

## Investigating the Effect of Loss-of-Coolant on the Stability of Water-Cooled Reactor Design Models

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### Abstract

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*Loss-of-coolant in the operating reactor usually causes instability in the reactor. To investigate the cooling problem of the reactor, two tests were conducted on safety margin. The first was applied to design temperature and design coolant flow rates while, the second was applied to design temperature and design volume of the fuel in the reactor core. The tests were carried out on some typical Water-Cooled Reactor Design (WCRD) models using Linear Regression Analysis Techniques. The results of the statistical analyses on these types of nuclear reactor models reveal that the WCRD models promises stability under application of small size of uranium (fuel) at 9g and below than large size of uranium (fuel) at 12g and above. Meanwhile, at 9g of fuel element the reactor seems to be most stable and safer as the regression plot was optimized. The safety margin prediction of up to 1.25% was validated for a typical WCRD model as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit. The implication of this research effort to Nigeria's nuclear power project development.*

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**Keywords:** Linear Regression Analysis, Water-Cooled Reactor Design Model with Normal Pressure, Water-Cooled Reactor Design Model with Abnormal Pressure, Safety Factor,  $\dot{Y}$ , Optimization, Stability Margin in Nuclear Power Reactor Designs

### 1.0 Introduction

The unsteady state of the coolant can affect the reactor stability since the rate of coolant flow is either low or high nor abnormal, the reactor may be overheated-up and hydrogen could built-up within reactor core of nuclear power plant while the steady state of the coolant will provide stability in the reactor that will minimize heat generation in the reactor core and also disallow fuel melting that may produce decay heat in the core assemblies which may degenerated to hydrogen built-up that can make reactor to melt or fail. As identified in the case of Fukushima Daiichi Nuclear accident March 2011, Reuters reported that Fukushima Daiichi Unit 3 has lost cooling capability and may be experiencing melting of the core, eventually, reactor 1- 4 was written-off.

The overheating of the reactor core could lead to temperature rise and gradual pressure built-up in the system. Removal of residual heat could not be assured in the case of Fukushima Daiichi nuclear plant accident in Japan, as reported to Reuter on 11<sup>th</sup> March 2011 by Mark Hibbs, a nuclear expert at the Carnegie endowment for international peace. The cooling of reactor core could not be assured in the case of Fukushima Daiichi nuclear plant accident. Seawater was being pumped into the reactor in an attempt to cool down the radioactive core. A catastrophic built-up of hydrogen gas inside the building that housed the reactor sparked an explosion, destroying the structure and throwing radioactive debris into the air. Also, this

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phenomenon of malfunctioning of cooling system has led to the Three Mile Island Nuclear Power Plant accident near Harrisburg, Pennsylvania in USA, in which TMI-2 reactor was destroyed in 1979. Chernobyl disaster in 1986 rated worst nuclear accidents in history.

A **loss-of-coolant accident (LOCA)** is a mode of failure for a nuclear reactor; if not managed effectively, the results of a LOCA could result in reactor core damage. Each nuclear plant's emergency core cooling system (ECCS) exists specifically to deal with a LOCA.

Nuclear reactors generate heat internally; to remove this heat and convert it into useful electrical power, a coolant system is used. If this coolant flow is reduced, or lost altogether, the nuclear reactor's emergency shutdown system is designed to stop the fission chain reaction. However, due to radioactive decay the nuclear fuel will continue to generate a significant amount of heat. The decay heat produced by a reactor shutdown from full power is initially equivalent to about 5 to 6% of the thermal rating of the reactor. If all of the independent cooling trains of the ECCS fail to operate as designed, this heat can increase the fuel temperature to the point of damaging the reactor.

The fuel and reactor internals may melt; if the melted configuration remains critical, the molten mass will continue to generate heat, possibly melting its way down through the bottom of the reactor. Such an event is called a nuclear meltdown and can have severe consequences. The so-called "China syndrome" would be this process taken to an extreme: the molten mass working its way down through the soil to the water table (and below) - however, current understanding and experience of nuclear fission reactions suggests that the molten mass would become too disrupted to carry on heat generation before descending very far; for example, in the Chernobyl disaster the reactor core melted and core material was found in the basement, too widely dispersed to carry on a chain reaction (but still dangerously radioactive).

Under operating conditions, a reactor may passively (that is, in the absence of any control systems) increase or decrease its power output in the event of a LOCA or of voids appearing in its coolant system (by water boiling, for example). This is measured by the coolant void coefficient. Most modern nuclear power plants have a negative void coefficient, indicating that as water turns to steam, power instantly decreases. Two exceptions are the Russian RBMK and the Canadian CANDU. Boiling water reactors, on the other hand, are designed to have steam voids inside the reactor vessel.

In practical term the fuel expands on heating the core of the pellet more than the rim (the upper or outer circular edge of the pellet). Because of the thermal stress thus formed the fuel cracks, the cracks tend to go from the center to the edge in a star shaped pattern. The cracking of the fuel has an effect on the release of radioactivity from fuel both under accident conditions and also when the spent fuel is used as the final disposal form. The cracking increases the surface area of the fuel which increases the rate at which fission products can leave the fuel.

