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Tidal flow in the Escravos Bar, Warri, Nigeria

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Abstract

Nature has placed a lot of resources at the disposal of mankind to annex for his uses. Day by day man is grasping the enormity of the potential at his disposal. It is now possible to use tidal flows in the generation of electricity. Escravos Bar, Warri is one of many estuaries in the Nigeria coastline where tidal flows occur. We have derived the profile of the tidal flow in terms of the tidal current and surface elevation in the estuary. The nodal line does not exist within the Gulf. Therefore, tides throughout the length of the Bar are at high water (or low water) as the oceanic tides and the height of the tides increases (or decreases) from the entrance of the gulf to the head of the gulf.

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1.0 Introduction

Tidal flow is the movement of open water from one place to another. It occurs due to the gravitational pull effects of the sun and the moon [1]. Nigeria is bounded in the South by a large expanse of the Atlantic Sea and there are many estuaries to the sea. One of these estuaries is the Escravos River. The Escravos Bar, Warri is an inlet of the Sea Water into the Coastland to form a Gulf.

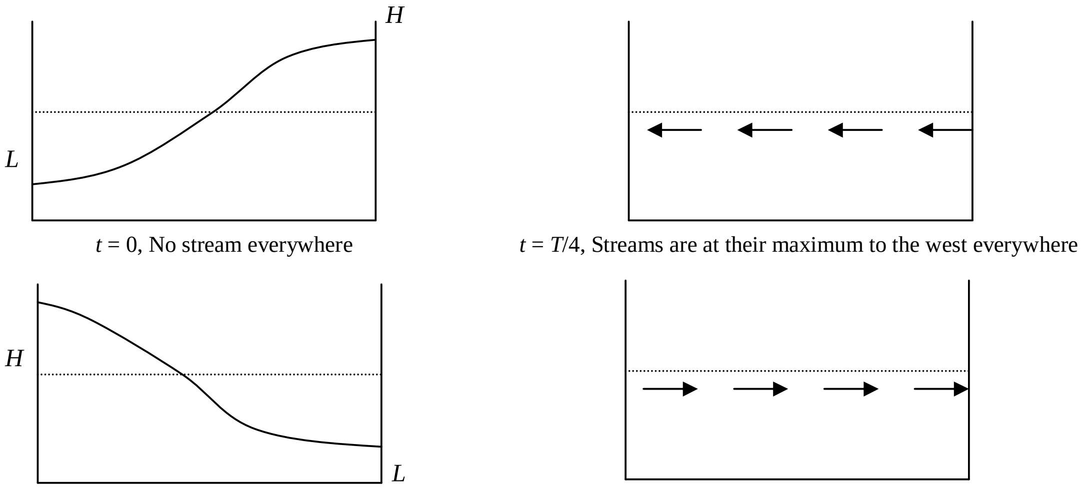
All over the world attempts are made everyday to annex nature to the benefit of mankind. One area receiving great attention presently is looking for ways of increasing power generation. Large amount of energy can potentially be derived from tides where there are large tidal ranges or narrow channels with fast tidal currents. The British Broadcasting Corporation on Sunday, February 9, 2003 reported plans to build underwater electric generator in the Severn Estuary [2]. The generators were expected to use giant propellers that are fixed on the seabed and turned by the passing tide in the estuary, which has one of the highest tides in the world. To prove this point, Marine current and Tidal flow turbines for renewable energy reported Tidal flow to power New York City [3].

In this work we consider one of our many Estuaries, the Escravos Bar, to determine the tidal flow therein and hoping that in future the benefit of nature can be properly annexed.

2.0 Theory of oscillations in water

Consider an elongated basin in the East West direction. Consider a periodic motion of period T, which oscillates in a seesaw fashion. The vertical movement will be greatest at the extreme edges of the basin. If the basin has a simple shape there will be a line near the centre of the basin corresponding to the pivot of the seesaw. Here thus there is no rise or fall of the surface on this line. Such a line is called a nodal line.

