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Semistate Equations of Duffing Van Der Pol and Bonhoffer Van Der Pol Electronic Circuits With Memristor

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Abstract

In this paper, we derived and solved the Semistate equations obtainable from both memristive extended Duffing Van Der Pol "DVPo" oscillator and extended Bonhoeffer Van Der Pol "BVPo" oscillator respectively. The aim is to characterize and compare the tractability index of these models under passivity assumptions, a key issue for the numerical and experimental simulation of circuit dynamics. We also extend these index analyses to strictly passive circuits including memristors, and a projector-based tractability index along the image of the leading matrix which was used as mathematical discussion. The solutions obtained showed that the extended BVPo and the extended DVPo equations were of index two models due to their combination of memristors' fluxes and charges, and which also indicate the property of being a chaotic system.

Keywords: Semistate equation , Memristor, Duffing Van Der Pol, Bonhoeffer Van Der Pol, Tractability index, Index characterization, Index two model.

1.0 Introduction

Memristor theory was formulated and named byLeon Chua [1]. A team at HP Labs announced the development of a switching memristor based on a thin film of titanium dioxide in which a regime of operation with an approximately linear charge-resistance relationship as long as the time-integral of the current stays within certain bounds. These devices are being developed for application in a nanoelectronic memories, computer logic, and neuromorphic computer architectures[1,2].

Like other two-terminal components (*e.g.*, resistor, capacitor, inductor), real-world devices are never purely memristors ("ideal memristor"), but will also exhibit some amount of capacitance, resistance, and inductance. Other scientists had already proposed fixed nonlinear flux-charge relationships, but Chua's theory introduced generality[3,4].

The systematic use of memristors within power electronics engineering will require some effort at the circuit modeling level and the formulation of these models is feasible because of the low number of devices used in the circuits[5]. However in integrated electronics, where millions of devices are involved, the formulation of state models is not easily automatable. Instead, semistate models based on differential-algebraic equations(DAEs) are preferred[6,8,9], following the seminal work of Dziurla and Newcomb[7,10].

A major problem in DAE modeling of electrical and electronic circuits is the characterization of the model index. The index can be seen as a measure of the system sensitivity to input perturbations, and also as a measure of the difficulties to be found in the numerical simulation of the system dynamics.In the strictly passive context, the index of different DAE models of circuits composed of resistors, inductors, capacitors, voltage and current sources is known to be no greater than two[11,12].

In this paper we extend these index analyses to an extended Duffing Van Der Pol and extended Bonhoffer Van Der Pol systems with memristor as the best component to replace the non linear parts of the systems.

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2.0 Models

2.1Memristor- Mathematical Formulations

The memristor is a two terminal element, in which the magnetic flux (Φ) between the terminals is a function of the electric charge that passes through the device [3]. The memristor M used in this work is a flux controlled memristor that is characterized by its incremental menductance (1) function M(Φ) which is the inverse of memristance $W(\Phi)$,describing the flux-dependent rate of change of charge:

$$M(\Phi) = \frac{dq(\Phi)}{d\Phi} \quad (1)$$

The relationship between the voltage across$(v(t))$ and the current through $(i(t))$the memristor is thus given by:

$$i(t) = W\big(\Phi(t)\big)v(t) \quad (2)$$

Memristor is an acronym for memory-resistor, and the integral operator on the menductance function(3) means the function remembers the past history of voltage values. In equation (3), memristor is simply a resistor.

$$W(\Phi(t)) = W(\textstyle\int v(t)) \quad (3)$$

A memristor is an electrical device governed by a nonlinear flux-charged relation of the type

$$g(\Phi, q) = 0 \quad (4)$$

If we assume the device to be charge-controlled, the increment memristance takes the form

$$W(q) = \frac{d\Phi(q)}{dq} \quad (5)$$

Considering elements such as the resistor, capacitor, and inductor. The common thread that binds these four elements together as the four basic elements of circuit theory is the fact that the characteristics of these elements relate the four variables in electrical engineering (voltage, current, flux and charge) intimately. Fig. 1 shows this relationship graphically:

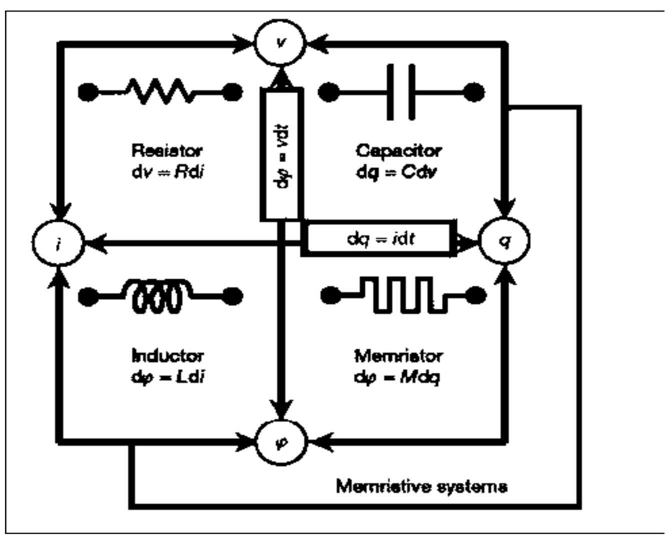

Fig 1:The Four Basic Elements

2.2 Device Models

I. q-devices

Definition 1: A q-device is a circuit element which admits a C^1 description of the form

$$v = \eta(q, i, t). \quad (6)$$

The terminology comes from the fact that q-devices may be controlled by the charge q (this will be the case for capacitors), its time derivative $q' = i$ (for current-controlled resistors) or both (for q-memristors). Voltage sources will be included in this group for the sake of completeness.

Depending on the actual form of the map η in (6), q-devices particularize to the ones listed below. It is worth noting that η need not be a scalar map; this accommodates coupling effects within the different types of q-devices. The derivative of η does not vanish identically; in the first case, it may find a zero at certain points, though; for instance, it vanishes at $i = 0$ in Chua's memristor ($v = M(q)i$).

1. A q-memristor is a q-device for which

$$\frac{\partial \eta}{\partial q} \not\equiv 0, \qquad \frac{\partial \eta}{\partial i} \not\equiv 0.$$

The map governing the set of q-memristors will be denoted by $v_{m=}\eta_1(q_m, i_m, t)$.

2. A capacitor is a q-device for which

$$\frac{\partial \eta}{\partial q} \not\equiv 0, \qquad \frac{\partial \eta}{\partial i} \equiv 0.$$

The relation governing the set of capacitors will be denoted by $v_c = \eta_2(q_c, t)$.

3. A current-controlled resistor is a q-device for which

$$\frac{\partial \eta}{\partial q} \equiv 0, \qquad \frac{\partial \eta}{\partial i} \not\equiv 0.$$

The map governing the set of current-controlled resistors will be written as $v_r = \eta_3(i_r, t)$.

4. An independent voltage source is a q-device for which

$$\frac{\partial \eta}{\partial q} \equiv 0, \qquad \frac{\partial \eta}{\partial i} \equiv 0.$$

The relation governing independent voltage sources will be denoted by $v_u = \eta_4(t)$. In a time-invariant setting, if η is linear in i then we get Chua's memristor, for which $v = M(q)i$. For this reason we use the expression "fully nonlinear" to label memristors with the general form $v = \eta(q, i)$. This should cause no misunderstanding with the nonlinear nature of the original flux-charge relation φ=ϕ(q) in Chua's setting. Note also that these memristors are referred to in the literature either as charge-controlled memristors or as current-controlled memristors; with the expression "q-memristor" we want to emphasize that the charge q is the dynamic variable actually involved in the description of the device, as it will happen with the flux φ in φ-memristors.

II. φ-devices

Definition 2: A φ-device is a circuit element which admits a C^1 description of the form

$$i = \zeta(\varphi, v, t). \qquad (7)$$

The different types of φ-devices are enumerated below:

1. A φ -memristor is a φ-device for which

$$\frac{\partial \zeta}{\partial \varphi} \not\equiv 0, \qquad \frac{\partial \zeta}{\partial v} \not\equiv 0.$$

The map governing the set of φ-memristors will be denoted by $i_w = \zeta_1(\varphi_w, v_w, t)$.

2. An inductor is a φ-device for which

$$\frac{\partial \zeta}{\partial \varphi} \not\equiv 0, \qquad \frac{\partial \zeta}{\partial v} \equiv 0.$$

The relation governing the set of inductors will be denoted by $i_l = \zeta_2(\varphi_{l,} t)$.

