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Temperature-Dependent Variable Viscosity of a Laminar Flow in a Channel Filled With Saturated Porous Media

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Abstract

In this paper, we examined temperature-dependent variable viscosity of laminar flow in a channel filled with porous media. We employed Galerkin weighted residual method to solve the resulting non-linear equation. The results show the effects of viscosity parameter, Reynolds number ,Darcy number and Brinkman number on the flow system.

Keywords: Laminar flow, Darcy number, Variable viscosity , Weighted residual method.

1.0 Introduction

Non-Newtonian fluids have received much attention than Newtonian fluids in the recent years due to its practical importance, rapid development of modern industrial materials and technological applications. The flow of reactive viscous fluids in porous media presents a theoretically challenging problem and has a broad range of scientific, technological and engineering applications. Real life areas where such flow systems are encountered including drying of food, geothermal energy extraction, nuclear waste disposal, the flow of heat and fluid inside human organs, insulation of buildings, groundwater movement, astrophysical plasmas, magnetohydrodynamic (MHD) pumps and generators, oil and gas production, to mention but just a few applications.

Heat transfer problem of second and third grade fluids have been studied by several authors: Hayat et al [1] considered partial slip effect on the flow and heat transfer characteristics in a third grade fluid. Fosdick and Rajagopal [2] performed a complete thermodynamic analysis of constitutive equations for the third grade fluid involving heat transfer process. Massoudi and Christie [3] analyzed numerically the flow of a third grade fluid in a pipe without heat source where the shear viscosity was assumed to be temperature dependent . Olajuwon [4] examined the flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation. Yurusoy et al [5],Nadeem et al [6] analytically considered the effects of partial slip on a fourth grade fluid with variable viscosity and Makinde [7] employed Hermite-Pade approximations to evaluate thermal radiation effect of inherent irreversibility in a variable viscosity channel flow. Massoudi and Christie [8] studied the effects of variable viscosity and viscous dissipation on the flow of third grade fluid in a pipe. Nadeem et al [9], Nadeem et al [10] examined the influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube.

The fundamental importance of convective flow in porous media has been established in the recent books by Nield and Bejan [11], Bejan and Kraus [12], Ingham et al.[13] and Bejan et al.[14]. The above studies of free and mixed convection flow in vertical channels are based on the hypothesis that the fluids are non-Newtonian. Moreover, because of their technological importance, studies involving free, forced and mixed convection flow of non-Newtonians in channels are very important in several industrial processes. Bhargava et al. [15] have studied the effects of magnetic field on the free convection flow of micropolar fluid between two parallel porous vertical plate.

Furthermore, Lazarus [16] investigated on laminar flow in a channel filled with saturated porous media. Lamidi and Ayeni [17] examined analytical solution of a steady non-reacting laminar fluid flow in a channel filled with saturated porous mediawith two distinct horizontal impermeable wall conditions. Yang and Hwang [18] studied a numerical simulation of turbulent fluid flow on the heat transfer characteristics in heat exchangers fitted with porous media. Yiotis et al.[19] examined a lattice Boltzmann study of viscous coupling effects in immiscible two phase flow in porous media. Motivated by the work of Lazarus [16], we considered the effects of reactive variable viscosity model and Brinkman number on the flow system.

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2.0 Governing Equations

The basic governing equations are continuity, momentum and energy equations. Flow of Newtonian fluids in porous media governed by Darcy's law is $\nabla P = -(\mu/k)u,$ implying a linear relationship between Darcy flux u and the gradient of the generalized pressure $P = p + \rho g z,\ p$ being the pressure, z the vertical coordinate and Brinkman momentum equation $\nabla P = -(\mu/k)u + \mu_{ef}\nabla^2 u.$ The model parameters which arise from the flow system are Da porous medium parameter, Reynolds number, Prandtl, Brinkman number and γ_1 reactive variable viscosity parameter. Numerical solutions are obtained using Galerkin weighted residual method. Modifying Brinkman momentum equation we have

$$\rho\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{d}{d\bar{y}}\left[\mu(T)\frac{d\bar{u}}{d\bar{y}}\right] - (\mu_{ef}/k)\bar{u} - \frac{\partial \bar{p}}{\partial z} \tag{2.1}$$

$$\rho cp\frac{\partial \bar{T}}{\partial \bar{t}} = K\frac{d^2\bar{T}}{d\bar{y}^2} + \mu(T)\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 \tag{2.2}$$

Together with the appropriate boundary conditions

$$\bar{u}(0,t) = 0, \bar{u}(h,t) = 0, \bar{T}(0,t) = T_0, \bar{T}(h,t) = T_1 \qquad t > 0 \tag{2.3}$$

where

K -Thermal conductivity, $\rho - Density$, $k -$ is the porous medium permeability, C_p -Specific heat at constant pressure,

μ -Dynamic viscosity, $\mu\left(\frac{\partial h}{\partial r}\right)^2$ is the viscous heating effect, $\bar{y}$ -Component of velocity in the vertical direction, μ_{ef} -Effective viscosity , γ -variable viscosity expansion exponent, T_0 -Initial temperature and it is the reference temperature, T -Temperature within the boundary layer, $T_1,\ T_2,.....T_\infty$ - Temperature at the plate, $p = \rho g h$,

the above equation is valid in the limit of very small and very large porous medium permeability.