Consider a sequence of events commencing at $t = 0$ and let the water be at rest but the surface not level. Let there be an elevation of the surface on the east of the basin and a depression on the surface on the west. In this condition there will be pressure gradient so that the currents from the east are about to start. Therefore at high water, the volume of the water in the east must begin to diminish so that the stream across the nodal section turns at high water. By the time $t = T/4$ (where T is the period of oscillation) the current will have reached their maximum values and the surface of the water will level and the volume of the water on the west is about to rise. At $t = T/2$, the surface of the water will have maximum elevation on the west and a maximum depression on the east and the current then is zero everywhere. Thus slack water occurs at high and low waters. Since there will be pressure gradient, this will then start the current from west to the east. By $t = 3T/4$, these currents will hall reached their maximum values and the surface of the water will again be level and the volume of the water on the east is about to increase such that by $t = T$, there is a maximum depression on the west. These conditions are illustrated in Figure 1.



t = 0, No stream everywhere; t = T/4, Streams are at their maximum to the west everywhere

t = T/2, No stream everywhere; t = 3T/4, Streams are at their maximum to the east everywhere

Figure 1: Oscillation in a basin.

Commencing from when t = 0, there is high water H on the east and low water L on the west of the nodal line. Also there are no streams everywhere. At t = T/4, the surface is level and the streams are at their strongest everywhere though the rates vary according to the distance from the end of the basin. The greatest rate occurs over the nodal section and the streams are weakest near the boundary ends across which no flow can take place. Analogous to this is the movement of water in the ocean. The rising of the water is called a flood tide and the falling is called an ebb tide. The movement of water with the flood and ebb tides creates tidal currents that reverse direction with the rising and falling water levels. At high and low tides when the tidal currents stop temporarily before reversing their flows, there is a period of slack water.

3.0 Tidal co-oscillation in a gulf

Consider the basin in the above illustration and suppose the west of the lake is replaced by the open sea so that the remaining part of the lake becomes a gulf as shown in Figure 3 where B denotes the transverse section which bounds the remaining part of the lake. Consider an oscillation, which to the west of B is the same as the longitudinal oscillation. Then in the open sea, there must be an oscillation, which at B produces the same elevations and current as the original oscillation. This oscillation is the harmonics constituent of the tides and the oscillation in the gulf is called a tidal co-oscillation. For a gulf the period T of the oscillation will not depend on the dimension of the gulf but it is prescribed by the tide at the end of the channel [4].

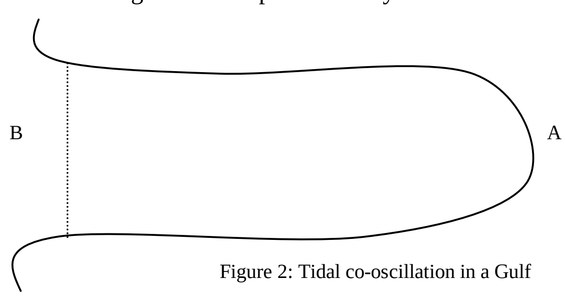


Figure 2: Tidal co-oscillation in a Gulf

The elevation along B is determined by the tides of the open sea however the features of the co-oscillation will depend on the dimension of the gulf. The first nodal line N from the head A of the gulf may be on either side of B. If N is further from A than B is then at any one time the elevation will have the same phase at all points of the gulf. In particular high water will everywhere within the gulf occur at the same time as it does at B. This is shown in Figure 3(a).

If N is nearer to A than B is, the nodal line will divide the area of the gulf into two parts such that when it is high water over one part, it is low water over the other part. Therefore high water occurs at the same moment everywhere on one side of the nodal line as shown in Figure 3b).

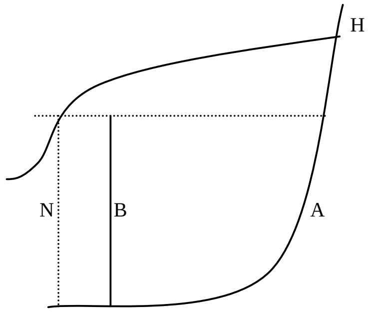


Figure 3(a): Position of Nodal line outside the Gulf

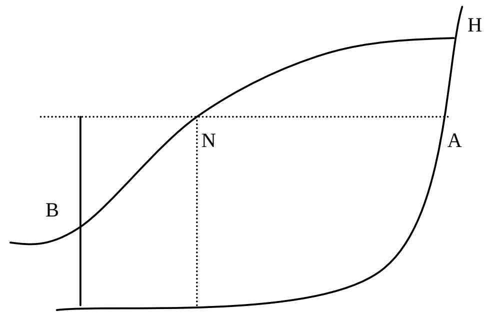


Figure 3(b): Position of Nodal line within the Gulf.