The temperature of the fuel varies as a function of the distance from the center to the rim. At distance  $d$  from the center the temperature ( $T_d$ ) is described by the equation where  $\rho$  is the power density ( $W m^{-3}$ ) and  $K_f$  is the thermal conductivity.

$$T_d = T_{Rim} + \rho (r_{pellet}^2 - d^2) (4 K_f)^{-1} \tag{1}$$

When the nuclear fuel increases in temperature, the rapid motion of the atoms in the fuel causes an effect known as Doppler broadening. When thermal motion causes a particle to move towards the observer, the emitted radiation will be shifted to a higher frequency. Likewise, when the emitter moves away, the frequency will be lowered. For non-relativistic thermal velocities, the Doppler shift in frequency will be:

$$f = f_0 \left( 1 + \frac{v}{c} \right) \tag{2}$$

where  $f$  is the observed frequency,  $f_0$  is the rest frequency,  $v$  is the velocity of the emitter towards the observer, and  $c$  is the speed of light.

Since there is a distribution of speeds both toward and away from the observer in any volume element of the radiating body, the net effect will be to broaden the observed line.

If  $P_v(v)dv$  is the fraction of particles with velocity component  $v$  to  $v + dv$  along a line of sight, then the corresponding distribution of the frequencies is

$$P_f(f)df = P_v(v_f) \frac{dv}{df} df \tag{3}$$

where

$v_f = c \left( \frac{f}{f_0} - 1 \right)$  is the velocity towards the observer corresponding to the shift of the rest frequency  $f_0$  to  $f$ .

Therefore,

$$P_f(f)df = \frac{c}{f_0} P_v \left( c \left( \frac{f}{f_0} - 1 \right) \right) df \tag{4}$$

We can also express the broadening in terms of the wavelength  $\lambda$ . Recalling that in the

non-relativistic limit  $\frac{\lambda - \lambda_0}{\lambda_0} \approx -\frac{f - f_0}{f_0}$ , we obtain

$$P_\lambda(\lambda)d\lambda = \frac{c}{\lambda_0} P_v \left( c \left( 1 - \frac{\lambda}{\lambda_0} \right) \right) d\lambda$$

In the case of the thermal Doppler broadening, the velocity distribution is given by the Maxwell distribution

$$P_v(v)dv = \sqrt{\frac{m}{2\pi kT}} \exp\left(-\frac{mv^2}{2kT}\right) dv$$

where  $m$  is the mass of the emitting particle,  $T$  is the temperature and  $k$  is the Boltzmann constant. Then,

$$P_f(f)df = \left(\frac{c}{f_0}\right) \sqrt{\frac{m}{2\pi kT}} \exp\left(-\frac{m \left[ c \left( \frac{f}{f_0} - 1 \right) \right]^2}{2kT}\right) df$$

We can simplify this expression as

$$P_f(f)df = \sqrt{\frac{mc^2}{2\pi kT f_0^2}} \exp\left(-\frac{mc^2 (f - f_0)^2}{2kT f_0^2}\right) df$$

which we immediately recognize as a Gaussian profile with the standard deviation

$$\sigma_f = \sqrt{\frac{kT}{mc^2}} f_0$$

and full width at half maximum (FWHM)

$$\Delta f_{FWHM} = \sqrt{\frac{8kT \ln 2}{mc^2}} f_0$$

The fuel then sees a wider range of relative neutron speeds. Uranium-238, which forms the bulk of the uranium in the reactor, is much more likely to absorb fast or epithermal neutrons at higher temperatures. This reduces the number of neutrons available to cause fission, and reduces the power of the reactor. Doppler broadening therefore creates a negative feedback because as fuel temperature increases, reactor power decreases. All reactors have reactivity feedback mechanisms, except some gas reactor such as pebble-bed reactor which is designed so that this effect is very strong and does not depend on any kind of machinery or moving parts.

There have been several report and analysis on the safety of these reactors taking into account the specific design features of these reactors, these include “Safety of Nuclear Power Reactors”[1], ‘Loss-of-Coolant Accidents (LOCAs) in BWRs and PWRs’ [2], ‘Accident Analysis for Nuclear Power Plants with Pressurized Water Reactors’[3] and “Probability of Failure in BWR Reactor Coolant Piping”[4]. Also, there have been several report and analysis on the safety of these reactors with respect to the nuclear fuel these include; “Nuclear Fuel Safety Criteria Technical Review”[5], “Safety of Nuclear Power Reactors”, ‘Current Trends in Nuclear Fuel for Power Reactors’[6]. “Review of Fuel Failures in Water Cooled Reactors”[7], Furthermore, there have been researches into preventing Loss-of-Coolant-Accidents these include; “Strategies for Reactor Safety: Preventing Loss of Coolant Accidents”[8], “Strategies for Reactor Safety”[9]. These accidents may perhaps be as a result of design concept process of BWR and PWR(which could involve novel technologies) that have inherent risk of failure in operation and were not well studied/understood.

