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Computational Approach for Reactive Third Grade Fluid Flow through A Non-Darcian Porous Medium

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Abstract

In this paper, a successive approximation method is presented to study the steady flow of a reactive third grade fluid under Arrhenius kinetics through parallel isothermal walls filled with non-Darcian porous medium. Approximate solutions to the strongly nonlinear ordinary differential equations arising from the model are obtained by using Adomian decomposition method (ADM). Parametric study of the solutions are presented and discussed, including the convergence analysis of the series solution.

Keywords: Third grade fluid, ADM, non-Darcian porous medium, reactive fluid.

1.0 Introduction

It is an established fact that non-Newtonian fluids are quite significant in several industrial and medical applications. For instance, in bio-fluid and haematology, building and confectionery industries, food industries, chemical engineering, bubble columns, polymer solution etc. It is well known fact that the classical Navier-stokes equations are inadequate to describe the complex rheological properties of some non-Newtonian fluids [1]. In this article, of interest is the third grade fluid model that has the ability to predict the shear thinning/shear thickening of these complex fluids. In the last few years, a reasonably good progress has been made on the third grade fluid under different flow conditions.

From application point of view, some of these non-Newtonian fluids are highly reactive and could undergo spontaneous heating even in the absence of naked fire. To ensure safety of lives and properties several studies have been conducted. For instance, Makinde [2] investigated the thermal criticality of a reactive third-grade liquid flowing steadily between two parallel plates in which the reaction is exothermic under Arrhenius kinetics. In addition, the reactive third grade flow through a cylindrical pipe has been studied in [3]. Makinde [4] studied the thermal criticality for a reactive gravity driven thin film flow of a third-grade fluid with adiabatic free surface down an inclined isothermal plane. In [5], a mathematical investigation on the effect of convective cooling on a reactive third-grade fluid flowing steadily through a cylindrical pipe was performed, under the assumption that the system exchange heat with the ambient. The author assumed the Newton's cooling law and the reaction is exothermic under Arrhenius kinetics. Moreover, Makinde and Chinyoka [6] examined the unsteady hydromagnetic generalized Couette flow and heat transfer characteristics of a reactive variable viscosity incompressible electrically conducting third grade fluid in a channel with asymmetric convective cooling at the walls in the presence of uniform transverse magnetic field. In addition, Makinde and Chinyoka [7] studied the transient problem of Generalized Couette flow and heat transfer in a reactive variable viscosity third grade liquid with asymmetric convective cooling. It was assumed that exothermic chemical kinetics took place in the flow system and the convective heat exchange with the ambient temperature at the channel surface follow Newton's law of cooling at the isothermal plates. Makinde et al [8] examined the thermal effects in an unsteady flow of a pressure-driven, reactive, variable viscosity, and third-grade fluid through a porous saturated medium with asymmetrical convective boundary conditions. In a related study, Okoya [9] studied the thermal transition of a reactive flow of a third-grade fluid with viscous heating and chemical reaction between two horizontal flat plates, where the top plate moved with a uniform speed while the bottom plate was fixed in the presence of imposed pressure gradient. In addition, Okoya [10] dealt with the effect of dimensionless non-Newtonian coefficient on the thermal stability of

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a reactive viscous liquid in steady flow between parallel heated plates. He assumed that the liquid is symmetrically heated and the flow fully developed. Okoya [11] presented a numerical integration to determine the critical and transitional values of parameters for steady, reactive, viscous, one dimensional plane Couette flow of an incompressible, homogeneous fluid of third-grade with the lower plate at rest while the upper is in uniform motion. Okoya [12] considered the reactive and fully developed fluid flow with exponential viscosity of a third-grade fluid flowing between parallel plates under the action of externally imposed pressure gradient. Ajadi [13] investigated the effect of variable viscosity and viscous dissipation on the thermal stability of a one-step exothermic, reactive non-Newtonian flow in a cylindrical pipe by assuming negligible reactant consumption.

In all the above studies, the flow of reactive third grade fluid through non-Darcian porous has not been investigated. However, the study is important in understanding the dynamics of reactive fluid through non-Darcian porous medium. Common example can be found in the recovery of heavy crude oil by in-situ combustion method. In this work, a more general Darcy–Forchheimer–Brinkman flow model will be used [14]. Specifically, the objective of this paper is to investigate the effect of the non-Darcian parameter on the reactive fluid flow which was not accounted for in the previous models in literature. To achieve this a computational method based on the rapidly convergent Adomian decomposition method [15]-[21] will be used to obtain an approximate solution to the nonlinear problem.