3.0 Method of Solution

This section is focused on obtaining solutions for reactive variable viscosity flow problem (2.1)-(2.2) subject to (2.3) as follows:

From equations (2.1)-(2.2),we consider a reactive variable viscosity (Reynolds model) of the form $\mu(T) = \mu_0 e^{-\gamma(T-T_0)}$ so that Eqs. (2.1)-(2.2) becomes

$$\rho\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{d}{d\bar{y}}\left[\mu_0 e^{-\gamma(T-T_0)}\frac{d\bar{u}}{d\bar{y}}\right] - (\mu_{ef}/k)\bar{u} - \frac{\partial \bar{p}}{\partial z} \tag{3.1}$$

$$\rho cp\frac{\partial \bar{T}}{\partial \bar{t}} = K\frac{d^2\bar{T}}{d\bar{y}^2} + \mu_0 e^{-\gamma(T-T_0)}\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 \tag{3.2}$$

We now introduce the following non-dimensional variables

$$\frac{\bar{y}}{l_0} = y, \frac{\bar{u}}{u_0} = u, \theta(y,t) = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0}, Da = \frac{k\rho}{\mu_{ef}}, t = \frac{u_0\bar{t}}{l_0}, \frac{\partial \bar{p}}{\partial z} = C \tag{3.3}$$

Substituting (3.3) into (3.1)-(3.2) we obtain

$$\frac{\rho}{l_0}\frac{\partial u}{\partial t} = \frac{\mu_0 u_0}{l_0^2}\frac{d}{dy}\left[e^{-\gamma(T-T_0)}\frac{du}{dy}\right] - u_0(\mu_{ef}/k)u - C \tag{3.4}$$

$$\frac{\partial u}{\partial t}=\frac{1}{\mathrm{Re}}\frac{d}{dy}\left[e^{-\gamma_1\theta}\frac{du}{dy}\right]-\frac{u}{Da}-C_1 \qquad (3.5)$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\mathrm{Pr}}\frac{d^2\theta}{dy^2}+Bre^{-\gamma_1\theta}\left(\frac{du}{dy}\right)^2 \qquad (3.6)$$

where

$$\mathrm{Pr}=\frac{l_0\rho c_p}{K},\ Br=\frac{\mu_0 u_0}{l_0\rho c_p\left(\overline{T_1}-\overline{T_0}\right)},\ \mathrm{Re}=\frac{l_0{}^3\rho}{\mu_0 u_0},\ C_1=\frac{C}{l_0\rho},\ \gamma_1=\gamma\left(\overline{T_1}-\overline{\mathrm{T}_0}\right) \qquad (3.7)$$

We now consider when the flow is steady we obtain

$$\frac{1}{\mathrm{Re}}\frac{d}{dy}\left[e^{-\gamma_1\theta}\frac{du}{dy}\right]-\frac{u}{Da}-C_1=0 \qquad (3.8)$$

$$\frac{1}{\mathrm{Pr}}\frac{d^2\theta}{dy^2}+Bre^{-\gamma_1\theta}\left(\frac{du}{dy}\right)^2=0 \qquad (3.9)$$

The corresponding dimensionless boundary conditions

$$u(0)=0,u(1)=0,\theta(0)=0,\theta(1)=0 \qquad (3.10)$$

We now proceed to solve equations (3.8) and (3.9) subject to (3.10) numerically using Galerkin-Weighted Residual Method as follows:

let $f=\sum_{i=0}^{2}A_i e^{(-i/4)\eta},\ \phi=\sum_{i=0}^{2}B_i e^{\left(-i/5\right)\eta}$

