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Musculoskeletal Magnetic Resonance Imaging Segmentation Using Finite Element Method

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Abstract

This research work presents a method for segmentation of medical images based on a deformable contour (active contour) paradigm. The deformable contour is a novel approach in image segmentation. A type of active contour is the Snake. Snake is a parametric curve defined within the domain of the image. Snake properties are specified through a function called energy functional. This means they consist of packets of energy which expressed as partial Differential Equations. The partial Differential Equation is the controlling engine of the active contour since this project, the Finite Element Method (Standard Galerkin Method) implementation for deformable model is presented.

Keywords: Active Contour,Musculoskeletal,Magnetic Resonance, Imaging Segmentation, Finite Element Method (Standard Galerkin Method), Partial Differential Equation.

1.0 Introduction

Diagnostic imaging is an invaluable tool in medicine today. Magnetic Resonance Imaging (MRI), Computed Tomography (CT), digital mammography, and other imaging modalities provide an effective means for noninvasively mapping the anatomy of a subject. These technologies have greatly increased knowledge of normal and diseased anatomy for medical research and are a critical component in diagnosis and treatment planning. With the increasing size and number of medical images, the use of computers in facilitating their processing and analysis has become necessary. In particular, computer algorithms for the delineation of anatomical structures and other regions of interest are a key component in assisting and automating specific radiological tasks. These algorithms play a vital role in numerous biomedical imaging applications such as the quantification of tissue volumes, diagnosis, localization of pathology, study of anatomical structure, treatment planning, partial volume correction of functional imaging data, and computer integrated surgery. With medical imaging playing an increasingly prominent role in the diagnosis and treatment of diseases, the medical image analysis community has become preoccupied with the challenging problem of extracting, with the assistance of computers, clinically useful information about anatomic structures imaged through the various imaging modalities. Although modern imaging devices provide exceptional views of internal anatomy, the use of computers to quantify and analyze the embedded structures with accuracy and efficiency is limited.

State-of-the-art medical images generate massive databases of static volume (3D) and dynamic volume (4D) images. These data sets, which usually suffer from sampling artifacts, spatial aliasing, and noise, are essentially “blocks of granite” with meaningful embedded structures. The problem is to extract thee surface elements belonging to an anatomical structure (the segmentation step) and to integrate these surface elements into a globally coherent surface model of the structure (the reconstruction step). Certain diagnosis procedures also require the tracking and deformation analysis of nonrigidly moving anatomical surfaces. The ease and accuracy of such procedures can be critically dependent upon the model used. Dynamic models are needed which are robust against noise-corrupted data and which are capable of accurately representing the complex geometries of anatomical surfaces while permitting the quantitative measurement of highly nonrigid tissue kinematics.

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Musculoskeletal disorders are certainly the most notorious and common cause of severe long-term pain and physical disability. For their diagnosis, the surgical planning and their postoperative assessment, the patient specific modeling of the musculoskeletal system from medical images is an important problem [1]. The musculoskeletal system exhibits a complex geometry, difficult to model realistically (multiple organs in contact), a complicated mechanical behavior (viscoelastic, anisotropic, hyperelastic and non linear); and complex interactions (e.g. confined cartilages within articulation).while simplified representation such as stick-figures and action lines [1,2] have proven to be useful for many application in biomechanics. They have limited accuracy [3,4] and they are unable to represent large attachment areas and accurately simulate global constraints such as volume preservation and non-penetration.