The rest of the paper is as follows. Section two presents the formulation and non-dimensionalization of the problem. In section three, the method of solution is described while section four deals with the discussion of results based on the physics of the problem. Finally, section five contains final remarks.

2.0 Mathematical Formulation

Consider the steady flow of an incompressible, viscous, heat generating and reactive third grade fluid flow through infinite parallel plates with isothermal temperature filled with non-Darcian porous medium. The flow is induced by an axial pressure gradient and the heat generation term is assumed to be an exponential function of temperature. The flow is assumed to be fully developed hence the governing equations depend on only y in the form. Neglecting the reactant consumption the flow governing equation can be written as [2]:

$$0 = -\frac{dp}{dx} + \mu\frac{d^2u'}{dy'^2} + 6\beta_3\frac{d^2u'}{dy'^2}\left(\frac{du'}{dy'}\right)^2 - \frac{\mu u}{k_p} - \frac{\rho b u^2}{k_p} \qquad (1)$$

$$0 = k\frac{d^2T'}{dy'^2} + \mu\left(\frac{du'}{dy'}\right)^2 + 2\beta_3\left(\frac{du'}{dy'}\right)^4 + QC_0Ae^{\frac{E}{RT}} \qquad (2)$$

The additional term in the momentum equation is due to Khani *et al* [14] which represents Darcian bulk impedance and quadratic Forchheimer porous resistance respectively. The no slip condition for the fluid velocity at the walls together with the exchange of heat with the ambient at the walls is

$$u' = 0, T' = T_0 \quad on \quad y = a \qquad (3)$$

and symmetric condition along the channel centreline is given as follows

$$\frac{du'}{dy'} = 0 = \frac{dT'}{dy'} \quad on \quad y = 0 \qquad (4)$$

Where u' - the fluid velocity, T' - the fluid temperature, p' - the pressure, β_3 - is the material coefficient, k - the thermal conductivity, k_p - is the porous permeability of the channel, ρ - is the fluid density, b - is the term representing the quadratic porous resistance, μ - is the dynamic viscosity, C_0 - is the initial concentration of the reactant specie, R - is the universal gas constant, Q - is the heat of reaction, A - is the rate constant and E - is the activation energy. Introducing the following dimensionless variables

$$y = \frac{y'}{a},\ u = \frac{u'}{U},\ \gamma = \frac{\beta_3 U^2}{a^2\mu},\ \varepsilon = \frac{RT_a}{E}, \theta = \frac{E(T - T_a)}{RT_a^2}, \lambda = \frac{QEAa^2C_0e^{\frac{E}{RT_a}}}{RT_a^2k} \qquad (5)$$

$$\alpha = \frac{E\mu U^2}{kRT_a^2},\ G = -\frac{a^2}{\mu U}\frac{dp}{dx}, P = \frac{a^2}{k_p}, H = \frac{\rho bUa^2}{\mu k_p}$$

we obtain the following problems

$$G+\frac{d^2u}{dy^2}+6\gamma\frac{d^2u}{dy^2}\left(\frac{du}{dy}\right)^2-Pu-Hu^2=0;\ \frac{du}{dy}(0)=0=u(1) \qquad (6)$$

$$\frac{d^2\theta}{dy^2}+\lambda e^{\frac{\theta}{1+\varepsilon\theta}}+\alpha\left[\left(\frac{du}{dy}\right)^2\left(1+2\gamma\left(\frac{du}{dy}\right)^2\right)\right]=0;\ \frac{d\theta}{dy}(0)=0=\theta(1) \qquad (7)$$

here γ - is the dimensionless third grade material parameter, λ - is the frank kameneskii parameter, α - viscous heating parameter, θ -is the dimensionless temperature, u -is the dimensionless velocity and ε - activation energy.

