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On Application of Matlab to the Solution of One Dimensional Finitely Damped String

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Abstract

The problem of vibration of plucked strings was investigated. The objective is to determine the vibrational behaviour and the decay rate of a guitar using nylon and steel strings. The derived Newton's second law of motion with damping coefficient was considered. The method of separation of variables was applied to solve the resultant equation. Using Matlab, the graphical result shows that nylon strings decay mainly as a result of internal damping in the strings while steel strings decay due to air viscosity. Therefore we conclude that nylon strings are under less tension than steel strings.

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1.0 Introduction

In this paper, we considered a mechanical system where its dynamic behaviour is resolved by interaction of several components. Virtually every system possesses the capability of vibration, and most systems can vibrate freely in a large variety of ways [1] . The plucked strings only radiate a small amount of sound directly, but they excite the bridge and the top plate, which in turn transfers energy to the air cavity, ribs and black plates [2]. Sound is radiated efficiently by the vibrating plates and through the sound hole [3].

There are three major techniques through which string instruments produce sound from one or more vibrating strings, and which is then transferred to the air by the body instrument: plucking, bowing and striking.

Here, we are concerned with plucking technique since guitar undergoes that in the model of vibrating string. N.C Pickering investigated the effect of pluck point and string and string properties on the sound of the string[4]. T.D Rossing also looked at how the sound of the guitar string evolves over time from the pluck to when the sound has finally died out [5]. We also used laboratory experiment values generated by R. Storjohann [6] to run a graphical solution of the second order partial differential equation that results, using MATLAB programming to determine the vibrational behaviour and decay rate of nylon and steel string(fig1). In the derivation of model of a damped guitar string, V.E Howel and L.N Trefethen assumed there is an additional force in the vertical direction that is proportional to the infinitesimal piece of string's velocity with width Δx[7].

2.0 Methodology

The derived Newton's second law of motion with the damping coefficient was applied

$$-T(x)\tan\theta + T(x+\Delta x)\tan\beta - \omega\Delta x U_t = mU_{tt} \qquad (1)$$

Where ω is the constant of proportionality, $\tan\theta$ and $\tan\beta$ are the slopes of the tangent lines.

Solving the equation above led us to a system of second order partial differential equation when solved. The analytical method of separable variable of solving a problem of one-dimensional finitely damped string was used.

Our interest is not in the musical effect but the basic mechanical fact that a string with both ends fixed has a number of well-defined states of natural vibrations. This produces a simple wave which must be subject to a restoring force that continually pulls the system forward to equilibrium position. We usually get the partial differential equations of the form.

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$U_{xx} - KU_t = \frac{1}{C^2} U_{tt} \ldots$ (2)

The solutions of the differential equation are used to develop theories which can be used to analyze general solution. vibration deals with oscillatory motion of dynamic systems and forces associated with them.

3.0 Main Result

Solving the equation (1), we obtained (2)
Where k is the damping coefficient.
Solving (2) using separable variable method of a boundary value problem,
Let $U(x,t) = X(x)T(t)$
$U_x = X'T;\ U_{xx} = X''T; U_t = XT';\ U_{tt} = XT''$
Using the initial conditions,

$$U(x,0) = \begin{cases} \frac{h}{b}x; & 0 \le x \le b \\ \frac{h}{b-L}(X-L); & b \le x \le L \end{cases} \ldots \ (3)$$

Applying (3) on (2), we obtain a general solution which is our main result:
$U(x,0) = \sum_{n=1}^{\infty} e^{-\alpha_n t} (a_n \cos \mu_n t + b_n \sin \mu_n t). \sin \frac{n\pi x}{L} \ldots (4)$

$$U(x,0) = \sum_{n=1}^{\infty} \frac{e^{-\alpha_n t}}{b(b-L)n^2\pi^2} \left\{ 2\cos \mu_n t + \frac{KC^2}{\mu_n} \sin\mu_n t \right\}. \sin \frac{n\pi b}{L}. \sin \frac{n\pi x}{L} \qquad (5)$$

Where $b_n = \frac{KC^2 a_n}{2\mu_n}$ and $a_n = \frac{-2hL^2}{b(b-L)(n\pi)^2} \sin \frac{n\pi b}{L}$.

4.0 Discussion

The derived Newton's second law of motion which resulted to second order partial differential equation was solved.

MATLAB Programming was used to run a graph(figure1) on the general solution of damped one-dimensional wave equation for nylon and steel strings at each pluck points ($x = 0.12, 0.225, 0.35, 0.485, 0.6, 0.725,$ and 0.83) but our main focus is on the pluck point at the middle($x = 0.485$). This authenticates the statement made by R. Storjohann that in a nylon string guitar, the higher modes decay mainly as a result of internal damping in the string, but a steel string guitar decay as a result of air viscosity. This is to say that the steel strings are actually damped less by air viscosity than nylon strings since they are thinner. We also discovered from figure1 that as the time(in seconds) increases, the Nylon string keep decaying, but that was not the case for Steel string.

5.0 Conclusion

The importance of decay rate is to make sure that the sound produced by the guitar dies out. The classical guitar can never die out if damping is not applied.

The way forward to this paper is to determine how the decay rate can be captured mathematically since N.C Pickering could only list the experimental values without showing any mathematical proof. These values helped us to run a Matlab graphical solution for Nylon and steel strings as shown in figure1.

It is therefore true that nylon strings are under less tension than steel strings.

The greater tension would be requiring more pressure to depress the strings.