The results are presented in Figures 1-4

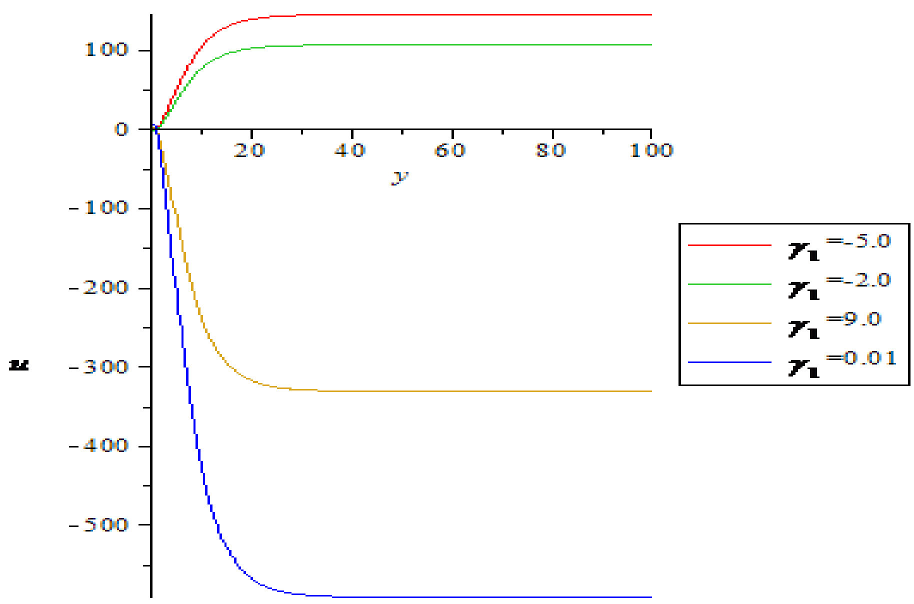

Fig.1: Graph of the velocity function u for various values of $C_1=0.5, \mathrm{Re}=Da=1.0$ of Eq. (3.8).

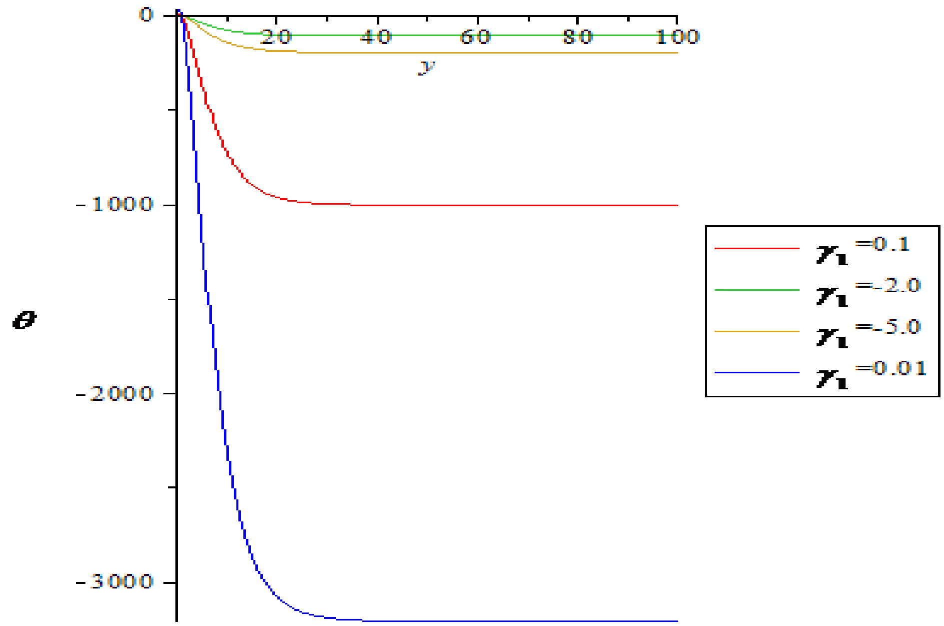

Fig.2: Graph of the temperature function θ for various values of $\Pr = Br = 1.0$ of Eq. (3.9)

