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Calculation of the Radial Oscillator Function of The Angular Momentum Quantum Number

Fatima S. K. and Said R. S.

Department of Physics,
Bayero University, Kano

Abstract

The main characteristic of harmonic oscillation is the fact that the oscillation frequency does not depend on the amplitude of the displacement. Thus all the circular and elliptical orbits have the same frequency. In considering harmonic oscillations, therefore, the three-dimensional quantum mechanical oscillator is usually employed. In this paper, we calculate the radial oscillation function for the angular momentum and radial momentum quantum number for a specified interval using a FORTRAN program (kap14) to compute the oscillation functions.The motivation of this study is to illustrate the power of code Kap 14 for the teaching of quantum mechanics.

1.0 Introduction

In considering the three-dimensional motion of the harmonic oscillator, all the circular and elliptical orbits have the same frequency. We will consider stationary solutions; the Eigen-solutions of the time–dependent Schrodinger equations[1] We shall note that the quantum mechanical oscillator displays a special simplicity. We note that the quantum mechanical oscillator is also one of the few systems for which the Schrodinger equations can be solved analytically. The Eigen values are usually equally spaced and the Eigen states form a complete basis that can be used for the representation of the operators[2]

2.0 The Governing Equations

The spherically symmetric oscillator potential is given by

$$V(r) = cr^2 = \frac{1}{2}m\omega^2 r^2 \tag{2.1}$$

In the right-hand side of equation (2.1), the constant c has been expressed by the mass m of the moving body and the angular frequency of the corresponding classical motion. The radial Schrodinger equation then becomes

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}v_{l(r)} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}v_{l(r)} + \frac{1}{2}m\omega^2 r^2 v_{l(r)} = Ev_{l(r)} \tag{2.2}$$

The Eigen-solutions of this equation are then considered with the following parameters

$$a = \frac{m\omega}{2\hbar} \tag{2.3}$$

$$r_0 = \frac{1}{(2a)^{\frac{1}{2}}} = \left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}} \tag{2.4}$$

We can arrange equation (2.2) in the form

$$v_l''(\mathrm{r}) - \left(\frac{r^2}{r_0^4} + \frac{l(l+1)}{r^2}\right)v_l(r) = \frac{2mE}{\hbar^2}v_{l(r)} \tag{2.5}$$

The Eigen-solutions of (2.5) can be obtained analytically[3] while the Eigen energy values are given by

$$E_{nl} = \left(2n + l - \frac{1}{2}\right)\hbar\omega \tag{2.6}$$

The radial quantum number n takes values n=1,2,3,…..The associated Eigen-functions $v_{nl}(\mathrm{r})$ are the radial oscillator functions and they satisfy the conditions:

$$v_{nl}(0) = 0 \tag{2.7}$$

$$\int_0^\infty \left|v_{nl(r)}\right|2\, dr = 1 \tag{2.8}$$

3.0 Methodology

We calculate functions and develop a connection between the quantum mechanical states and the classical motions[4]. For states with low excitation energies, the connections are not easily recognizable because of the Heisenberg uncertainty principle[5]. Connections, however exists and are recognizable for high excitation energies.

Corresponding author: ***Fatima S. K.,*** E-mail: skfatima@yahoo.com, Tel.: +2348064120841

3.1 Description of the code

Program KAP14 6](Schmid& Spitz, 1987) calculates the radial oscillator function for the angular momentum quantum number l and the radial quantum number n in the interval [0, b].
The Fixed quantities in the input parameters in the code are:

- IB: Number of sub-intervals in the interval [0,b] (IB=480)
- B: Upper interval limit b in units of 1fm (B=12)
- NMAX: Maximum radial quantum number (NMAX=20)

The Input quantities are, depending on the systems used, A is the Oscillator width parameter in units of 1/fm, l is the Angular momentum quantum number n is the Radial quantum number
The Output quantities are:

- r = Distance in units of 1fm where $r(I) = I * H,\ \ H = B/IB,\ and\ \ I = 0,1,2,\ldots,IB$
- $v_{nl}(I,n)$ = Radial oscillator function for the radial quantum number n and the angular momentum quantum number l, and is the function value at the point r(I)
- $W_{nl}(I,n)$ = Radial position probability density,$W_{nl}(I,n) = V_{nl}^2(I,n)$