Improvement in terms of accuracy could be achieved using surface models [1] or equivalent reduced representations such as medial axis [3]. Current interactive modeling methods [5, 6, 7, and 2] remain time-consuming and are not suitable for clinical use. Indeed, orthopaedists, biomechanicians and kinesiologists would like to simulate, visualize and navigate through articulation with a minimum amount of manual tasks. Diagnosis tools used in the daily medical practice, especially medical scanner, are becoming increasingly precise, available, standardize, as well as less and less invasive. Besides traditional 3D images available from virus modalities such as computed tomography (CT) and magnetic resonance imaging (MRI), kinematical data are now getting more accessible (e.g. cine MRI, real-time MRI ultrasound). MRI is a flexible modality for imaging both soft and bony tissues non-invasively [9], and has been chosen for this project. The potentially large amount of data makes data difficult to exploit. In this context, 3D anatomical models (e.g. shapes, surface, volumes) and 4D kinematical models (e.g. joint angles, deformation maps) could prove more insight and help in fusing (registering) data from devices modalities or scan sessions. This project surveys deformable models, a promising and vigorously researched model-based approach to computer-assisted medical image analysis. The widely recognized potency of deformable models stems from their ability to segment, match and tract images of anatomy structures by exploiting (bottom-up) constraints derived from the image data together with (top-down) a priori knowledge about the location, size, and shape of this structure. Deformable models are capable of accommodating the often significant variability of biological structure over time and across different individuals. Furthermore, deformable models support highly intuitive interaction mechanisms that allow medical scientists and practitioner to bring their expertise to bear on the model-based image interpretation task when necessary.

MATERIALS AND METHOD

2.0 Magnetic Resonance Imaging

Magnetic Resonance Imaging [MRI] is a medical diagnostic technique that combines strong magnetic fields, radio waves, and computer technology to create images of the body using the principle of nuclear magnetic resonance. A versatile, powerful, and sensitive tool, MRI can generate thin-from computerized images of any part of the body-including the heart, arteries, and veins-from any angle and direction, without surgical invasion and in a relatively short period of time. MRI also creates “maps” of biomedical compounds within any cross section of the human body. These maps give basic biomedical and anatomical information that provides new knowledge and may allow early diagnosis of many diseases. In 2003 Paul Lauterbur of the United States and Sir Peter Mansfield of the United Kingdom shared the Nobel Prize in physiology or medicine for their contributions to MRI technology.

MRI is possible in the human body because the body is filled with small biological “magnets,” the most abundant and responsive of which is the proton, the nucleus of the hydrogen atom. The principles of MRI take advantage of the random distribution of protons, which possess fundamental magnetic properties. Once the patient is placed in the cylindrical magnet, the diagnostic process follows three basic steps. First, MRI creates a steady state within the body by placing the body in a steady magnetic field that is 30,000 times stronger than Earth’s magnetic field. Then MRI stimulates the body with radio waves to change the steady-state orientation of protons. It then stops the radio waves and “listens” to the body’s electromagnetic transmissions at a selected frequency. The transmitted signal is used to construct internal images of the body using principles similar to those developed for computerized axial tomography, or CAT scanners. In current medical practice, MRI is preferred for diagnosing most diseases of the brain and central nervous system. MRI scanners provide equivalent anatomical resolution and superior contrast resolution to that of X-ray CAT scanners. They produce functional information similar to that of positron emission tomography (PET) scanners but with superior anatomical detail. MRI scanners also provide imaging complementary to X-ray images because MRI can distinguish soft tissue in both normal and diseased states. Although an MRI scan is relatively expensive, it may actually reduce costs to patients and hospitals by providing diagnostic evaluation to outpatients and thereby frequently limiting more expensive hospitalization. Because it does not use ionizing radiation, MRI is risk free except for patients with cardiac pacemakers, patients who might have iron filings next to their eyes (for example, sheet metal workers), patients with inner ear transplants, and patients with aneurysm clips in their brains. In the early 2000s open MRI scanners were introduced as an alternative to the standard MRI machine, which encloses the body, requires the patient to lie immobile for 45 minutes, and makes disturbing, loud noises. The open MRI scanners are much less confining and far quieter. While the scans they provide bare not as detailed as traditional MRI scans, the open MRI is a highly effective device for patients who fear the loud, dark experience of the closed, cylindrical MRI machine.