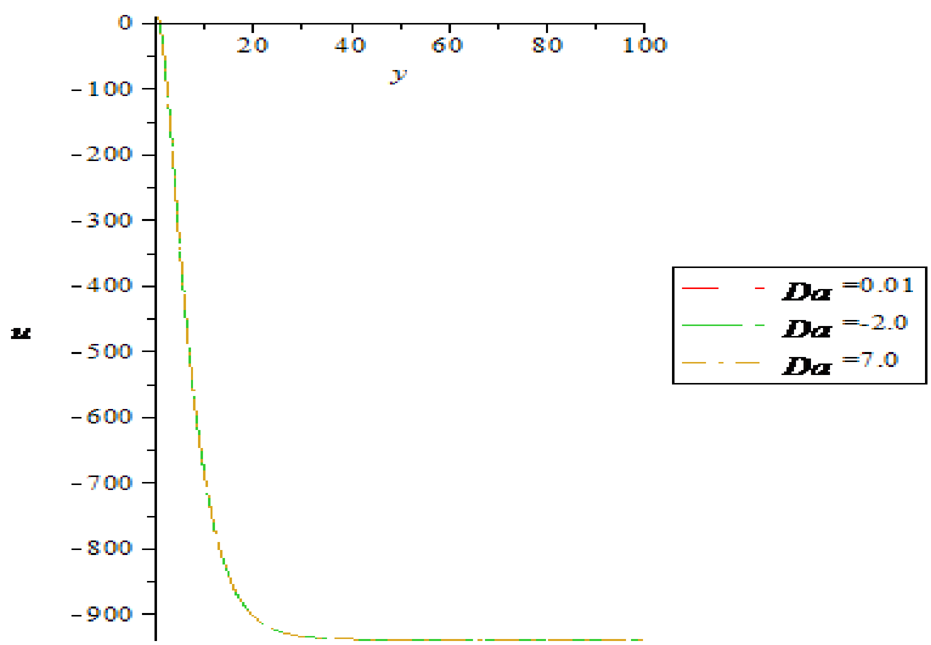

Fig.3: Graph of the velocity function u for various values of $C_1 = 0.5, \mathrm{Re} \geq 0.1, \gamma_1 = 0$ of Eq. (3.8).

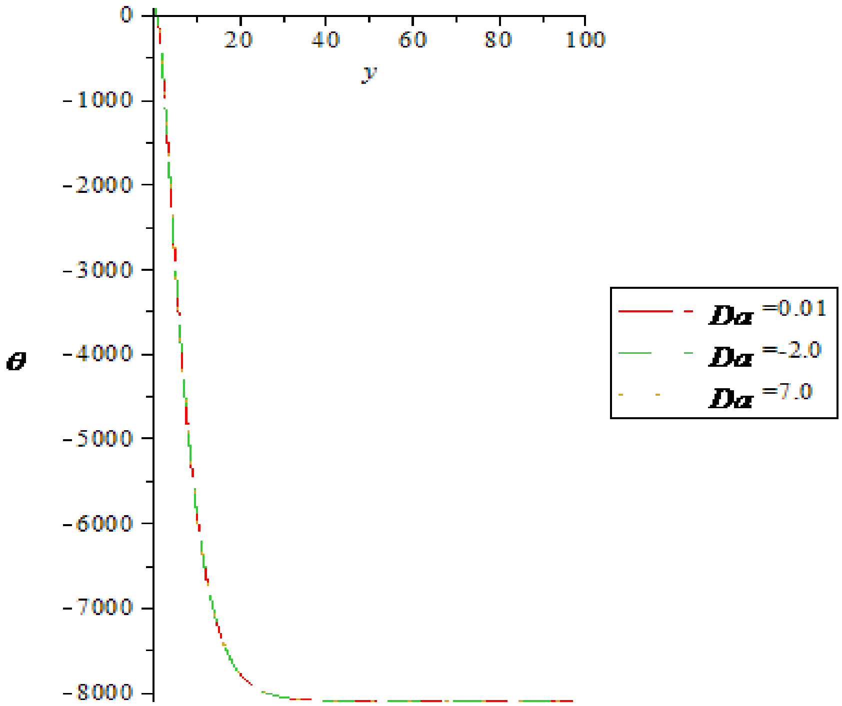

Fig.4: Graph of the temperature function θ for various values of $\Pr = Br \geq 0.01, \gamma_1 = 0$ of Eq. (3.9)

4.0 Discussion of Results

The study of flow of non-Newtonian fluids in porous media is very important due to its wide variety of practical applications in processes such as enhanced oil recovery from underground reservoirs, filtration of polymer solutions and soil remediation through the removal of liquid pollutants to mention but just a few. From Fig.1 the result shows that the velocity decreases for negative values of γ_1 variable viscosity parameter as Reynolds number and Darcy number increases. It is noted from figure 1 that the velocity profile increases for positive values of γ_1 viscosity parameter. From Figs.2 & 4 the results show that the temperature profile decreases as Prandtl number, Darcy number and γ_1 viscosity parameter increases. From Fig.3 the results show that the velocity profile decreases as Reynolds number and Darcy number increases. It is observed that γ_1 viscosity parameter has a significant effect on the temperature profile.

Conclusion

We computationally investigated the transient flow of a reactive variable viscosity non-Newtonian fluid through a porous saturated medium. We observed that there is a transient increase in fluid velocity with an increase in the fluid viscosity parameter (which decreases the viscosity). A transient decrease in both the fluid velocity and temperature is observed with increase in γ_1 viscosity parameter, Prandtl number, Reynolds number and Darcy number which decreases the porosity in the flow.

For engineering purpose, the results of this problem are of great interest in petroleum applications as in reservoir of oil or gas flows, natural gas production and enhanced oil production. The flow of the petroleum through the porous ground represents an example of the motion of our fluid, especially in the motion of the fluid in the earth's core. Hence, viscosity parameter and viscous dissipation affects both the velocity as well as the temperature of the flow significantly. We expect some of the results of this work to be useful for field and production engineers in the oil fields during the processes of oil recovery. Finally, the above numerical computational procedure is advocated as effective tool for investigating several other parameter dependent non-linear boundary-value problem.

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