MATLAB CODES FOR ONE-DIMENSIONAL FINITELY DAMPED STRING(NYLON & STEEL)

```
Clear
clc
Alpsteel = [0.1 0.13 0.16 0.18 0.21 0.22 0.23 0.25 0.27 0.28];
Alnylon = [0.4 0.55 0.7 0.9 1.05 1.15 1.3 1.45 1.55 1.65];
n=1:10;
K=0.026;
C=11.7;
X= [0.12 0.225 0.35 0.485 0.60 0.725 0.83];
L=0.965;
h=1;
b=0.4825;
Yn=n*pi/L
Un=sqrt ((( 4*Yn.^2*C^2) – (K^2*C^4))/4)
An= (2*h*L^2*sin(n*pi*b))./(b.*(L−b).*((n.*pi).^2).*L)
```

```
Bn= (K*C^2./2*Un).*An
i=1
for j= 1:7
for t= 0.01:0.01:1
Unylon (i)= sum (exp(−Alpnylon.*t).*((An.*cos(Un.*t)) + (Bn.*sin (Un.*t))).*sin (n*pi*X (j) /L))
Usteel (i)= sum (exp(−Alpsteel.*t).*((An.*cos(Un.*t)) + (Bn.*sin (Un.*t))).*sin (n*pi*X (j) /L))
i= i+1
end
t= 0.01:0.01:1
figure
plot (t, Unylon)
title (sprintf ( ′Nylon \n Displacement at x=%12.8f ′ ,
x (j)))
xlable ( ′ Time (secs) ′ )
figure
plot (t, Usteel)
title (sprintf ( ′Steel \n Displacement at x=%12.8f ′ ,
x (j)))
xlable ( ′ Time (secs) ′ )
i=1
end
```

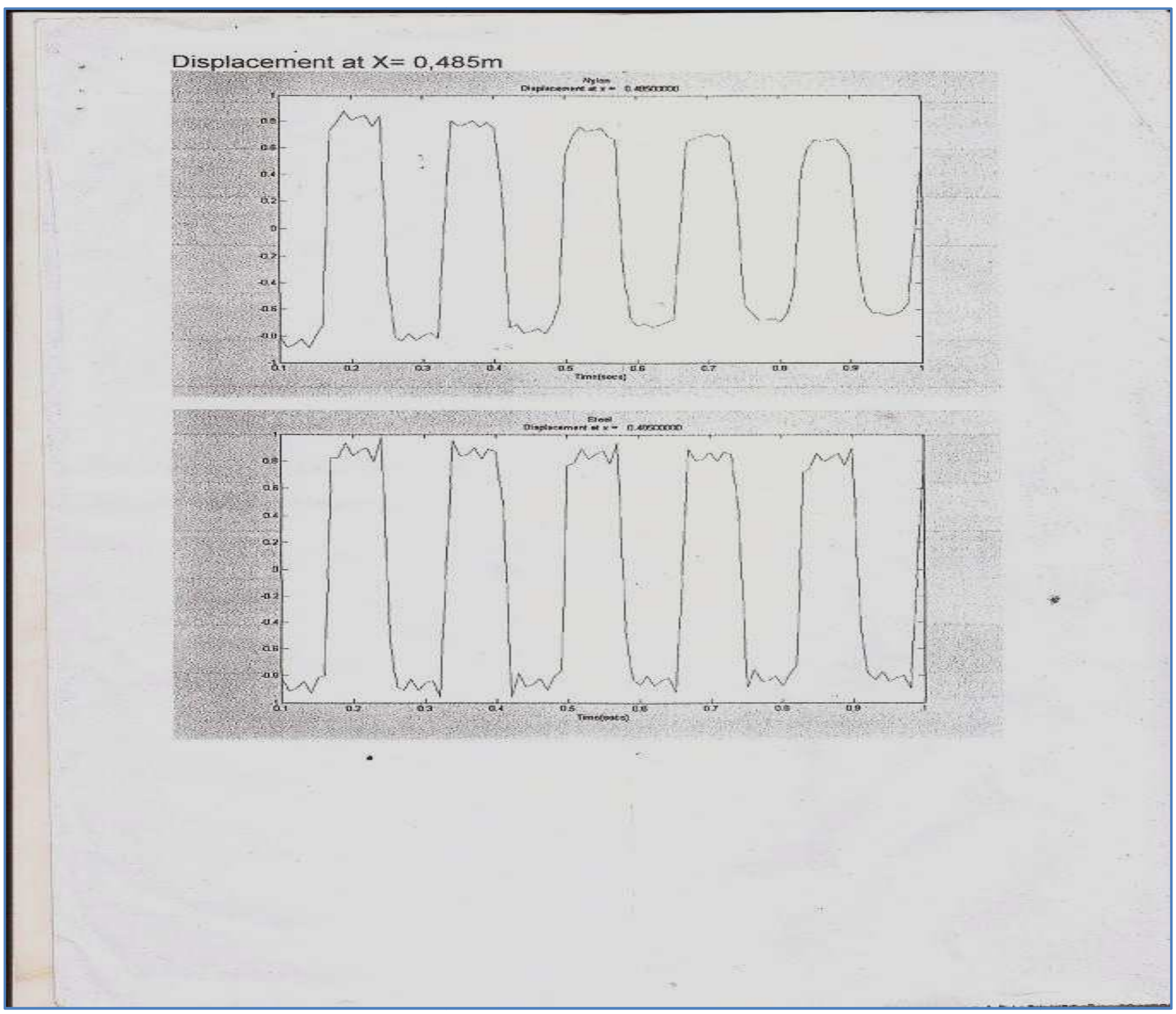

Figure 1 : matlab graph for nylon and steel string

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