4.0 Mathematical formulation

For a basin with regular shape the flow is governed by the one-dimensional equations of continuity and motion:

$$\frac{\partial (AU)}{\partial x} + b\frac{\partial \eta}{\partial t} = 0 \quad (4.1)$$

$$\frac{\partial U}{\partial t} + g\frac{\partial \eta}{\partial x} = 0 \quad (4.2)$$

where x is the coordinate normal to the tidal crest or shoreline, A is the area of the transverse section of the water, t > 0 is the time, $U(x,\ t)$ is the current, b the breadth of the basin, $\eta(x, t)$ the elevation of the surface from the undisturbed level (this describes high and low water levels in the estuary) and g is the acceleration due to gravity. We can eliminate $\eta(x, t)$ from equations (4.1) and (4.2) by differentiation of equation (4.1) with respect to x and equation (4.2) with respect to t and subtracting to get

$$b\frac{\partial^2 U}{\partial t^2} - g\frac{\partial^2 (AU)}{\partial x^2} = 0 \quad (4.3)$$

If b is uniform and h is the uniform depth of the water from the undisturbed surface

$$A = bh \quad (4.4)$$

Equation (4.3) then reduces to

$$\frac{\partial^2 U}{\partial t^2} - gh\frac{\partial^2 U}{\partial x^2} = 0 \quad (4.5)$$

Similarly we can follow the same order to eliminate $U(x,\ t)$ in equations (4.1) and (4.2) to get

$$\frac{\partial^2 \eta}{\partial t^2} - gh\frac{\partial^2 \eta}{\partial x^2} = 0 \quad (4.6)$$

Equations (4.5) and (4.6) are the well-known simple harmonic wave equations satisfied by $U(x, t)$ and $\eta(x, t)$. We now superimpose two of such waves moving in opposite directions. Each of these represents simple harmonic oscillation of a progressive wave. If we consider a uni-nodal oscillation which is at high water at t = 0, the boundary conditions on $U(x,\ t)$ and $\eta(x, t)$ are given as

$$U(x, 0) = U(x, T/2) = 0 \quad (4.7a)$$

$$U(0, t) = U(\lambda/2, t) = 0 \quad (4.7b)$$

$$\eta(x, T/4) = \eta(x, 3T/4) = 0 \quad (4.8a)$$

$$\eta(\lambda/4, t) = \eta(3\lambda/2, t) = 0 \quad (4.8b)$$

where λ is the wavelength of the oscillation

By the principle of superposition and considering the dominant mode the solutions of equations (4.5) and (4.6) subject to the boundary conditions (4.7) and (4.8), we have

$$U(x, t) = A\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T} \quad (4.9)$$

$$\eta(x, t) = B\frac{Th}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T} \quad (4.10)$$

and

$$T = \frac{\lambda}{\sqrt{gh}} \tag{4.11}$$

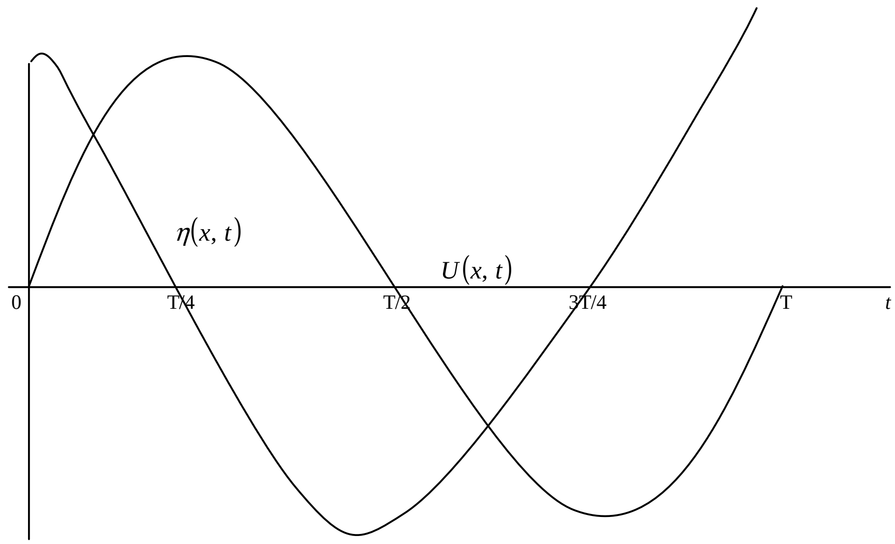


Figure 4: The graph of $\eta(x, t)$ and $U(x, t)$ against t

If the basin is of length L, and we are considering only one nodal line within the basin then