3. A voltage-controlled resistor is a φ-device for which

$$\frac{\partial \zeta}{\partial \varphi} \equiv 0, \qquad \frac{\partial \zeta}{\partial v} \not\equiv 0.$$

The map governing the set of voltage-controlled resistors will be written as

$$i_g = \zeta_3(v_g, t).$$

4. An independent current source is a φ-device for which

$$\frac{\partial \zeta}{\partial \varphi} \equiv 0, \qquad \frac{\partial \zeta}{\partial v} \equiv 0.$$

The relation governing independent current sources will be denoted by $i_j = \zeta_4(t)$.

Again, Chua's flux-controlled memristor [10], for which $i = W(\varphi)v$, is obtained in particular when ζ is time-independent and linear in v. We do not impose any condition on the time derivatives, either for q- or φ-devices. If they vanish in memristors, resistors, capacitors and inductors we would be led to a time-invariant setting, with $\eta_k, \zeta_k (k = 1, 2, 3)$ being independent of t. For sources this assumption would model DC ones. It is also worth remarking that current-controlled voltage sources (CCVS's)could be easily included in the above taxonomy as q-devices and, similarly, voltage-controlled current sources (VCCS's) can be modeled as φ-devices.

2.3 Digraphs

Many properties of an electrical circuit can be expressed in terms of its underlying directed graph or digraph. We compile below, for later use, some elementary notions coming from digraph theory: details can be found e.g. in [6,11,12].

Let n and b stand for the number of circuit nodes and branches. After choosing a reference node, the reduced incidence matrix $A \in \mathbb{R}^{(n-1)xb}$ is defined as (a_{ij}) with

$$a_{ij} = \begin{cases} 1 & \text{if branch j leaves node i} \\ -1 & \text{if branch j enters node i} \\ 0 & \text{if branch j is not incident with node i.} \end{cases}$$

Provided that the circuit is connected, the reduced incidence matrix has maximal row rank. The matrix A is split as (Ar Ac Al Am Av Ai), where Ar (resp. Ac, Al, Am, Av, Ai) describes the incidence between resistive (resp. capacitive, inductive, memristive, voltage source, current source) branches and nodes[6,11,12].

A key role in our analysis will be played by certain types of loops and cutsets. A subset K of the set of branches of a connected digraph is a cutset if the deletion of K results in a disconnected digraph, and it is minimal with respect to this property (namely, the deletion of any proper subset of K does not disconnect the digraph). Loops and cutsets defined by specific types of branches can be characterized in terms of the incidence matrix, as stated in Lemmas 1 and 2 below. Both are standard results in graph theory. We denote by A_{κ}(resp.$A_{\mathcal{G}-\kappa}$) the submatrix of the reduced incidence matrix formed by the columns defined by the branches in K (resp. not in κ).

2.4 The Tractability Index

The index of DAE models arising from a very broad class of strictly passive circuits is known to be no greater than two, and for this reason we restrict the introduction of the tractabil- ity index to problems with index ≤ 2, avoiding some technical difficulties which arise in the general case. The importance of the matrices $E_i(x)$ introduced below emanates from the fact that their invertibility (which defines the tractability index of the DAE) supports a decoupling procedure which unveils the solutions of the system. Assume that $C(v_c)$ and $L(i_l)$ are non-singular; this condition will be met in our working setting because of the assumption that these matrices are positive definite. It follows that the kernel of the leading matrix E(x) is constant. Write $F(x) = -f^{'}(x)$ and let Q be a constant projector onto kerE(x) . The quasilinear DAE is said to have tractability index one if

$$E_1(x) = E(x) + F(x)Q \qquad (8a)$$

is non-singular. Suppose now that $E_1(x)$ has constant (non-maximal) rank and that there exists a continuous projector $Q_1(x)$ onto ker$E_1(x)$. Write $F_1(x) = F(x)P$, with $P = I - Q$. System is then said to have tractability index two if

$$E_2(x) = E_1(x) + F_1(x)Q_1(x) \qquad (8b)$$

is non-singular. Note that this definition of the index is simpler than the one for general nonlinear DAEs, being feasible because of the special form of the circuit equations.

These notions mean that, in this context, the computation of the tractability index relies on the construction of the matrices E_1 and (when E_1 is singular) E_2, by means of the projectors Q, Q_1 onto the kernel of the leading matrices E, E_1. From a local point of view this is equivalent to the computation of the Kronecker index of the matrix pencil {E(x∗), F(x∗)} at any x = x∗, as stated in the following results.