Failure may be recognized by measures of risks which include performance, design fault, obsolete components, wrong application, human errors and accident. These risks can be defined and quantified as the product of the probability of an occurrence of failure and a measure of the consequence of that failure. Since the objective of engineering is to design and build things to meet requirements, apart from cost implication, it is important to consider risk along with performance, and technology selections made during concept design. Engineering council guidance on risk for the engineering profession defined “Engineering Risk” as “the chance of incurring a loss or gain by investing in an engineering project”. Similar definitions are given by Modarres, Molak and Blanchard, that risk is a measure of the potential loss occurred due to natural or human activities.

In this work, Ordinary Least Square (OLS) methodology, which is largely used in nuclear industry for modeling safety, is employed. Some related previous works on the application of regression analysis technique include: “Optimization of The Stability Margin for Nuclear Power Reactor Design Models Using Regression Analyses Techniques”[10], “Modeling and Simulation of an Industrial Trickle-Bed Reactor for Benzene Hydrogenation: Model Validation against Plant Data”[11], “Statistical Models for Reliability and Management of Ultrasonic Inspection Data”[12], “Stochastic Modeling of

Deterioration in Nuclear Power Plants Components”[13],“The Correlation of Intergranular Stress Corrosion Cracking (IGSCC) Propagation With the Power Plant Transient History”[14]. Others are, “A Procedure for Estimation of Pipe Break Probabilities Due to ZGSCC”[15], “Investigation of Fundamental Thermal- Hydraulic Phenomena in Advanced Gas-Cooled Reactors”[16], ‘Quantitative functional failure analysis of a thermal-hydraulic passive system by means of bootstrapped Artificial Neural Networks’[17], these are materials where the effective used of Regression Analyses Techniques ‘RAT’ in the Optimization of the Safety in Nuclear Reactor Design Model has been established.

This study provides a mathematical expression for predicting “Safety Margin”,  $\hat{Y}$ , (dependent variables) given the values of independent variables or input parameters for a typical reactor design model. Furthermore, the mathematical expression can be used to determine the effect of decay heat (which is the independent variables) to the nuclear power reactor stability given the value of dependent variable.

**2.0 The Research Objectives**

The purpose of this work is to assist countries wishing to include nuclear energy for the generation of electricity, like Nigeria, to secure a reactor that is better and safe. The achievement here is to make worldwide contribution to knowledge. The studies intended to provide guidance in developing practical catalytic materials for power generation reactor and to help researchers make appropriate recommendation for Nigeria nuclear energy proposition as one of the solutions to Nigeria energy crisis.

**3.0 Research Design/Approach**

Theory and experiment have shown that for a water-cooled reactor, the coolant flow rate determines the amount heat or decay heat within reactor core. Therefore, the mass of the fuel plays significant role in the safety of the reactor during operation in preventing overheating of the reactor and reactor damage during accident. Hence, in this work, an assessment of the rise in fuel temperature in the reactor is considered of a typical boiling/pressurized water reactor designs. More specifically, the studies concentrate on technical factors that limit the achievement of higher burn-up of fuel, such as the fuel size mechanical interaction. Detailed investigations of fuel behaviour under reactor accident conditions are also included.

The research approach involves adjusting the parameters of a model function to best fit a data set. A simple data set consists of  $n$  points (data pairs)  $(x_i, y_i), i = 1, \dots, n$ , where  $x_i$  is an independent variable and  $y_i$  is a dependent variable whose value is found by observation. The model function has the form  $f(x, \beta)$ , where the  $m$  adjustable parameters are held in the vector  $\beta$ . The goal is to find the parameter values for the model which "best" fits the data. The least squares method finds its optimum when the sum,  $S$ , of squared residuals

$$S = \sum_{i=1}^n r_i^2$$

is a minimum.

The Tables 1 and 2 presented the values of design fuel input parameters in an operating reactor. For each of these different designs, a linear regression analysis technique was applied using statistical power analysis software, NCSS.

**Table 1:** The conceptual design of the coolant flow rates in the reactor

Nos. of trial (j)	Coolant (Water) flow rate in kg/sec.	Coolant (Helium) flow rate in kg/sec.	Coolant (Helium) flow rate in %	Heat Generated °C
1	40	8	20	200
2	60	24	40	270
3	80	48	60	300
4	100	80	80	440
5	120	120	100	480
6	140	168	120	520
7	160	184	140	600
8	180	288	160	760

Source : [18]

**Table 2:** Input data for safety margin against fuel size in a typical BWR similar to Fukushima Daiichi damaged reactor 1-4, in Japan and similar to BWR at Chernobyl Nuclear Power Reactor no. 4 in Russia accident meltdown and similar to PWR at Three Mile Island Unit 2 damaged reactor near Pennsylvania in USA.

