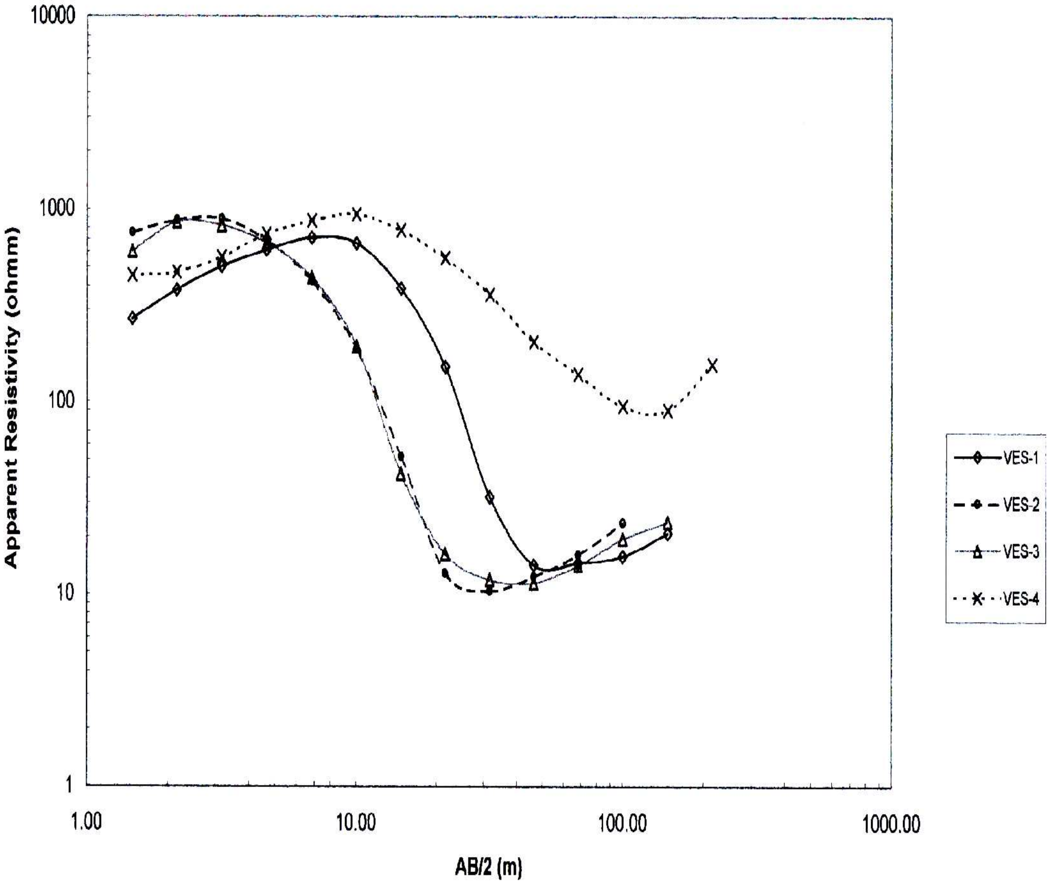

Figure 2: VES curves for Lonla.

Table 5: The two-geoelectric sections for Lonla, VES 1.

Layer	Model 1		Model 2	
	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	165.00	0.73	165.60	0.73
2	1889.80	2.48	1889.80	2.48
3	32.00	1.13	32.00	1.13
4	8.04	44.78	8.40	18.00
5	367.20	∞	12.36	45.47
6	-	-	408.32	∞
Total Depth	49.12 -		67.81 -	
RMS Error (%)	5.18		4.97	

Table 6: The two-geoelectric sections for Lonla, VES 2.

Layer Layer	Model 1		Model 2	
	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	537.00	0.73	537.00	0.73
2	1899.00	1.50	1899.00	1.50
3	19.45	5.80	19.45	5.08
4	5.76	23.10	5.76	23.10
5	408.00	∞	32.00	12.78
6	-	-	408.02	∞
Total Depth	31.13 -		43.19 -	
RMS Error (%)	4.15		4.32	

Table 7: The two-geoelectric sections for Lonla, VES 3.

