

## Brinkman-Forchheimer Extended Model in Co-Axial Horizontal Cylinders Filled With Porous Material: An Approximate Analytical Approach

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### *Abstract*

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*An approximate analytical solution is presented for pressure driven flow between co-axial horizontal cylinders filled with porous material. The Brinkman-Forchheimer extension of Darcy equation is utilized to model the fluid flow in the annulus. The result obtained is compared to those obtained from an implicit finite-difference solution of the corresponding time dependent flow problem. It was noticed that the time dependent flow solution yields almost same with steady state values as obtained by using the proposed approximate analytical solution approach.*

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**Keywords:** Forchheimer term, Injection, Pressure-driven, Analytical solution.

### Nomenclature

$G$  = Dimensionless Pressure gradient

$r$  = Dimensional co-ordinate

$R$  = Dimensionless co-ordinate,  $\left(\frac{r}{H}\right)$

$n$  = Index

$C^*$  = Inertia coefficient

$C$  = Dimensionless inertia coefficient,  $\left(C^* \nu^{n-2} H^{\left[3-\frac{3n}{2}\right]}\right)$

$\alpha$  = permeability of the porous medium

$Da$  = Darcy number,  $\left(\frac{K}{H^2}\right)$

$u'$  = Velocity of the fluid

$u$  = Dimensionless velocity of the fluid,  $\left(\frac{u'H}{\nu}\right)$

$d$  = Dimensionless ratio of the radii of the cylinders

Greek symbols

$\nu_{eff}$  = Effective kinematics viscosity of porous medium

$\nu$  = Kinematics viscosity of fluid

$\gamma$  = Ratio of kinematics viscosities

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**1.0 Introduction**

Studies of fluid flow through annulus filled with porous media considering the effect of inertia have being presented in the literature. These are because of its applications in Thermal insulation, crude oil extraction, and solidification of castings among others. In describing the fluid flow in channel filled with porous media normally inertia term comes into picture. The model used here is usually the Brinkman-Forchheimer extension Darcy which is nonlinear differential equation. Its solution is normally obtained utilizing finite difference method which is approximate solution. An example is the results presented utilizing Brinkman-Forchheimer extension Darcy in [1]. So far no result presented with analytical solution to Brinkman-Forchheimer extension Darcy in annulus.

Recently, Jha and Kaurangini proposed and presented approximate solution of the model in horizontal channel [2]. The results so obtained were compared with the finite difference solution to validate its accuracy. It is found to be same. Similarly Jha and Kaurangini presented approximate solution of model in composite horizontal channel [3].

In this work is an attempt was made to present a solution of the Brinkman-Forchheimer Extended Darcy Model using approximate analytical approach solution for pressure driven flow between co-axial horizontal cylinders filled with porous material.

**2.0 Mathematical Model**

The present study considers the fully developed laminar pressure driven flow of a viscous incompressible fluid between two co-axial horizontal cylinders. The fluid is assumed to be Newtonian with uniform properties and the porous medium is isotropic and homogeneous. Using these assumptions, the mathematical model representing fluid flow situation in dimensionless form is

$$\gamma \left[ \frac{d^2 u}{dR^2} + \frac{1}{R} \frac{du}{dR} \right] - \frac{u}{Da} - \frac{C}{\sqrt[4]{Da^n}} u^n = -G. \tag{1}$$

The Forchheimer term is when  $n = 2$ , and the momentum transfer which is governed by steady Brinkman-Forchheimer extended Darcy model.

The boundary conditions in dimensionless form are:

$$\begin{aligned} u &= 0 \text{ at } R = 1 \\ u &= 0 \text{ at } R = d \end{aligned}$$

**3.0 Analytical Solutions**

Using the assumption,  $\alpha = u^{n-1}$  as suggested in [2] and [3] and taking  $\alpha\lambda = -\frac{1}{\sqrt{\gamma}} \left[ Da + \frac{\alpha C}{\sqrt[4]{Da^n}} \right]^{\frac{1}{2}}$  in the equation (1), then its solution becomes

$$u(R) = \frac{G}{X} \left[ 1 - \frac{I_0(\alpha\lambda R)}{I_0(\lambda\alpha)} \right] \tag{2}$$

Where  $X = \frac{1}{Da} + \frac{\alpha C}{\sqrt[4]{Da^n}}$   
and  $I_0$  is the usual Bessel function

**4.0 Numerical Solution**

To explore the validity of the proposed analytical solution to the Brinkman-Forchheimer extended Darcy model in co-axial horizontal cylinders, the numerical solution of the same problem is obtained using implicit finite difference approach [1, 4-6]. The same method and computation procedure are adopted here for numerical solutions of equations to validate the accuracy of the proposed solution approach.

**5.0 Conclusion**

Pressure-driven flow between co-axial horizontal cylinders filled with saturated porous material was presented utilizing the Brinkman-Forchheimer extension of Darcy equation.

For ( $n = 2$ ),  $d = 2.0$ ,  $\gamma = 1.0$ ,  $C = 0.52$  for both  $Da = 0.1$  and  $0.01$ , the result of the equation (1) has been compared with the implicit finite-difference solution of the equation in Table I.

An approximate analytical solution was obtained and compared with the numerical finite difference solutions. The solution obtained is exactly same as the numerical finite difference to 1 decimal place as noticed in the Table.

**Brinkman-Forchheimer Extended Model in Co-Axial... Jha and Kaurangini J of NAMP**

**TABLE I Comparison of Results for  $\gamma = 1.0, C = 0.52, G = 10$   
Da=0.01**

d=2.0			d=1.8		
R	Analytical	Numerical	R	Analytical	Numerical
1.0	0.00000	0.00000	1.0	0.00000	0.00000
1.1	0.06471	0.06462	1.1	0.06463	0.06453
1.2	0.08724	0.08719	1.2	0.08699	0.08694
1.3	0.09507	0.09505	1.3	0.09441	0.09439
1.4	0.09767	0.09766	1.4	0.09592	0.09591
1.5	0.09818	0.09817	1.5	0.09358	0.09355
1.6	0.09728	0.09727	1.6	0.08513	0.08508
1.7	0.09410	0.09408	1.7	0.06193	0.06183
1.8	0.08539	0.08533	1.8	0.00000	0.00000
1.9	0.06210	0.06200			
2.0	0.00000	0.00000			

Da=0.1

d=2.0			d=1.8		
R	Analytical	Numerical	R	Analytical	Numerical
1.0	0.00000	0.00000	1.0	0.00000	0.00000
1.1	0.26193	0.25802	1.1	0.23712	0.23507
1.2	0.42545	0.42312	1.2	0.37747	0.37679
1.3	0.52176	0.52198	1.3	0.44821	0.44906
1.4	0.56944	0.57140	1.4	0.46434	0.46564
1.5	0.57848	0.58085	1.5	0.43194	0.43246
1.6	0.55269	0.55415	1.6	0.34993	0.34899
1.7	0.49079	0.49036	1.7	0.21070	0.20886
1.8	0.38665	0.38413	1.8	0.00000	0.00000
1.9	0.22889	0.22560			
2.0	0.00000	0.00000			

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