Nos. of trial (j)	Fuel size in Mass (g)	Heat Generated °C
1	2.8	200
2	3.5	270
3	4.2	300
4	5.0	440
5	5.7	480
6	6.0	520
7	7.4	600
8	8.3	760
9	9.0	900
10	10.6	1050
11	11.0	1100
12	12.0	1200

Source : [19]

#### 4.0 Results and Analyses

##### 1. Water-Cooled Reactor Design (WCRD)

The results of the application of the linear regression analysis of the data in Table 1 for a typical BWR and PWR are presented as follows:

##### (i) Empirical Expression for Safety Factor, $\hat{Y}$

Investigating the effect of coolant flow rates on the Stability and Safety of the nuclear reactor during operation. The data obtained in Table 1 which represents a typical parameter for Water-Cooled Reactor Design (WCRD) and a typical parameter for Gas-Cooled Reactor Design (GCRD) was modified in order to obtain the best fit for the model. The new conceptual design reactor model optimizes the performance of the Fukushima Daiichi damaged reactor 1-4 in Japan, Chernobyl Nuclear Power Reactor no. 4 disaster accident melt-down in Russia, (TM1-2) damaged reactor in near Pennsylvania in USA and a typical GCRD of Pebble Bed Reactor (PBR).

The linear regression model equation to be solved is given by:

$$\hat{Y} = B_0 + B_1X_j + e_j \tag{5}$$

where,  $B_0$  is an intercept,  $B_1$  is the slope,  $X_j$  is the rate of increase in fuel volume  
 $e_j$  = error or residual,  $j = 1,2,3,\dots,k$  and  $k$  the last term.

##### Empirical Expression for Safety Factor, $\hat{Y}$ on Temperature and Coolant Flow Rates Effect

The model empirical expression is the equation of the straight line relating heat in the reactor and the coolant flow rates as a measure of safety factor estimated as:

$$\hat{Y} = (-91.9048) + (1.8810)*(X_j) + e_j \tag{6}$$

- Estimated model or predicted  
 where,

$\hat{Y}$  = heat, -91.9048 is the intercept, 1.8810 is a slope,  
 $X$  = the value of input parameter ( is the volume of coolant),  
 $e$  = error or residual and  $j = 1,2,3,\dots,8$

The Figure 1 shows the linear regression plot section of volume of coolant flow rate during reactor operation

##### (ii) Linear regression plot section

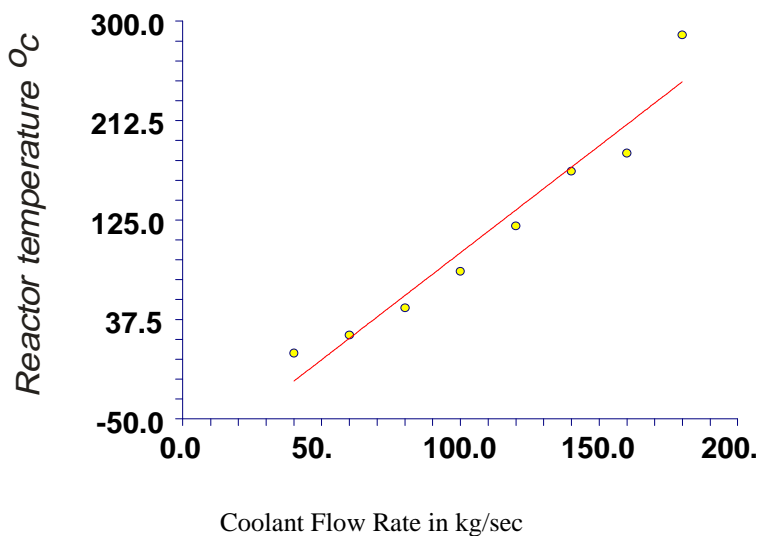


Figure 1: Temperature and Coolant effect on the stability of operating reactor

(iii) **F-test Result**

**Table 3: Summary of F-test Statistical Data**

Parameter	Value
Dependent Variable	Heat
Independent Variable	Volume
Intercept( $B_0$ )	-91.9048
Slope( $B_1$ )	1.8810
R-Squared	0.9439
Correlation	0.9716
Mean Square Error (MSE)	$5.883174 \times 10^{-2}$
Coefficient of Variation	0.2109
Square Root of MSE	24.25526

❖ Siegel ([20], P. 577) has shown that  $R^2$  can be used to test the validity of a model since it can be tested directly in this manner. If  $R^2$  calculated value is smaller than the critical value in the  $R^2$  table then the model is not significant in that case we accept  $H_0$ . But, if the  $R^2$  value is larger for the calculated value, then the model is significant at the given significant level. The critical value for  $n-12$  and  $k-1$  is 0.673 or 67.3%. Thus the model equation is significant at the given significant level of 5%.