3.0 Musculoskeletal Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is a highly reputed medical technology for studying the human brain topology and malfunctions. It can also be extended to the human musculoskeletal system [1]. This research provides a rich exposition for more detail information on human musculoskeletal anatomy based on two-dimensional image segmentation. Image segmentation is a method that is used to extract the region of interest (ROI) from images. In the scientific world, an image is an information carrier which constituent physical properties may be perceived by human eye only. This is due to some inherent corruption termed noise which may entangle or blotted out the detail information of ROI. The specification of this area to extract detail information from ROI is known as image analysis. The idea of segmentation is generally known as active contour (also known as contour deformation). Active contour is a parametric curve within the domain of the image. Active contour properties and its inherent behavior is specified through a function called energy function. This indicates that the contour consists of packages of energy; these packages of energy are expresses in mathematics formation *as partial differential equations.* The partial differential equation is the controlling engine of the active contour as it involves; this regulates the contour and at the same time reduce its energy constituent to the minimum. From the foregone, active contours are curves around the boundaries of objects or ROI. They have broad range of applications in such areas as medical science, motion tracking and image analysis.

4.0 Active Contours (Snakes) and Equation Formulation

Active contour or deformable models are energy-minimising curves that deform to fit image features. Snakes are a special case of deformable models [10]. Snake is an energy-minimizing parametric contour that deforms over a series of time step. Each element along the contour **u** depends on two parameters, where the space parameter *s*is taken to very between 0 and $N-1$, and t is time (iteration): Introduced by Kass et al, snakes are method of attempting to provide some of the post-processing that our own visual system performs. A snake is a continuously curve (possible closed) that attempts to dynamically position itself from a given starting position in such a way that it 'clings' to edges in the image. It has built into it various properties that are associated with both edges and the human visual system (E.g. continuity, smoothness and to some extent the capability to fill in section of an edge that have been occluded).

Snake Properties

From now on snake will be referred to as the parametric curve
$u(s) = (\mathrm{x(s)}, \mathrm{y(s)})$ Where s is assumed to very between 0 and 1.

(a) The snake must be 'driven' by the image. That is, it must be able to detect an edge in the image and align itself with the edge. One way of achieving this is to try to position the snake such that the average 'edge' (however that may be measured) along the length of the snake is maximized. If the measure or edge strength is F (x, y) > 0 at the image point (x, y) then this amount to saying that the snake u(s) is to be chosen in such a way that the functional

$$\int_{s=0}^{s=1} \mathrm{F}: (x(s), y(s))ds \qquad (1)$$

is maximized. This will ensure that the snake will tend to mound itself to edge in the image if it finds them, but does guarantee that it will them in the first place. Given an image the functional may have many local minima (a static problem); finding them is where the 'dynamics' arises. An edge detector applied to an image will tend to produce an edge map consisting of mainly thin edge.

(b) If an elastic band were held around a convex object and then let go, the band would contract until the object prevent it from doing so further. At this point the band would be molded to the object, thus describing the boundary. Two forces are at work here, first that providing the natural tendency of the band to contract, and secondly the opposition force provided by the object. The band contracts because it tries to minimize its elastic energy due to stretching. If the band were describe by the parametric curve $u(s) = (\mathrm{x(s)}, \mathrm{y(s)})$ then the elastic energy at any point *I* is proportional to

$$\left(\frac{du}{dx}\mathrm{l}_\iota\right)^2 = \left(\frac{dx}{ds}\mathrm{l}_\iota\right)^2 + \left(\frac{dy}{ds}\mathrm{l}_\iota\right)^2 \qquad (2)$$

That is, the energy is proportional to the square of how much the curve is being stretched at that point. The elastic band will take up a configuration so that the elastic energy along its entire length, given the constraint of the object, is minimized. Hence the band assumes the shape of the curve $\mathrm{u(s)} = \mathrm{u((s)}, \mathrm{y(s))}$ where u(s) minimizes the functional

$$\int_{s=0}^{s=1} \left\{ \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 \right\} ds \qquad (3)$$

subject to the constraints of the object.

(c) One of the properties of edges that are difficult to model is their when they can no longer be seen. If we were looking at a car and a person stood in front of it, few of us would have any difficulty imagining the contours of the edge of the car that were occluded. There would be 'smooth' extension of the contours on either side of the person. If the above elastic band approach were adopted it will be found that the formed a straight line were the car was occluded (because it tries to minimize energy, and thus length in this situation). If however the band had some stiffness (that is a resistance to bending as for example displayed by s flexible bar) then it would tend to form a smooth curve in the occluded region of the image and be tangential to the boundaries on either side.