Layer	Model 1		Model 2	
	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	484.64	0.73	484.64	0.73
2	1899.00	1.50	1899.00	1.50
3	19.45	5.08	19.45	5.08
4	6.91	23.10	6.91	23.10
5	43.81	∞	16.73	39.00
6	-	-	408.00	∞
Total Depth	30.41 -		69.41 -	
RMS Error (%)	6.07		6.78	

Table 8: The two-geoelectric sections for Lonla, VES 4.

Layer	Model 1		Model 2	
	Resistivity (Ωm)	Thickness (m)	Resistivity (Ωm)	Thickness (m)
1	471.00	0.66	471.00	0.66
2	283.50	0.80	283.50	0.80
3	1823.00	4.37	1823.00	5.10
4	242.04	25.00	144.21	46.26
5	7.80	∞	14.74	18.90
6	-	-	1336.65	∞
Total Depth	40.83 -		71.72 -	
RMS Error (%)	2.94		3.50	

Equivalence in the interpretation of these curves lies in the determination of the layer parameters of the intermediate layer. The two models gave theoretical curves that practically fit the field curve at approximately the same RMS error level.

Note that the change in the specific resistivity does not affect the lithological equivalent of the layer. However, the bedrock is seriously affected and this can have serious consequences on conclusions drawn especially if the study involves the determination of depth to the bedrock or any geologic layer of interest.

5.0 Conclusion

The results indicate that it may not always be possible to model accurately and uniquely the depth to the bedrock or any geologic layer for one VES. This work suggests that geophysicists should conduct 4 or more VES in the neighbourhood of 50 – 100m apart, correlate the thicknesses of each geoelectric layer so as to confirm the true depth

indices and resistivity contrasts among successive horizons (layers) in each VES, and use the thicknesses and apparent resistivities to compute the corresponding curve.

Acknowledgements

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References

[1] Ehrenburg,D.O.,and Watson,R.J.,(1932).Mathematical theory of electrical flow in stratified media with horizontal homogeneous and isotropic layers.Trans.Amer.Inst.Min.Eng.Geophysical Prospecting,Vol.97 , pp.423-442.

[2] Frohlich, R.K, (1973) Detection of fresh water aquifers in the glacial deposits of northwestern Mossouri by geoelectrical methods. Water Resources Bulletin.Vol. 9, No. 4, pp 723-734.

[3] Ghosh, D.P, (1971.)Inverse filter coefficient for the computation of apparent resistivity standard curves for a horizontally stratified earth.Geophs.Prosp., Vol.19,No.4., pp.769-775.

[4] Koefoed, O.(1979). Geosounding Principles1: resistivity sounding measurements. Elsevier Science Publishing Co.

[5] Olayinka, A.I., (2000). Aspects of equivalence in two-dimensional resistivity imaging and modeling. Jounal of Mining and Geology.Vol.36, No.1, pp.67-76.

[6] Simms, J. E., and Morgan, F.D., (1992). Comparison of four least – squares inversion schemes for studying equivalence in one – dimensional resistivity interpretation. Geophysics, Vol. 57, pp.1282-1293.

[7] Stefanesco, S.,Schlumberger, C., and Schlumberger, M., (1930).Sur.la distribution electrique potentielle en tour d'une prise de terre pon etuelle dens un terrain a couches horizontales, homogene et isotropes, J. de Physique et.le Radium. Series 7, Vol. 1. pp.132-140.

[8] Van Overmeeren,R.A., (1989). Aquifer boundaries explored by geoelectrical measurements in the coastal plain of Yemen: A case of equivalence. Geophysics. Vol. 54. No. 1, pp. 38-48.

[9] Zhdanov, M.S., and Keller, G.V., (1994). The geoelectrical method in geophysical exploration. Elsevier Science.