3.0 Method of Solution

The Integral form of (6) and (7) leads to

$$u(y)=a_0+\int_0^y\int_0^y\left\{-G+Pu+Hu^2-6\gamma\frac{d^2u}{dY^2}\left(\frac{du}{dY}\right)^2\right\}dYdY \qquad (8)$$

$$\theta(y)=b_0-\int_0^y\int_0^y\left\{\lambda e^{\frac{\theta}{1+\varepsilon\theta}}+\alpha\left(\frac{du}{dY}\right)^2\left(1+2\gamma\left(\frac{du}{dY}\right)^2\right)\right\}dYdY \qquad (9)$$

where the unknown constants a_0, b_0 will be determined using the other boundary conditions. To obtain the approximate solution of the nonlinear equations (8) and (9), we assume series solutions in the form

$$u=\sum_{n=0}^{\infty}u_n\ ,\ \theta=\sum_{n=0}^{\infty}\theta_n, \qquad (10)$$

so that using (10) in the integral equations (8)-(9) leads to the following recursive relations

$$\begin{aligned} u_0(y)&=a_0+\int_0^y\int_0^y(-G)dydy \\ u_{n+1}(y)&=\int_0^y\int_0^y(Pu_n+QA_n-6\gamma B_n)dydy \qquad n\geq 0 \end{aligned} \qquad (11)$$

the zeroth component of (9) divided into two, to get

$$\begin{aligned} \theta_0(y)&=b_0 \\ \theta_1(y)&=-\int_0^y\int_0^y\left\{\alpha\left(\frac{du}{dY}\right)^2\left(1+2\gamma\left(\frac{du}{dY}\right)^2\right)+\lambda C_0\right\}dYdY \\ \theta_{n+1}(y)&=-\int_0^y\int_0^y\lambda C_n dYdY \qquad n\geq 1 \end{aligned} \qquad (12)$$

where $A=u^2, B=\frac{d^2u}{dY^2}\left(\frac{du}{dY}\right)^2$ and $C=e^{\frac{\theta}{1+\varepsilon\theta.}}$ are the nonlinear terms that can be evaluated using the Adomian polynomials

$$\begin{aligned} A_0&=u_0^2 \\ A_1&=2u_0u_1 \qquad , \\ A_2&=2u_0u_2+u_1^2 \\ &\ldots\ldots \end{aligned} \qquad (13)$$

$$
\begin{aligned}
B_0 &= \frac{d^2u_0}{dY^2}\left(\frac{du_0}{dY}\right)^2 \\
B_1 &= 2\frac{d^2u_0}{dY^2}\left(\frac{du_0}{dY}\right)\left(\frac{du_1}{dY}\right) + \frac{d^2u_1}{dY^2}\left(\frac{du_0}{dY}\right)^2 \qquad (14) \\
B_2 &= \frac{d^2u_0}{dY^2}\left(\frac{du_1}{dY}\right)^2 + 2\frac{d^2u_0}{dY^2}\left(\frac{du_0}{dY}\right)\left(\frac{du_2}{dY}\right) + 2\frac{d^2u_1}{dY^2}\left(\frac{du_0}{dY}\right)\left(\frac{du_1}{dY}\right) + \frac{d^2u_2}{dY^2}\left(\frac{du_0}{dY}\right)^2
\end{aligned}
$$

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and

$$
\begin{aligned}
C_0 &= e^{\frac{\theta_0}{1+\varepsilon\theta_0}.} \\
C_1 &= \frac{\theta_1}{(1+\varepsilon\theta_1)^2} e^{\frac{\theta_0}{1+\varepsilon\theta_0}.} \qquad (15) \\
C_2 &= \frac{\{(1-2\varepsilon-2\varepsilon^2\theta_0)\theta_1^2 + 2(1+\varepsilon\theta_0^2\theta_2)\}}{2(1+\varepsilon\theta_1)^4} e^{\frac{\theta_0}{1+\varepsilon\theta_0}.}
\end{aligned}
$$

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summing up the iterations, the approximate solution for the velocity distribution is obtained as the partial sum

$$
u = \sum_{n=0}^{q} u_n \,,\; \theta = \sum_{n=0}^{j} \theta_n \qquad (16)
$$

where q, j are truncation point.