Again a flexible bar tends to form a shape so that its elastic energy is minimized. The elastic energy in bending is dependent on the curvature of the bar that is second derivatives. To help force the snake to emulate this type of behavior the parametric curve u(s) = (x(s), y(s)) is chosen so that it tends to minimized the functional

$$\int_{s=0}^{s=1} \left\{ \left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 \right\} ds \qquad (4)$$

Which represent a pseudo-bending energy term. Of course, if a snake were made too stiff then it would be difficult to force it to conform to highly curved boundaries under the action of the forcing term in (1).

Three desirable properties of snakes have now been identified. To incorporate all three into the snake at once the parametric curve

$u(s) = (\mathrm{x(s)}, \mathrm{y(s)})$ representing the snake is chosen so that it minimizes

$$I(x(s), y(s)) = \left\{ \int_{s=0}^{s=1} a(s) \left\{ \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 \right\} + \beta\,(\mathrm{S}) \left\{ \left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 \right\} - \mathrm{F}\,(x\,(\mathrm{s}), y\,(\mathrm{s})) \right\} ds \qquad (5a)$$

Equation (5a) can be better written as

$$I = \left\{ \int_{s=0}^{s=1} a(s) \left\{ \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 \right\} + \beta\,(\mathrm{S}) \left\{ \left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 \right\} \mathrm{F}\,(x\,(\mathrm{s}), y\,(\mathrm{s})) ds \right\} (5b)$$

Here the terms $a(s) \geq 0$ and $\beta(s) > 0$represent respectively the amount of stiffness and elasticity that the snake is to have. It is clear that if the snake approach is to be successful then the correct balance of these parameters is crucial.

5.0 Equation Formulation

Clearly the ideal situation is to seek a local minimum in the locality of the initial position of the snake. In practice the problem that is solved is

$$u(\mathrm{s}) = (u(\mathrm{s}), y(\mathrm{s})) \in H^2[0,1] \times H^2[0,1]$$

$$\frac{\partial I(u(s)+\varepsilon v(s))}{\partial \varepsilon}\Big|_{\varepsilon=0} = 0 \;\; v(s) \in H_0^2[0,1] \times H_0^2[0,1] \qquad (6)$$

Here $\mathrm{H}^2[0,1]$ denotes the class of real valued functions defined on $[0,1]$ that have 'finite energy' in the second derivatives (that is the integral or the square of the second derivatives exists) and $\mathrm{H_0}^2[0,1]$ is the class of functions in $\mathrm{H}^2[0,1]$ that are zero at s = 0 and s = 1.

To see how this relates to finding a minimum consider u(s) to be a local minimum and $u(s) + \varepsilon\,\mathrm{v}\,(\mathrm{s})$ to be a perturbation about the minimum that satisfies the same boundary conditions (i.e. $v(0) = \mathrm{v}(1) = 0$). clearly,

Considered as a function of$\varepsilon, \mathrm{I}\,(\varepsilon) = \varepsilon\mathrm{I}\,(u(\mathrm{s}) + \varepsilon\,\mathrm{v(s)}$, is a minimum at $\varepsilon = 0$

Hence the derivation of $I(\varepsilon)$ must be zero at$\varepsilon = 0$.. Equation (6) is therefore a necessary condition for a local minimum.