**Table 4: Descriptive Statistics Section**

Parameter	Dependent	Independent
Variable	Heat (°C)	Fuel (g)
Count	8	8
Mean	115.0000	110.0000
Standard Deviation	94.8442	48.9898
Minimum	8.0000	40.0000
Maximum	288.0000	180.0000

**Table 5: Regression Estimation Section**

Parameter	Intercept B(0)	Slope B(1)
Regression Coefficients	-91.9048	1.8810
Lower 95% Confidence Limit	-206.9666	1.4231
Upper 95% Confidence Limit	-37.3398	2.3389
Standard Error	22.2995	0.1871
Standardized Coefficient	0.0000	0.9716
T-Value	-4.1214	10.0514
Prob Level (T-Test)	0.0062	0.0001
Reject H0 (Alpha = 0.0500)	Yes	Yes
Power (Alpha = 0.0500)	0.9263	1.0000
Regression of Y on X	-91.9048	1.8810
Inverse Regression from X on Y	-104.1924	1.9927
Orthogonal Regression of Y and X	-101.5799	1.9689

**Table 6: Analysis of Variance Section**

Source	DF	Sum of Squares	Mean Squares	F-Ratio	Prob Level	Power(5%)
Intercept	1	105800	105800			
Slope	1	59438.09	59438.09	101.0306	0.0000	1.0000
Error	6	3529.905	588.3174			
Adj. Total	7	62968	8995.429			
Total	8	168768				
S = Square Root(588.3174) = 24.25526						

**2. Empirical Expression for Safety Factor,  $\dot{Y}$  on Temperature and Fuel Size Effect**

Investigating the effect of fuel size on the Stability and Safety of the nuclear reactor during operation. The data obtained in Table 1 which represents a typical parameter for Water-Cooled Reactor Design (WCRD) was modified in order to obtain the best fit for the model. The new conceptual design reactor model optimizes the performance of the Fukushima Daiichi damaged reactor 1-4 in Japan, Chernobyl Nuclear Power Reactor no. 4 disaster accident melt-down in Russia and (TM1-2) damaged reactor in Three Mile Island Unit 2 near Pennsylvania in USA.

**(i) Empirical Expression for Safety Factor,  $\dot{Y}$**

The model empirical expression is the equation of the straight line relating heat in the reactor and the volume of fuel in the reactor as a measure of safety factor estimated as:

$$\dot{Y} = (139.3887) + (110.9289)*(X_j) + e_j \tag{7}$$

- Estimated model or predicted where,

$\dot{Y}$  = heat, 139.3887 is the intercept, 110.9289 is a slope, X = the value of input parameter ( is the mass of fuel), e = error or residual and j = 1,2,3,...,12

The Figure 2 shows the linear regression plot section of volume of fuel effect on the reactor.

**(ii) Linear Regression Plot Section**

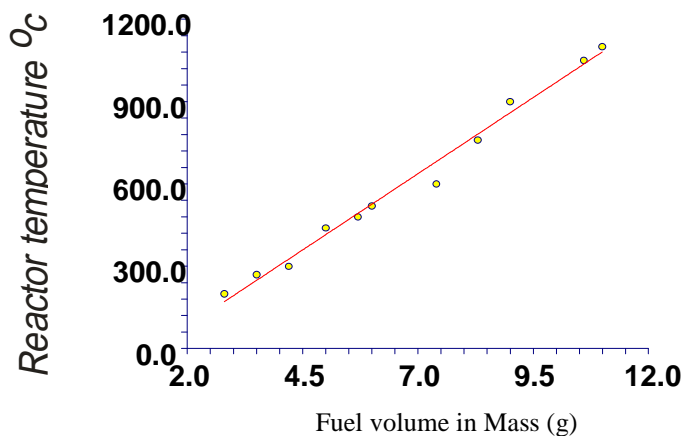


Figure 2: Temperature and Fuel Size effect on the stability of operating reactor

(iii) F-test Result

Table 7: Summary of F-test Statistical Data

Parameter	Value
Dependent Variable	Heat (°C)
Independent Variable	fuel
Intercept(B <sub>0</sub> )	139.3887
Slope(B <sub>1</sub> )	110.9289
R-Squared	0.9875
Correlation	0.9938
Mean Square Error (MSE)	1.347137 x 10 <sup>-3</sup>
Coefficient of Variation	0.0610
Square Root of MSE	36.70336

Table 8: Descriptive Statistics Section

Parameter	Dependent	Independent
Variable	Heat (°c)	Fuel (g)
Count	12	12
Mean	601.8182	6.6818
Standard Deviation	311.9878	2.7949
Minimum	200.0000	2.8000
Maximum	1100.0000	11.0000

Table 9: Regression Estimation Section

Parameter	Intercept B(0)	Slope B(1)
Regression Coefficients	-139.3887	110.9289
Lower 95% Confidence Limit	-206.9666	101.5348
Upper 95% Confidence Limit	-71.8107	120.3231
Standard Error	29.8732	4.1527
Standardized Coefficient	0.0000	0.9938
T-Value	-4.6660	26.7122
Prob Level (T-Test)	0.0012	0.0000
Reject H0 (Alpha = 0.0500)	Yes	0.0000
Power (Alpha = 0.0500)	0.9849	1.0000
Regression of Y on X	-139.3887	110.9289
Inverse Regression from X on Y	-148.7376	112.3281
Orthogonal Regression of Y and X	-148.7369	112.3280

**Notes:**

The above report shows the least-squares estimates of the intercept and slope followed by the corresponding standard errors, confidence intervals, and hypothesis tests. These results are based on several assumptions that are validated before they are used.