4.0 Results and Discussion

In this section, the effect of variations in flow parameter of the flow structure is conducted. To ensure the accuracy of the solution, a perfect agreement is observed between the present result and that obtained in Makinde [2] when $P = H = \lambda = 0$

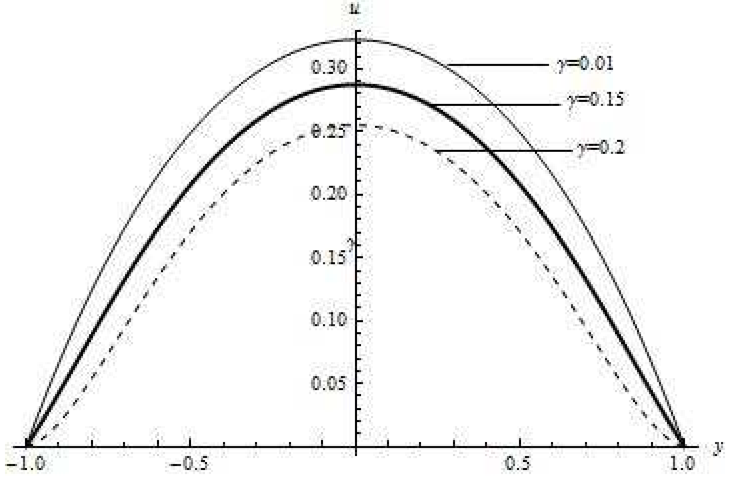

Figure 1:- Effects of non-Newtonian parameter on fluid velocity

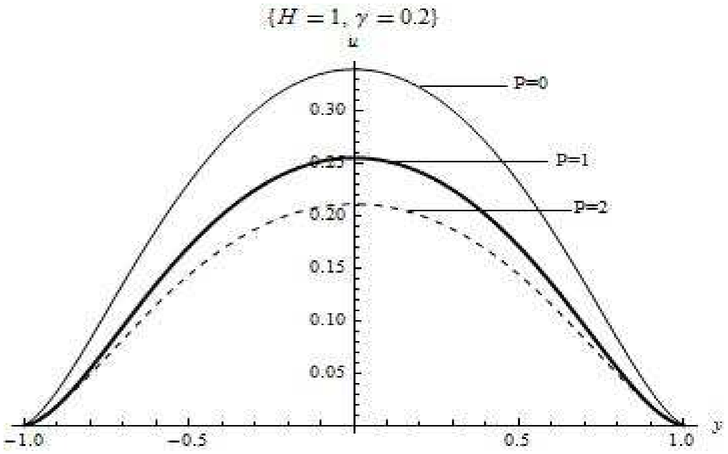

Figure 2:- Effects of Darcian porosity parameter on fluid velocity

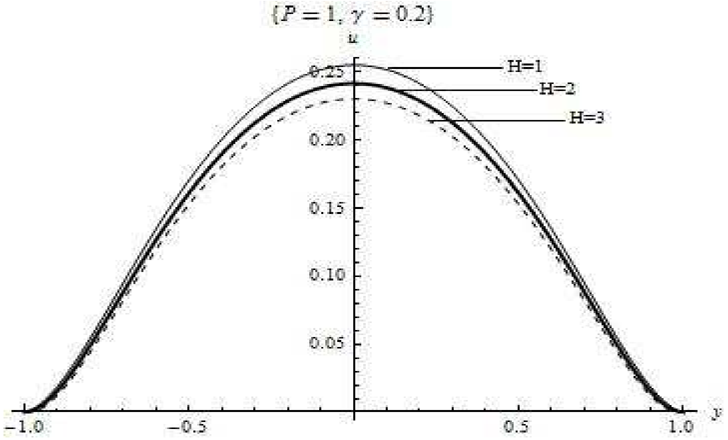

Figure 3:- Forchheimer effects on fluid velocity

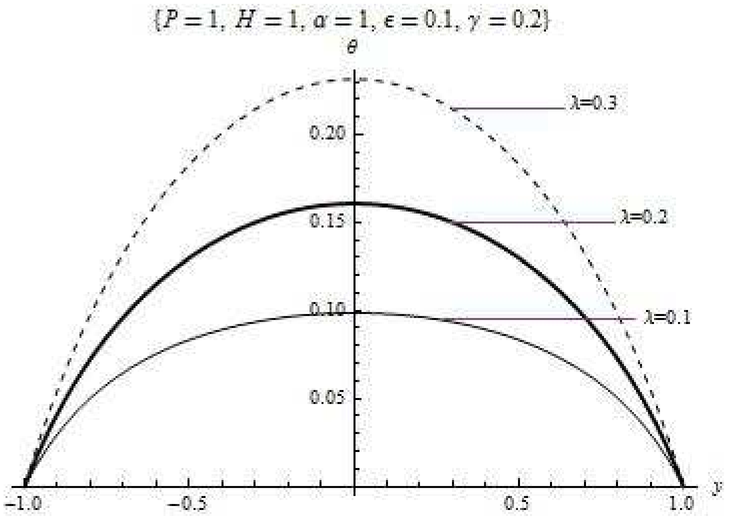

Figure 4: Effects of Frank-Kameneskii-parameter on fluid temperature.