Standard arguments in the calculus of variations show that problem (6) is equivalent to another problem, which is simpler to solve:

Find a curve $(\mathrm{x}\,(\mathrm{s}), \mathrm{y}\,(\mathrm{s}))\, \varepsilon \mathrm{C}^4[0,1] \mathrm{x}\, \mathrm{C}^4[0,1]$ that satisfy the pair of fourth order ordinary differential equations

$$\frac{d}{ds}\left\{\alpha(\mathrm{s})\left(\frac{dx}{ds}\right)\right\} - \frac{d^2}{ds^2}\beta(\mathrm{s})\left(\frac{d^2x}{\mathrm{ds}^2}\right) + \frac{dx}{ds} = 0 \qquad (7)$$

$$\frac{d}{ds}\left\{\alpha(\mathrm{s})\left(\frac{dy}{ds}\right)\right\} - \frac{d^2}{ds^2}\beta(\mathrm{s})\left(\frac{d^2y}{ds^2}\right) + \frac{dy}{ds} = 0 \qquad (8)$$

In practice,$\alpha(s)$ and $\beta(s)$ are usually taken to be constant, arbitrary chosen to be $\alpha = 1.0, \beta = 0.5$, hence equation (7) and (8) becomes

$$\frac{d^2x}{\mathrm{ds}^2} - (0.5)\frac{d^4x}{\mathrm{ds}^4} + \frac{dx}{ds} = 0 \qquad (9)$$

$$\frac{d^2y}{\mathrm{ds}^2} - (0.5)\frac{d^4y}{ds^4} + \frac{dy}{ds} = 0 \qquad (10)$$

Together with the boundary condition

x (0) = y(0) = 0 ; *x(1)* = *y(1)* = 1 (11)

The vectors *x* and *y* contain the x and y coordinates respectively of each clement in the model; they represent the position of the snake elements, both before and after adjustment to conform with the internal forces.

6.0 Finite Element Method for Active Contours

There are so many Galerkin approaches ranging fromH^0,H^1, H^2[11], H^{-1} etc. But this work employs the use of H^0 (Standard Galerkin Method) for its analysis.
We wish to minimize the residual

$$\int_a^b R\,(x_i, c_1, c_2, c_3 \ldots n)dx$$

$$I = min\int_a^b R\,(x_i, c_1, c_2, c_3 \ldots n)dx = 0$$

By using the appropriate value for $c_1, c_2, c_3 \ldots\ c_n$

$$\frac{\partial I}{\partial C_i} = \int_a^b U_i R\,(x_i, c_1, c_2, c_3 \ldots n)dx$$

This is the standard Galerkin method of the Finite Element Method.
From (9) above we consider an approximate solution of the form

$$U(x) = U_0(x) + C_1U_1(x) + C_2U_2(x) + C_3U_3(x) + \cdots C_nU_n(x) \qquad (12)$$

Where;
$U_0(x)$ Satisfies the non-homogenous boundary condition at the boundary and
$U_i(x), i = 1,2,3, \ldots n$ are basis functions which satisfies the homogenous boundary conditions $[\mathrm{a,b}][0,1]$ in equation (11) and the
$U_i(x), i = 1,2,3 \ldots n$ are such that

i. $U_i(x), i = 1,2,3 \ldots n$are linearly independent
ii. $U_i(x)$ satisfies the homogenous
iii. $U_i(x)$ is of the form $(x-a)^i(b-x)$ or $(x-a)(b-x)^i i = 1,2,3\,.\,.n$ but if it is in the interval $[0,1]$ then we have $x^i(1-x)$or $(1-x^i)$, where is frequently used.

To continue,
We substitute the appropriate solution *U*(*x*) into the b.v.p to obtain a residual in the form

$R\,(x_i, c_1, c_2 \ldots c_n = L[u] - f$

But our $U_0(x)$ can be obtained from $[0,1]$ as follows:

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{1-0}{1-0}$$

$\therefore y = x \Rightarrow U_0(x) = x$
$U_i = x^i(1-x), \Rightarrow U_1 = x(1-x) = x - x^2, U_2 = x^2(1-x) = x^2 - x^3,$
$U_3 = x^3(1-x), = x^3 - x^4, U_3 = x^4(1-x) = x^4 - x^5$
U(*x*) = $U_0(x) + C_1U_1(x) + C_2U_2(x) + C_3U_3(x) + \cdots C_nU_n(x)$
We find the values of C_i and then substitute and evaluate