Table 10: Analysis of Variance Section

Source	DF	Sum of Squares	Mean Squares	F-Ratio	Prob Level	Power(5%)
Intercept	1	3984036	3984036			
Slope	1	961239.4	961239.4	713.5426	0.0000	1.0000
Error	9	12124.23	1347.137			
Adj. Total	10	973363.6	97336.37			
Total	12	4957400				
S = Square Root(1347.137) = 36.703						

Table 11: Tests of Assumptions Section

Assumption/Test Residuals follow Normal Distribution?	Test Value	Prob Level	Is the Assumption Reasonable at the 20% or 0.2000 Level of Significance?
Shapiro Wilk	0.8921	0.147784	No
Anderson Darling	0.4847	0.227540	Yes
D'Agostino Skewness	-1.9311	0.053468	No
D'Agostino Kurtosis	1.4457	0.148249	No
D'Agostino Omnibus	5.8194	0.054492	No
<b>Constant Residual Variance?</b>			
Modified Levene Test	0.6445	0.442780	Yes
<b>Relationship is a Straight Line?</b>			
Lack of Linear Fit F(0, 0) Test	0.0000	0.000000	No

**Notes:**

A 'Yes' means there is not enough evidence to make this assumption seem unreasonable.

A 'No' means that the assumption is not reasonable

In Table 11 Anderson Darling method confirms the rejection of  $H_0$  at 20% level of significance but all of the above methods agreed that  $H_0$  Should not be rejected at 5% level of significance. Hence the normality assumption is satisfied as one of the assumptions of the Linear Regression Analysis is that the variance of the error variable  $\delta^2$  has to be constant.

**(iv) Residual Plots Section**

The plot section is used as further check on the validity of the model to satisfy all the assumptions of the linear regression analysis.

Amir ([21], P. 528) has stated that the normality assumption can be checked by the use of plot of errors against the predicted values of the dependent variable against each of the independent variable and against time (the order of selection of the data points) and on a probability scale. The diagnostic plot for linear regression analysis is a scatter plot of the prediction errors or residuals against predicted values and is used to decide whether there is any problem in the data at hand (Siegel F [20], p. 578).

The Figure 2 is for the plot of errors against the order to selection of the data points ( $e = 1, 2, \dots, 12$ ). Although the order of selection was not used as a variable in the mode, the plot reveal whether order of selection of the data points should have been included as one of the variables in our regression model. This plot shows no particular pattern in the error as the period increases or decreases and the residuals appear to be randomly distributed about their mean zero, indicating independence. The residuals are randomly distributed with no pattern and with equal variance as volume of fuel increases.

**Note:**

1. Residual = original value for heat (Y) minors predicted value for heat,  $\hat{Y}$
2. Count = the design number (design 1, 2, 3, ..., 12 )

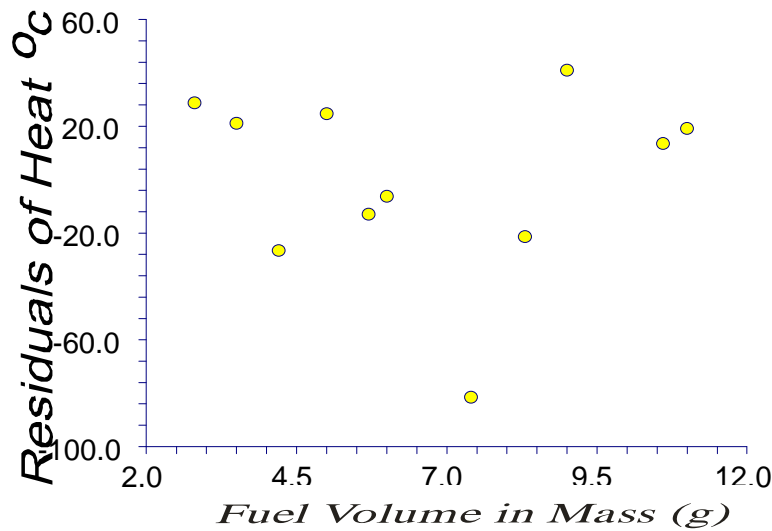


Figure 3: Residuals of Heat ( $^{\circ}\text{C}$ ) versus Fuel (g)

Figure 4 shows the histogram of residuals of error ( $e_t$ ) and this is nearly skewed to the right but the software used indicated that the plot is normal.