$$\lambda = 2L \tag{4.12}$$

$$U(x, t) = A \sin\frac{\pi x}{L} \sin\frac{2\pi t}{T} \tag{4.13}$$

$$\eta(x, t) = B\frac{Th}{2L} \cos\frac{\pi x}{L} \cos\frac{2\pi t}{T} \tag{4.14}$$

$$T = \frac{2L}{\sqrt{gh}} \tag{4.15}$$

Equation (4.15) is known as Merian formula and shows the relationship between the period of a uni-nodal oscillation *T* and the length of the basin [4]. The nodal line occurs at half the length of the basin

$$x = \frac{1}{4}T\sqrt{gh} \tag{4.16}$$

5.0 Mathematical analysis

We now consider the Escravos Bar as a gulf of rectangular shape and a constant depth. The profile of the tide in the gulf is dependent on the dimension of the gulf. The relationship between the length of the gulf and the period of oscillation is given as

$$L = \tfrac{1}{2}T\sqrt{gh} \tag{5.1}$$

and the nodal line occurs at

$$x = \tfrac{1}{4}T\sqrt{gh} \tag{5.2}$$

from the shore line.

The necessary condition for the existence of a nodal line in a gulf of length L' is that

$$L' \geq \tfrac{1}{2}T\sqrt{gh} \tag{5.2}$$

High water/m	Time	Period T/hr
1.4	10.10	
1.2	23.13	13.05
1.4	11.23	12.17
1.3	00.39	13.27
1.4	12.46	12.17
1.3	02.01	13.05
1.4	14.06	12.08
1.5	03.08	13.03
1.4	15.12	12.07
1.6	04.02	12.33
1.6	16.07	12.08
1.7	04.48	12.78
1.7	16.55	12.17
1.8	05.30	12.58
1.7	17.40	12.17
1.8	06.10	12.50
1.7	18.22	12.20
1.7	06.48	12.43
1.6	19.03	12.55
1.7	07.24	12.35
1.5	19.47	12.38
1.6	08.02	12.25
1.4	20.33	12.52
1.5	08.42	12.15
1.3	21.27	12.75
1.4	09.24	11.95
1.2	22.31	13.17
1.3	10.17	12.23
1.2	23.51	13.57

Table 1: High Water, High water time and periods of Oscillations.

The period of oscillation is the time interval between two consecutive high waters or low waters at a particular place. However due to the variations in the force generating tides, the period of oscillation is not constant. In this work we have considered the 28 oscillations in the first fifteen days of the month of September, 2003 as available from the Tides Table prediction of the Nigerian Navy [5]. The time of high waters is used in getting the period of each oscillation. The mean period of oscillation $\overline{T}$ is then calculated as the average of these periods. The periods of oscillations are as given in Table 1 from which the mean period $\overline{T} = 12.51\,hours$.
We also found out that this is the average mean period for the low waters.
The mean depth of the Escravos Bar is h = 4.33m and its length is 8667m [6]

$$L' = 8667m$$

$$L = \tfrac{1}{4}T\sqrt{gh}$$

$$= \tfrac{1}{4}\ x\ 12.51\ x\ 60\ x\ 60\ x\ \sqrt{9.81\ x\ 4.33}m$$

$$= 73380.84m$$

This shows that

$$L' < L \qquad (5.4)$$

This implies that the nodal line does not exist within the gulf but in the ocean. Thus tide throughout the length of the Bar is at high water (or low water) as the oceanic tide. Also the range of the tide increases (or decreases) from the entrance of the gulf to the head of the gulf.

5.0 Conclusion

We have derived the profile of the tidal flow in terms of the tidal current and surface elevation in the Estuary. The nodal line does not exist within the Gulf. Therefore tides throughout the length of the Bar are at High water (or Low water) as the oceanic tides and that the range of the tides increases (or decreases) from the entrance of the gulf to the head of the gulf.

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