$U\left({}^{1}/_{4}\right) = x_1, U\left({}^{1}/_{2}\right) = x_2, U\left({}^{3}/_{4}\right) = x_3, U(1) = x_4$

$$U(x) = x + C_1(x - x^2) + C_2(x^2 - x^3) + C_3(x^3 - x^4) + C_4(x^4 - x^5) \quad (12)$$

$$U^{\mathrm{I}}(x) = 1 + C_1(1 - 2x) + C_2(2x - 3x^3) + C_3(3x^2 - 4x^3) + C_4(4x^3 - 5x^4) \quad (13)$$

$$U^{\mathrm{II}}(x) = -2C_1 + C_2(2 - 6x) + C_3(6x - 12x^2) + C_4(12x^2 - 20x^3) \quad (14)$$

$$U^{\mathrm{III}}(x) = -6C_2 + C_3(6 - 24x) + C_4(24x - 60x^2) \quad (15)$$

$$U^{\mathrm{I}v}(x) = -24C_3 + C_4(24x - 120x) \quad (16)$$

Putting into $\frac{d^2y}{dx^2} - 0.5\frac{d^4y}{dx^4}\frac{dy}{dx} = 0\ gives$

$$-2C_1 + C_2(2 - 6x) + C_3(6x - 12x^2) + C_4(12x^2 - 20x^2) - 0.5[-24C_3 + C_4(24 - 120x)] + 11C_1(1 - 2x) + C_2(2x - 3x^2) + C_3(4x^3 - 5x^4) = 0 \quad (17)$$

Collecting like terms gives

$$C_1(-2 + 1 - 2x) + C_2(2 - 6x + 2x - 3x^2) + C_3(6x - 12x^2 + 12 + 3x^2 - 3x^2) + C_4(12x^2 - 20x^3 - 12 + 60x + 45x^3 - 5x^4) = -1 \quad (18)$$

$$\Rightarrow C_1[-1 - 2x] + C_2[2 - 4x - 3x^2] + C_3[12 + 6x - 9x^2 - 4x^2] + C_4[-12 + 60x + 12x^2 - 16x^3 - 5x^4] = -1 \quad (19)$$

Making (19) orthogonal by firstly multiplying each term by $(x - x^2)$ and integrating over $[0,1]$ gives

$$C_1\int_0^1(-1 - 2x)\,(x - x^2)dx + C_2\int_0^1(2 - 4x - 3x^2)\,(x - x^2)dx + C_3\int_0^1([12 + 6x - 9x^2 - 4x^3])\,(x - x^2)dx + C_4\int_0^1(-12 + 60x + 12x^2 - 16x^3 - 5x^4)\,(x - x^2)dx = -\int_0^1(x - x^2)\,dx \quad (20)$$

$$\Rightarrow C_1\int_0^1(-x - x^2 + 2x^3)dx + C_2\int_0^1(2x - 6x^2 + x^3 + 3x^4)\,dx + C_3\int_0^1(12x - 6x^2 - 15x^3 + 5x^4 + 4x^5)\,(x - x^2)dx + C_4\int_0^1(-12x + 72x^2 - 48x^3 - 24x^4 + 11x^5 + 5x^6)\,dx = \int_0^1(x^2 - x)\,dx \quad (21)$$

$$\Rightarrow C_1\left[{}^{-x}/_{2} - {}^{x^3}/_{3} + {}^{x^4}/_{2}\right]_0^1 + C_2\left[x^2 - 2x^3 + {}^{x^4}/_{4} + {}^{3x^5}/_{5}\right]_0^1 + C_3\left[6x^2 - 2x^3 + {}^{15}/_{4x^4} + {}^{4}/_{6x^6}\right]_0^1 + C_4\left[-6x^2 + 24x^3 - 12x^4 - {}^{28}/_{5x^5} - {}^{11}/_{6x^6} + {}^{5}/_{7x^7}\right]_0^1 = \left[{}^{x^3}/_{3} - {}^{x^2}/_{2}\right]_0^1 \quad (22)$$