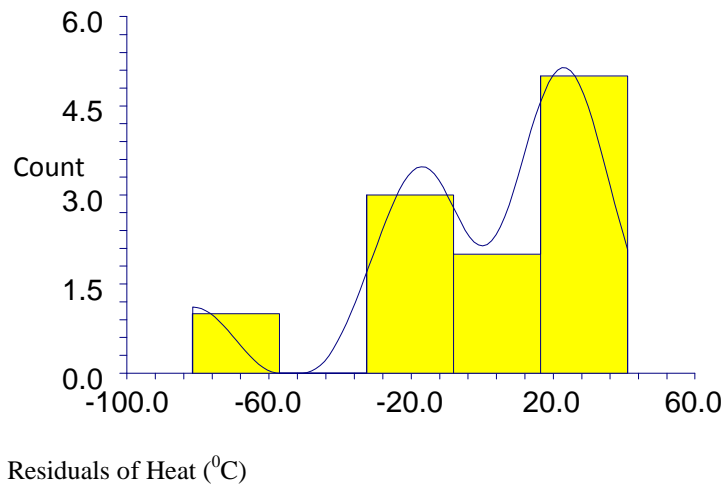


Figure 4: Histogram of Residuals of Heat ( $^{\circ}\text{C}$ )

The Figure 5 is the plot graph of experimental errors. The residuals are perfectly normally distributed as most of the error terms align themselves along the diagonal straight line with some error terms outside the arc above and below the diagonal line. This further indicates that the estimated model is valid.

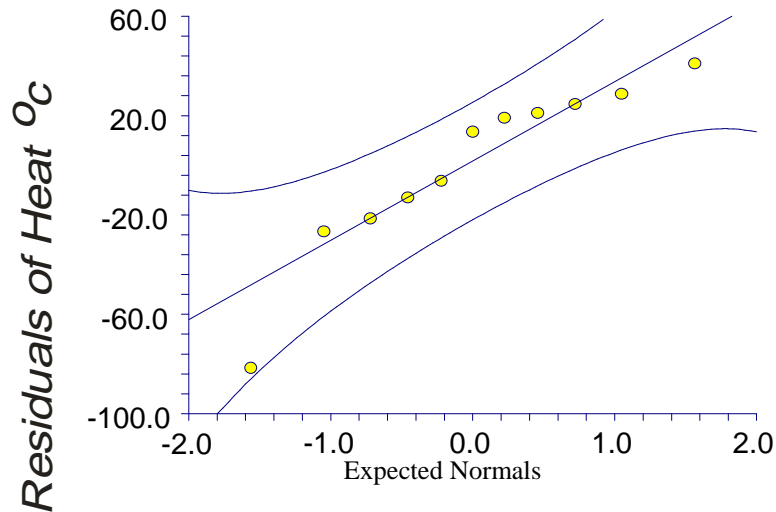


Figure 5: Normal Probability Plot of Residuals of Heat ( $^{\circ}\text{C}$ )

### 3. Application of Loss-of-Coolant-Accident Cases in Water-Cooled Reactor

The Loss-of-Coolant-Accident could be likened to a situation where the operating reactor gradually loses coolant and stops functioning, we could take example from the Three Mile Island nuclear power plant in USA as reported that “a cooling malfunction caused part of the reactor core to melt in the # 2 reactor, the reactor was destroyed”. As identified in the case of Fukushima Daiichi Nuclear Accident March 2011 that “the fuel became critical as it could not cool down”. Furthermore, Reuters reported that Fukushima Daiichi Unit 3 has lost cooling capability and may be experiencing melting of the core, eventually reactor 1- 4 was written-off. These situations can be applied to Figure 6. During operation the reactor is stable as water coolant flow rises from 200kg/sec to 600kg/sec, and operates steadily between the safety factor of 6 to 11 and maintains cooling at 600kg/sec, but later falls to 500kg/sec and suddenly drops from 100kg/sec to near zero level, at this point the reactor becomes unstable, safety is no longer guaranteed, as the reactor can start melting since cooling of the reactor is no longer taken place, though other parameters may be held constant e.g. power, control rod, etc.