Lemma 3: Let { E, F} be a matrix pencil with singular E, and assume that Q is any projector onto ker$E$. Then the pencil is regular with Kronecker index one if and only if

$$E_1 = E + FQ \qquad (9)$$

is a non-singular matrix. If E_1 is singular, denote $P = I - Q$ and let Q_1 be any projector onto kerE_1. Set $F_1 = FP$. Then the pencil is regular with Kronecker index two if and only if

$$E_2 = E_1 + F_1Q_1 \qquad (10)$$

is non-singular. These properties do not depend on the specific choices of the projectors Q and Q_1.Previous characterizations of the tractability index in circuit theory rely on this construction . However, the following result of Griepentrog and Marz [13] will turn out to be useful in our analysis.

Lemma 4: Let { E, F} be a matrix pencil with singular E, and assume that R is any projector along imE. Then the pencil is regular with Kronecker index one if and only if

$$E_1^R = E + RF \qquad (11)$$

is a non-singular matrix. If E_1^R is singular, let R_1 be any projector along imE_1^R . Then the pencil is regular with Kronecker index two if and only if

$$E_2^R = E_1^R + R_1(I - R)F \qquad (12)$$

is non-singular. These properties do not depend on the specific choices of the projectors R and R_1.

2.5 Analysis Assumptions

(a) Theorem1: *For nodal system of index 2, the conductance,memristance, capacitance and inductance memristances G, M, C, L will be assumed to be positive definite, and the circuits will have neither voltage sources loops nor current sources cutsets.*

(b) Theorem 2: *System is index 1 in the absence of VC-Loops and IL-Cutsets, and index two in the presence of at least one VC-Loop including capacitors and/or at least one IL-Cutset including inductor.*

3.0 Experimental Work

3.1Semistate Circuit Models of an Extended Duffing Van Der Pol System and its Index

The nodal equations for a circuit composed of resistors, memristors, inductors, capacitors, voltage and current sources can be then written in the following differential-algebraic form:

$$\begin{aligned}
CV_c &= i_c \\
Li_l &= e_1 \\
q_m &= i_m \\
0 &= i_l - i_v \\
0 &= i_l + i_m + i_c \\
0 &= V_s - e_1 \\
0 &= i_m - i_c \\
0 &= M(q_m)i_m - e_2
\end{aligned} \qquad (13)$$

Here $V_c, i_l, q_m, e_1, e_2, i_v, i_c, and\ i_m$ stand for voltage across the capacitor, current across the inductor, memristor charge, node voltages, current through the voltage, capacitor and inductor, respectively. Arranging the column according to the order $V_c,\ i_l,\ q_m,\ e_1,\ e_2,\ i_v,\ i_c,\ and\ i_m$. Choosing the projector Q. Therefore, from equation (1) we have:

$$E = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & M(q_m) \end{bmatrix}$$

From Lemma 3, E_1= E +FQ reads

$$E_1 = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & L & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & M \end{bmatrix} \qquad (14)$$

The matrix E_1 will be singular if and only if so it's determinant of

$$E_1 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & M \end{bmatrix} \neq 0 \qquad (15)$$

A vector (x, y, z, u) of the kernel of the matrix must verify

$$\begin{aligned} z + u &= 0 \\ -z + u &= 0 \\ -x + Mu &= 0 \end{aligned} \qquad (16)$$

Considering E_1 as a singular matrix having the form depicted in

$$Q_1 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 & -C^{-1} & -C^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & L^{-1} & L^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \qquad (17)$$

$$F_1 = FP = F[I - Q]$$

$$F_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_2 = E_1 + F_1Q_1 = E_1 + F[I - Q]Q_1$$

Then,
$$E_2 = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & L & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & (2L)^{-1} & 1+(2L)^{-1} & 0 & 0 \\ 0 & 1 & 0 & 0 & (2L)^{-1} & (2L)^{-1} & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & M \end{bmatrix} \quad (18)$$

The determinant of $E_2 = -\frac{1}{2}C[4L + M + 2]$, where the degree of freedom for E_2 is three i.e C, L and M.