$$\Rightarrow C_1\left({}^{-1}/_{2} - {}^{1}/_{3} + {}^{1}/_{3}\right) + C_2\left(1 - 2 + {}^{1}/_{4} + {}^{3}/_{5}\right) + C_3\left(6 - 2 - {}^{15}/_{4} + {}^{6}/_{4}\right) + C_4\left(-6 + 24 - 12 + {}^{28}/_{5} - {}^{11}/_{6} + {}^{5}/_{7}\right) = \left({}^{1}/_{3} - {}^{1}/_{2}\right) \quad (23)$$

$$\Rightarrow {}^{1}/_{3}\,C_1 - {}^{3}/_{20}\,C_2 + {}^{11}/_{12}\,C_3 + {}^{2971}/_{210}\,C_4 = -{}^{1}/_{6} \quad (24)$$

Multiplying (24) through by -1260, we have:

$$420C_1 + 189C_2 - 1155C_3 - 17826C_4 = 210 \quad (25)$$

Similarly, making (19) orthogonal by multiplying each term of (18) by$(x^2 - x^3)$, integrating over $[0,1]$ and multiplying result by -2520 we have,

$$231C_1 + 168C_2 - 1140C_3 - 34980C_4 = 105 \quad (26)$$

Again making (19) orthogonal by thirdly multiplying each term of (18) by$(x^3 - x^4)$, integrating over $[0,1]$ and multiplying result by -12600 we have,

$$294C_1 + 534C_2 - 1296C_3 - 3353C_4 = 126 \quad (27)$$

We do same for $(x^4 - x^5)$ and multiply result by -2520 to get

$$294C_1 + 534C_2 - 1296C_3 - 2432C_4 = 84 \quad (28)$$

Putting these equations in augmented matrix form we obtain below:

$$\begin{pmatrix} 420 & 189 & -1155 & -17826 & \vdots & 210 \\ 231 & 168 & -1140 & 34980 & \vdots & 105 \\ 294 & 534 & -1296 & -3353 & \vdots & 126 \\ 204 & 207 & -823 & -2432 & \vdots & 84 \end{pmatrix}$$

We now solve for C_1, C_2, C_3, C_4 using Gaussian Elimination method. We have the emerging final matrix as below to obtain

$$\begin{pmatrix} 1 & 0.45 & -2.75 & 42.442857 & \vdots & 0.5 \\ 0 & 1 & -7.895009 & 699.210263 & \vdots & -0.163936 \\ 0 & 0 & 1 & 101.250052 & \vdots & 0.016712 \\ 0 & 0 & & 01 & \vdots & 0.001134 \end{pmatrix}$$

$$\therefore \begin{cases} C_4 = 0.001134 \\ C_3 = -101.250052C_4 + 0.016712 \\ C_3 = 0.131530 \\ C_2 - 7.895009C_3 + 699.210263C_4 = -0.163936; \;\; C_2 = 0.08159 \end{cases}$$

$C_1 + 0.45C_2 - 2.75C_3 + 42.44287C_4 = 0.5$

$C_1 = 0.776862$

$\therefore U(x) = x + 0.776862(x - x^2) + 0.08009(x^2 - x^3) + 0.131530(x^3 - x^4) +$
$0.001134(x^4 - x^5)$

$= 1.776862x - 0.695272x^2 + 0.04994x^3 - 0.130396x^4 - 0.001134x^5$(29)

When our segmentation parameter is$h = \, ^1/_4$, then on the interval $[0,1]$we have

$U(0) = x_0 = 0.0000, U(^1/_4) = x_1 = 0.401031$

$U(^1/_2) = x_2 = 0.71267,$

$U(^1/_2) = x_3 = 0.92110$

$U(^1/_2) = x_4 = 1.0000$

But when our segmentation parameter is $h = 0.1$, then for the interval$[0,1]$we have,

$U(0.0) = x_0 = 0$

$U(0.1) = x_1 0.170770$

$U(0.2) = x_2 = 0.327752$

$U(0.3) = x_3 = 0.470774$

$U(0.4) = x_4 = 0.599348$

$U(0.5) = x_5 = 0.712670$

$U(0.6) = x_6 = 0.809619$

$U(0.7) = x_7 = 0.888751$

$U(0.8) = x_8 = 0.948303$

$U(0.9) = x_9 = 0986189$

$U(1.0) = x_{10} = 1.000000$

However, if we let our segmentation parameter $h = 0.01$ and divide the interval $[0,0.1]$ into10 equal halves then