4.0 Results

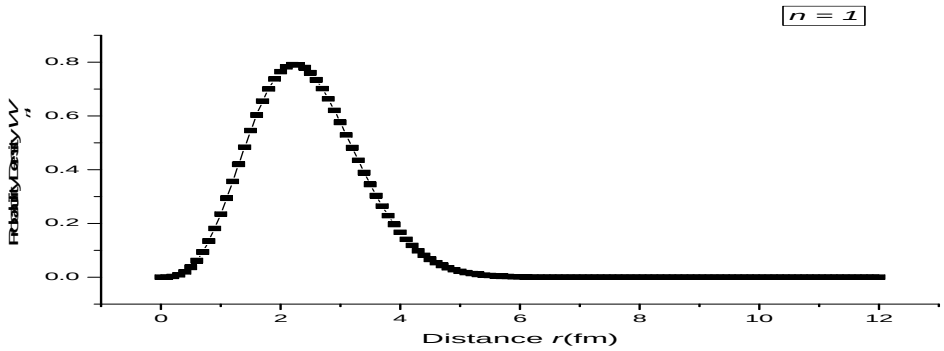

Fig 1. The plot of Distance against Radial position probability density

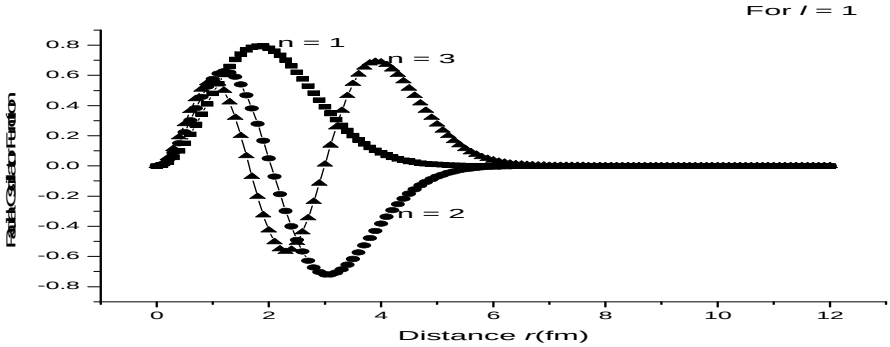

Fig 2. The plot of Distance against Radial oscillator function

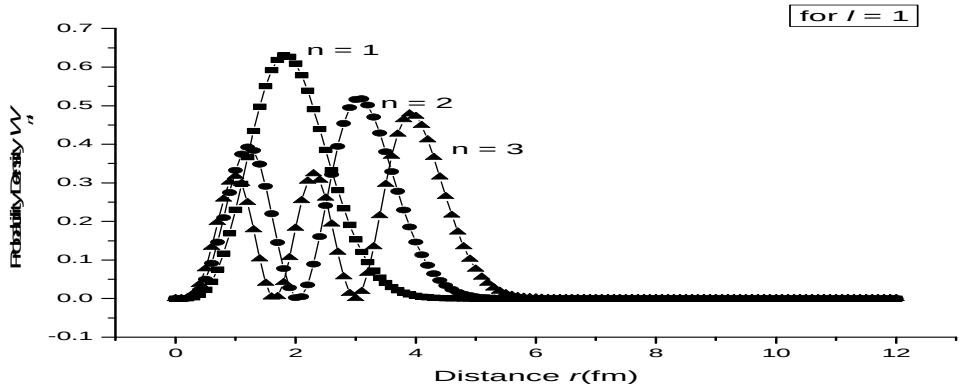

Fig 3. The plot of Distance against Radial position probability density

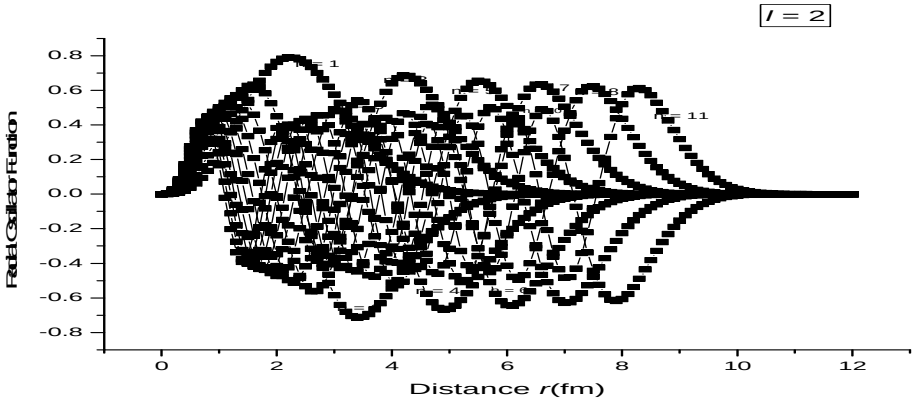

Fig 4. The plot of Distance against Radial oscillator function

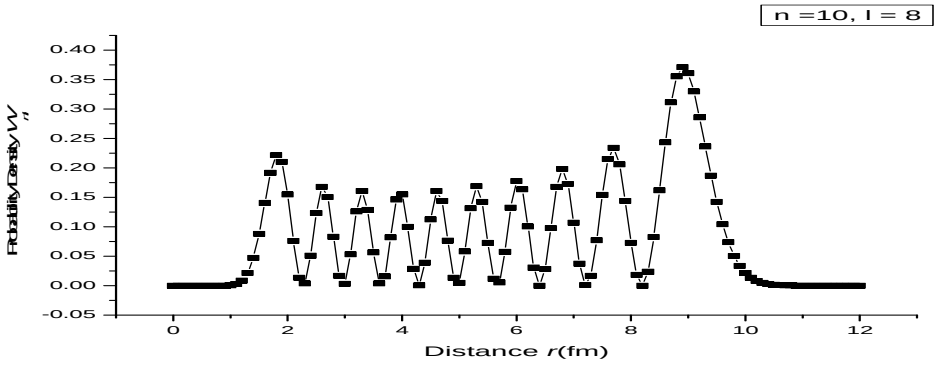

Fig 5. The plot of Distance against Radial position probability density

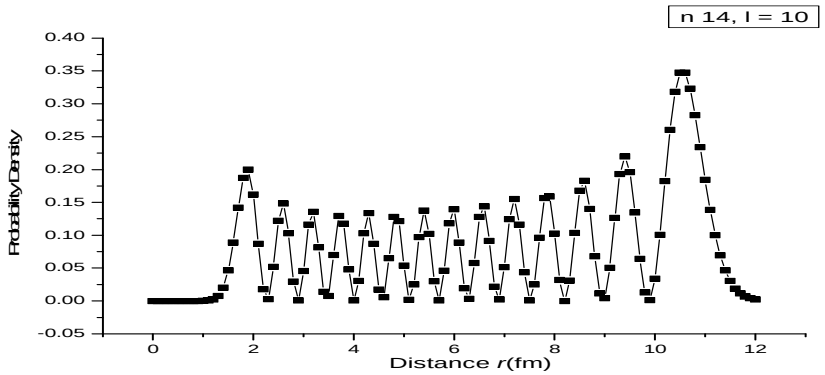

Fig 6. The plot of Distance against Radial position probability density

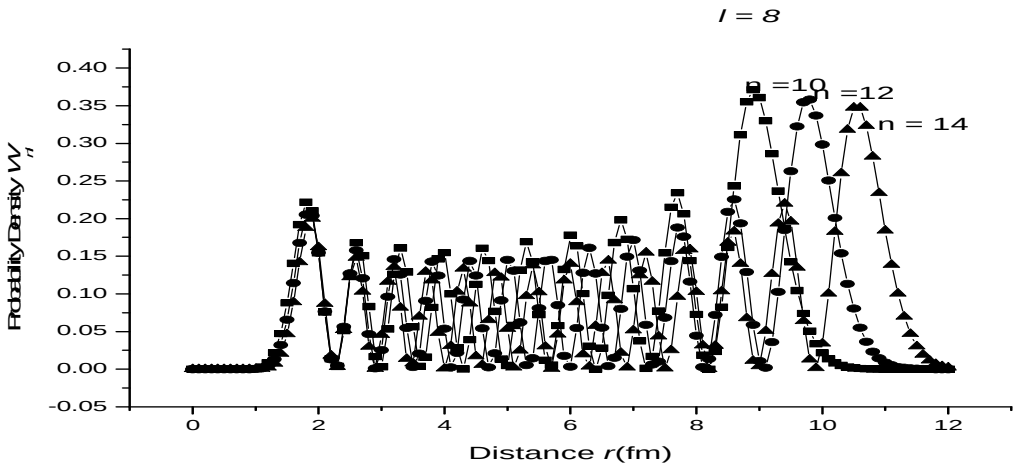

Fig 7. The plot of Distance against Radial position of probability density

5.0 Discussion

It can be observed that for the different systems used, the same results were obtained for same angular momentum quantum number l and the radial quantum number n in each case, which shows the radial quantum behaviour. For $n = 0$ and $l = 0$ the values of all Radial oscillator function $v_{nl}(I, n) = 0$ this means that there is no radial oscillation at the ground-state of all the systems. The radial oscillations can only be observed in the excited levels. The code gave the results at $l = 1$ for different values of$n$that run from 1 to 14 and results at $l = 2$ for different values of n that run from 1 to 14 and so on. The plots give results for $l = 1, 3, 8, 10,\ and\ 12$, the patterns are the same with increasing wavelength from $n = 1, n = 2$, and so on.

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