3.2Semistate Circuit Models of an Extended Bonhoeffer Van Der Pol System and its Index

The nodal equations for a circuit composed of resistors, memristors, inductors, capacitors, voltage and current sources can be then written in the following differential-algebraic form:

$$\begin{aligned} Li_l &= e_1 - e_2 \\ C\dot{e}_1 &= -i_l - i_m \\ C\dot{e}_2 &= i_l - e_2r^{-1} \\ q_m &= i_m \\ 0 &= i_c + i_l - i_m \\ 0 &= i_v + i_c \\ 0 &= V_s - e_1 \\ 0 &= -e_2 + e_3 + M(q_m)i_m \end{aligned} \quad (19)$$

Here $V_s, i_l, q_m, e_1, e_2, e_3, i_v, i_c, and\ i_m$ stand for voltage across the source, current across the inductor, memristor charge, node voltages, current through the voltage, capacitor and inductor, respectively,$\dot{e}_1 and\ \dot{e}_2$ also stands for the currents across the two nodes 1and 2 of the circuit. Arranging the column according to the order $i_l,\ q_m,\ e_1,\ e_2, e_3, i_c,\ i_v\ and\ i_m$. Choosing the projector Q. Then equation (7) becomes:

$$E = \begin{bmatrix} L & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -r^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & M(q_m) \end{bmatrix}$$

From Lemma 3, $E_1 = E + FQ$ reads

$$E_1 = \begin{bmatrix} L & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & C & -r^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & M \end{bmatrix} \quad (20)$$

$$F_1 = FP = F[I - Q]$$

$$F_1 = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -r^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

The matrix E_1 will be singular if and only if so it's determinant of

$$E_1 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & M \end{bmatrix} \neq 0 \quad (22)$$

A vector $[x, y, z, u]$ of the kernel of the matrix must verify

$$\begin{aligned} y - u &= 0 \\ y + z &= 0 \\ x + Mu &= 0 \end{aligned} \quad (23)$$

Considering E_1 as a singular matrix having the form depicted in

$$Q_1 = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 & -L^{-1} & -L^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & C^{-1} & L^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$F_1 = FP = F[I-Q]$

$$F_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$E_2 = E_1 + F_1Q_1 = E_1 + F[I-Q]Q_1$, then

$$E_2 = \begin{bmatrix} L & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & (2L)^{-1} & (2L)^{-1} & 0 & 0 \\ 0 & 0 & C & -r^{-1} & -(2L)^{-1} & -(2L)^{-1} & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -(2L)^{-1} & 1-(2L)^{-1} & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -0.5 & -0.5. & 0 & M \end{bmatrix} \quad (25)$$

The determinant of $E_2 = -\frac{1}{2}L^2[L+M+1]$, and the degree of freedom is three.

4.0 Result and Discussion

In Semistate equation (13) for an extended Duffing Van Der Pol system, non-varnishing in equation (16) of state variable x indicates presence of IL-cutset and presence of z which is inductor [Theorem 2], shows that the system is index two. Also, the determinant of E_1 in equation (15) is non-zero which indicate that the system is not index one, and the existence of determinant $E_2 \neq 0$ in equation (18) with three degree of freedom "C, L, M" which indicate the presence of index 2 (i.e the system is chaotic). While in Semistate equation (19) for Bonhoeffer Van Der Pol system, the non-varnishing in equation (23) of state variable x implies existence of IL-Cutset and which implies(y,z) are non-null vector indicating VC-Loops. Also, the determinant $|E_1|$ in equation (22) is non-zero which also implies that the system is not index 1, and the presence of index two in equation (25) with the determinant of $E_2 \neq 0$ with characteristics of three positive degree of freedom C,L and M.

5.0 Conclusion

In this paper we have discussed the Semistate equations of models generated from extended Duffing Van Der Pol(DVPs) and extended Bonhoeffer Van Der Pol(BVPs) electronic circuits arising from their nodal analysis. The introduction of the passive component "memristor" to replace the chua's diode with the application of tractability index and index characterization notion (that is, presence of IL-Cutsets and/or VC-Loops), shows that the models combine memristors fluxes and charges is index two for the two systems which is an indication of systems being chaotic.

This result should be of interest in both digital and analog domains with the inclusion of memristor. Analog applications of memristors are based on the possibility to continuously vary their resistances and , therefore, on their ability to store more information than in the digital regime.

Acknowledgement

Future research is open to the extention of index characterization and tracterbility index to other mem-devices (i.e mem-capacitor and mem-inductor) after memristor on chaotic systems.

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