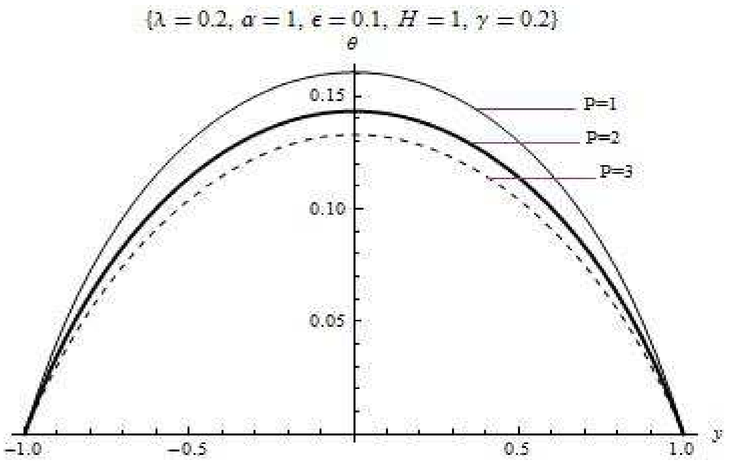

Figure 5:- Effects of Darcian porosity on fluid temperature

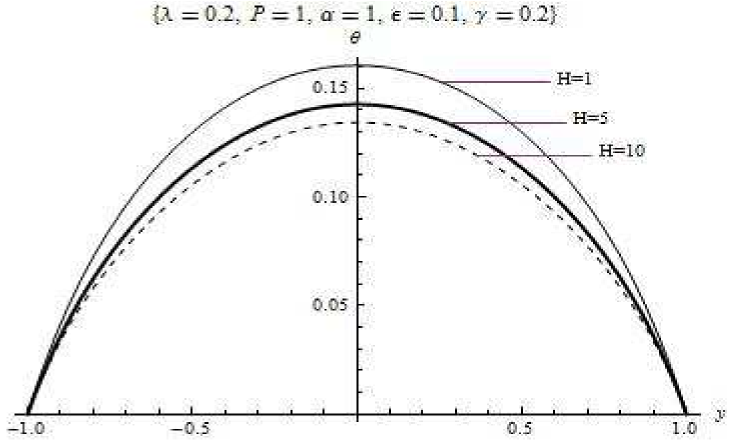

Figure 6: - Forchheimer effects on fluid temperature

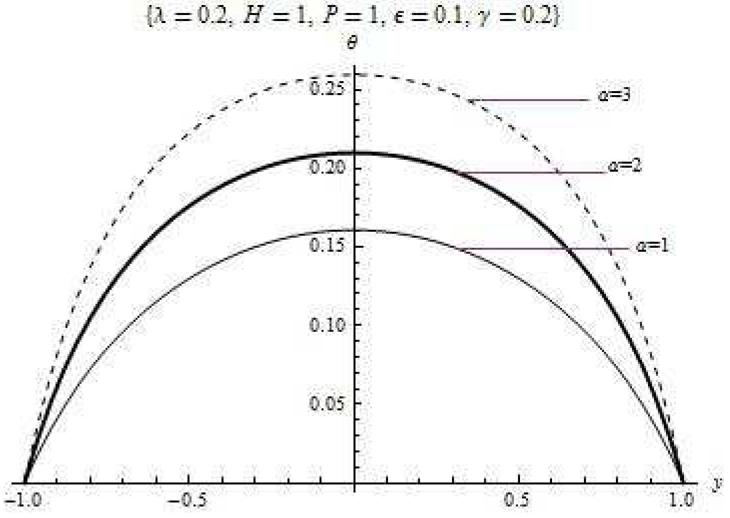

Figure 7: - Effects of viscous dissipation on fluid temperature

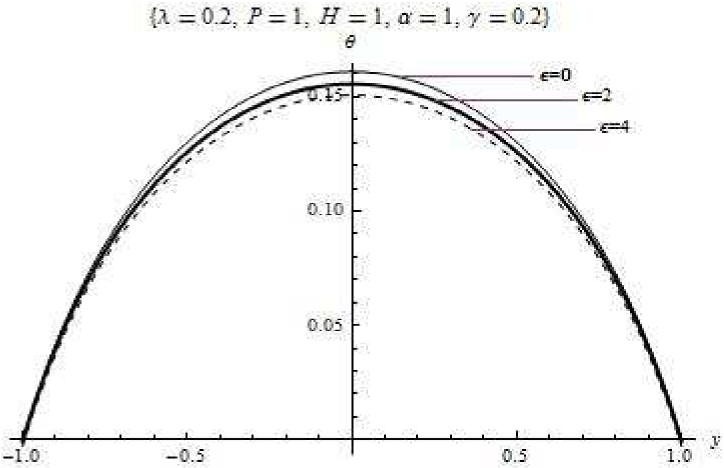

Figure 8: - Effects of activation energy on fluid temperature

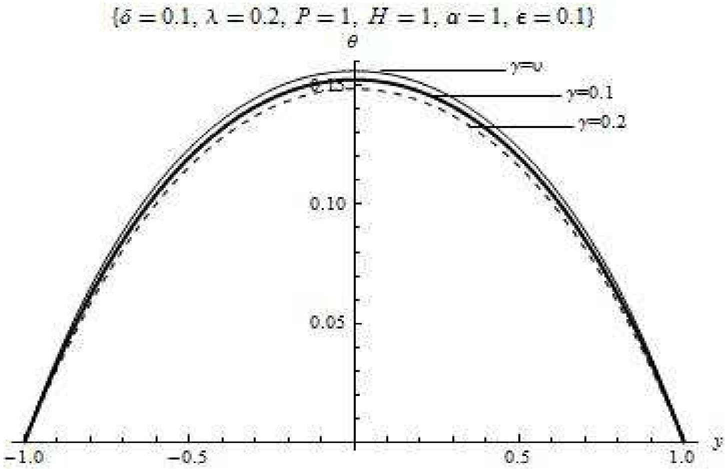

Figure 9: - Effects of non-Newtonian parameter on fluid temperature.

Parametric study of the solutions can be conducted for physical parameters like third grade material effect (γ), Darcian bulk impedance (P), Forchheimer porous resistance effect (H), Frank-Kameneskii parameter (λ), viscous dissipation/ heating (α) and activation energy (ε). Figure 1 shows the plot of variations in non-Newtonian material effect. As observed from the graph, an increase in the non-Newtonian material effects decreases the velocity maximum due to the thickening of the fluid. Figure 2 depicts the effect of Darcian parameter on the flow. From the result, it is observed that as the porous permeability of the channel increases there is reduction in the fluid flow maximum. Figure 3 represents the effect of Forchheimer parameter on the fluid flow velocity. Just like the Darcian parameter, the result shows that an increase in this parameter decreases the flow velocity maximum.