$U(0.00) = x_0 = 0.000000,$

$U(0.01) = x_1 = 0.017699$

$U(0.02) = x_2 = 0.035260,$

$U(0.03) = x_3 = 0.052681$

$U(0.04) = x_4 = 0.069965$

$U(0.05) = x_5 = 0.087110$

$U(0.06) = x_6 = 0.104118$

$U(0.07) = x_7 = 0.120988$

$U(0.08) = x_8 = 0.137719$

$U(0.09) = x_9 = 0.15314$

$U(0.10) = x_{10} = 0.170770$

Now, dividing the interval $[0.9,1.0]$into 10 equal halves with the same segmentation parameter of 0.01, we obtain;

$U(0.90) = x_0 = 0.9861893087$

$U(0.91) = x_1 = 0.9886963539$

$U(0.92) = x_2 = 0.9909603231$

$U(0.93) = x_3 = 0.9929785953$

$U(0.94) = \mathrm{x}_4 = 0.9947485172$

$U(0.95) = x_5 = 0.9962674029$

$U(0.96) = x_6 = 0.9975325340$

$U(0.97) = x_7 = 0.9985411592$

$U(0.98) = x_8 = 0.9992904949$

$U(0.99) = x_9 = 0.9997777248$

$U(1.00) = x_{10} = 1.000000000$

7.0 Remarks

From the above solution by Finite Element Method (Standard Galerkin Method to be precise), we can see that the smaller the value of the segmentation parameter h, the better the image segmentation. Hence the smaller value of h causes the segmentation steps/phases (in this case $x_0, x_1, x_2 \ldots$)to be closer to each other, thus eliminating possible errors which would otherwise occur in taking lumps of the image pixels or voxels at a time. Thus it is preferable to use a smaller value of h for a more efficient segmentation to take place.

Although the large value if h, faster the convergence speed, it may not be adequate to give a good curve of the snake. On the other hand, the smaller the value of h, the slower it takes to converge, but it gives a good curve around the region of interest.

8.0 Discussion/Analysis

In the application of image processing tools, an image is first passed through the Gaussian low pass filter which the low passed image then has its intensity adjusted by the image intensity adjustment tools. Then, finally Sobel edge Detector is used to enhance the outline of the image. This order is necessary for the processing because Sobel edge detector is very sensitive to noise. As such the noise need be filtered out before the edge detector application. More so, since the gradient of the Sobel edge detector is related to the change in intensity at the edge of an object the image intensity adjustment is used to produce a higher contrast image. As image intensity adjustment can improve the intensity of the image as well as noise within the image, the noise must be filtered out before the intensity adjustment. Gaussian low pass is employed to reduce the noise effect.

9.0 Conclusion

Further research in the segmentation of medical images will strive towards improving the accuracy, precision, and computational speed of segmentation methods, as well as reducing the amount of manual interaction. Accuracy and precision can be improved by incorporating prior information from atlases and by combining discrete and continuous – base segmentation methods. For increasing computational efficiency, multiscale processing and parallelizable methods such as neural networks appear to be promising approaches. Computational efficiency will be particularly important in – real time processing applications. Possibly the most important question surrounding the use of image segmentation is its application in clinical settings. Computerized segmentation methods have already demonstrated their utility in research applications and are now garnering increased use for computer aided diagnosis and radiotherapy planning. It is unlikely that automated segmentation methods will ever replace physicians but they will likely become crucial elements of medical image analysis. Segmentation methods will be particularly valuable in areas such as computer integrated surgery, where visualization of the anatomy is a critical component. Active contour are a method for segmentation based on minimizing the energy of a continuous spline contour subject to constraints on both its autonomous shape and external forces derived from a superposed image that pull the active contour toward Image features such as line and edges. The contour is initially place near an edge under consideration, and then image forces and the user-defined external constraints draw the contour to the edge in the image.

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