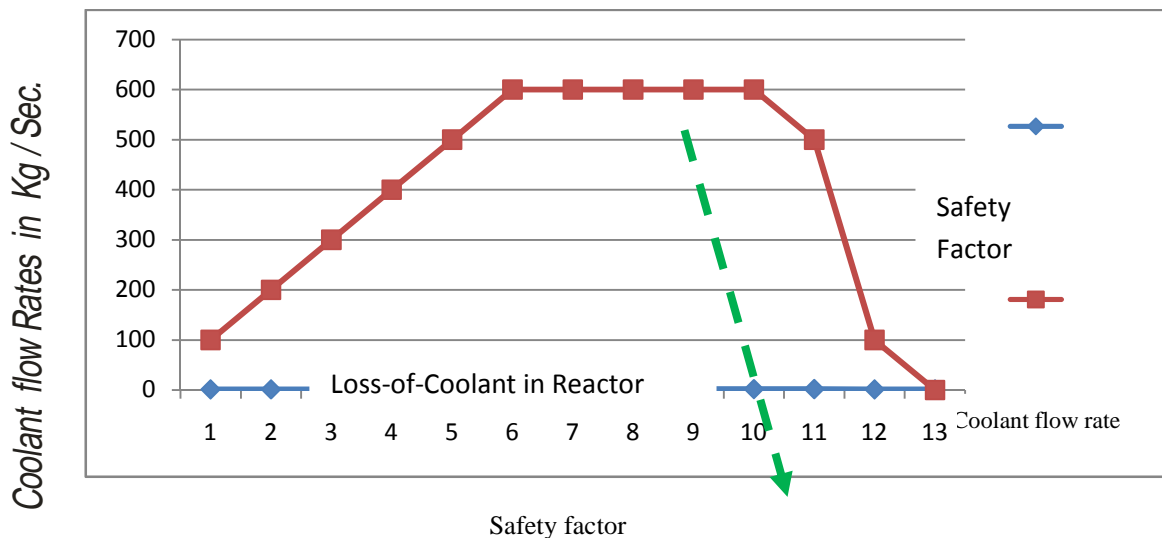


Figure 6: Demonstration of Loss-of-Coolant in Water-Cooled Reactor

**5.0 Summary/Conclusion**

The equation of the straight line relating heat in the reactor and the volume of fuel in the reactor is estimated as:

$$\hat{Y} = (139.388674238897) + (110.928917233032) * (X_j) + e_j$$

using the 11 observations in this dataset.

to 4 significant figure it becomes

$$\hat{Y} = (139.3887) + (110.9289) * (X_j) + e_j \quad \text{- this is the desire estimated model or predicted}$$

- (i) This is the model equation that could be applied to make predictions of the safety factor on these types of reactor design models as relating heat in the reactor and the volume of fuel in the reactor.
- (ii) The empirical expressions may also be used for the calculation of heat ( $^{\circ}\text{C}$ ),  $\hat{Y}$  in the reactors which in turn is a measure of the reactor's stability.
- (iii) Also, the empirical formula derived can be used to determine the contribution of heat or temperature ( $^{\circ}\text{C}$ ) to the stability of the reactor during operation or accident.
- (iv) The estimated value of  $\hat{Y}$  when  $X_j$  is zero is 139.3887 with a standard error of 29.8732.
- (v) The slope represent the estimated change in heat ( $\hat{Y}$ ) per unit change in fuel ( $X_j$ ), is given as 110.9289 with a standard error of 4.1527.
- (vi) The value of coefficient of determination(R-Squared ( $R^2$ )) explains the proportion of the variation in heat that can be accounted for by variation in fuel as 0.9875.
- (vii) The correlation between heat ( $\hat{Y}$ ) and fuel ( $X_j$ ) is 0.9938.
- (viii) A significance test that the slope is zero resulted in a t-value of 26.7122. The significance level of this t-test is 0.0000. Since  $0.0000 < 0.0500$ , the hypothesis that the slope is zero is rejected.
- (ix) The lower limit of the 95% confidence interval for the slope is 101.5348 and the upper limit is 120.3231. The estimated intercept is 139.3887. The lower limit of the 95% confidence interval for the intercept is -206.9666 and the upper limit is -71.8107.

In conclusion linear regression analysis is applied on typical Water-Cooled Reactor Design (WCRD) models to test for safety margin between decay heat and volume of the fuel in the reactor. Empirical expressions are obtained for the WCRD models. The results of the statistical analyses on these types of nuclear reactor models reveal that the WCRD models promises stable under application of small size of uranium (fuel) at 9g and below than large size of uranium (fuel) at 12g and above WCRD models. Meanwhile, at 9g of fuel element the reactor seems to be most stable and safer as the regression plot was optimized. Therefore, the least squares method finds its optimum when the sum,  $S$ , of squared residuals

$$S = \sum_{i=1}^n r_i^2$$

is a minimum at the given mass (9g) of fuel element.

In this method of regression analysis the safety margin prediction of up to 1.25% has been validated for reactor design models on water-cooled reactor as an advantage over the current 5.1% challenging problem for plant engineers to predict the safety margin limit. According to Xianxun Yuan ([13], P. 49) in "Stochastic Modeling of Deterioration in Nuclear Power Plants Components" a challenging problem of plant engineers is to predict the end of life of a system safety margin up to 5.1% validation. However, the current design limits for various reactors Safety in a nuclear power plant, defined by the relative increase and decrease in the parametric range at a chosen operating point from its original value, varies from station to station.

Finally, the discoveries on water-cooled reactor safety factor should provide a new method for reactor design concept taken cognizant of the fuel size effect and pressure built-up trouble in the reactor core. This shall also provide a good, novel approach and method for multi-objective decision-making based on six dissimilar objectives attributes: evolving technology, effectiveness, efficiency, cost, safety and failure.

It is therefore suggested that for countries wishing to include nuclear energy for the generation of electricity, like Nigeria, the parameters of the selected nuclear reactor should undergo analysis via RAT for optimization and choice.

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