Figure 4 shows the plot of Frank-Kameneskii parameter on the fluid temperature. The result shows that an increase in the Frank-Kameneskii parameter increases the fluid temperature due to reduction in the fluid thermal conductivity. As observed in Figures 4-6, an increase in the fluid linear and quadratic Darcian parameter decreases the fluid temperature within the channel. Figure 7 addresses the variations in the viscous heating parameter. As observed from the graph, an increase in the viscous heating parameter increases the fluid temperature within the channel. This is due to the fact that the kinetic energy of the moving fluid is converted to internal energy. Figure 8 shows the effect of activation energy on the fluid temperature. The decrease in the fluid temperature observed in the plot is due to rise in the dynamic viscosity of the fluid. Moreover, figure 9 shows the effect of the non-Newtonian material effect on the fluid temperature. The result shows that an increase in the parameter decreases the fluid temperature. This is physically true due to rise in the dynamic viscosity of the fluid.

5.0 Conclusion

The objective of the present study is to obtain an approximate solution of a strongly nonlinear boundary valued problem by using successive approximation technique based on Adomian decomposition method. The main contribution to knowledge is that decreasing porosity leads to increase in the non-Darcian porous medium parameters. This has decreasing effect on both the flow velocity and temperature maximum within the channel. In conclusion, the method is a promising analytical method for handling strongly nonlinear boundary-value problem. Area of future research on the work includes; thermal run away, disappearance of thermal criticality and multiplicity of solution.

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