Journal of the Nigerian Association of Mathematical Physics
Volume 25, No. 2 **(November, 2013), pp 199 – 202**

Physical Model of the Lumbar Spine

[1]*Mafuyai M. Y.*, [2]*Babangida B. G.*, [3]*Mador E. S. and* [4]*Jabil Y. Y.*

[1,4]Department of Physics,
University of Jos, PMB 2084 Jos.
[2]Department of Mathematics,
Kaduna State College of Education, Gidan Waya.
[3]Department of Anatomy, University of Jos.

Abstract

With the increasing incidences of the lower back pain and the attendant surgical consequences, various attempts have been made by researchers around the world to model the lumbar spine to enable detail studies of the surgical effects on the anatomy of the spine. Most of these models, developed from one or a combination of the four major approaches existing, suffer some common challenges which include variation in sizes of subjects, weight of subjects , posture of the lumbar spine, cost of obtaining modelling data and complexity of the muscles of the lumbar spine. This research is an attempt to present a different approach and model that hopes to eliminate most of the challenges hitherto.

Keywords: Lumbar spine, Flexion, Dorso, Action function and Parameter

1.0 Introduction

In human anatomy, the lumbar vertebrae are the five vertebrae between the rib cage and the pelvis. They are the largest segments of the vertebral column and are characterized by the absence of the foramen transversarium within the transverse process, and by the absence of facets on the sides of the body. They are designated L1 to L5, starting at the top. The lumbar vertebrae help support the weight of the body, and permit movement [1]. This area is commonly called the "lower back". The lumbar vertebrae are the largest of the vertebrae because of their weight-bearing function in supporting the torso and head [2].

Even though researchers have a considerable knowledge of the structural properties of the vertebrae, inter-vertebral joints and disc, and the mechanical limits of these structures [3], the problem still remains on ways of measuring how the forces, strains and stresses experienced come to these limits [3]. Biomechanical modelling has been an alternative method meant to resolve the challenges. In most of the models developed, researchers have made use of one or more of the following methods of allocating forces to the numerous components that make up the anatomical model; electromyography (EMG), optimization, combined EMG-optimization technique and neutral networks.

The models developed are not without limitations some of which are: inability to explain; the cause of increasing lower back pain around the world and mostly the developed nations of the world [4,5,6,7,8], increase in stress with back ward bending and decrease in stress with forward bending, high cost of generating data especially when using EMG method [3] of allocating forces etc.

2.0 Method

The physical model employed the method of calculus of variation and only a one-plane dynamics of the lumbar spine is considered.

Calculus of variation; it is method which depends on minimization or maximization of some certain quantities of interest such as maximum area under a curve, minimum line joining two points on a curve, potential minimization. It provides a powerful means of finding a function $Y(x)$ in which the appropriate quantity of interest to be maximised or minimised by an appropriate choice of the function $Y(x)$ may be expressed as an integral involving $Y(x)$ and the variables describing the geometry of the situation [9].

Corresponding author: ***Mafuyai M. Y.,*** E-mail: Conceptmaster1@yahoo.com, Tel.: +2347062942299

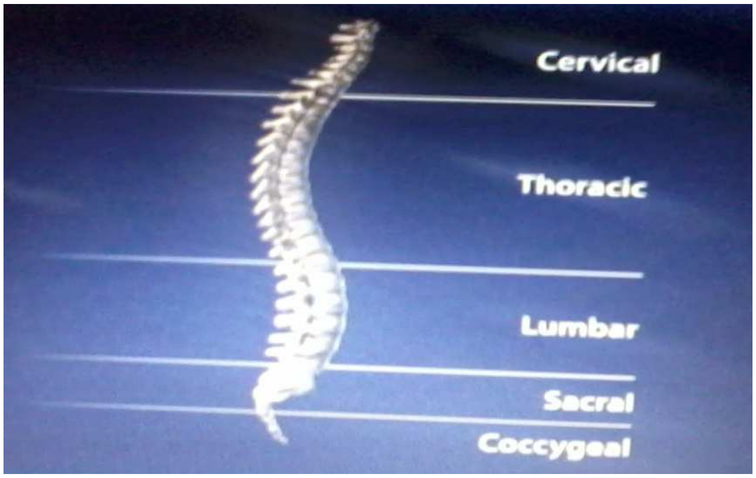

Figure 1.1: THE STRUCTURE OF THE SPINE SHOWING THE VARIOUS SECTIONS.

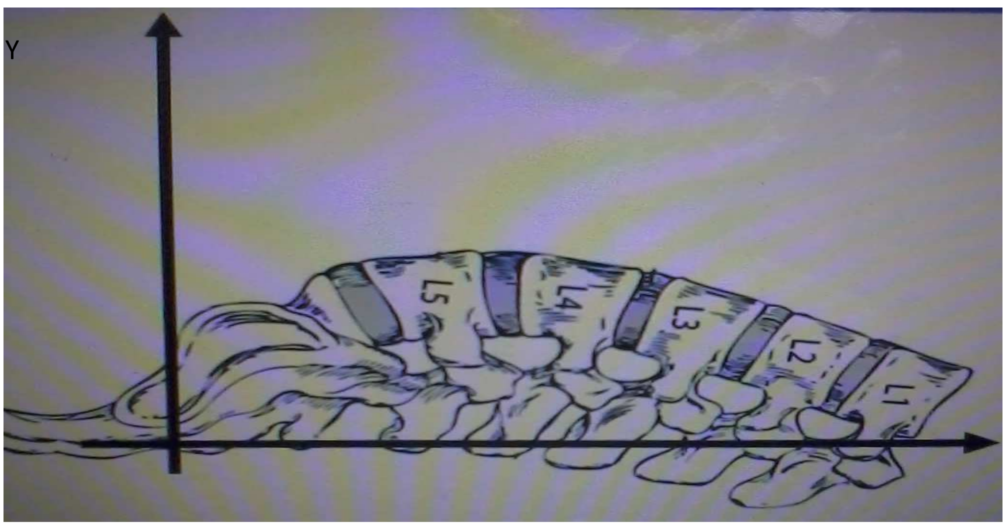

Figure 2.1: SCHEMATIC OF THE LUMBAR SPINE ORIENTED ON AN X-Y PLANE

2.1 Geometry of the lumbar spine; the geometry of the lumbar spine utilises the normal Cartesian coordinate system.

Definition 2.1; we define lumbar spine to include the last thoracic vertebra (T_{12}), the usual five lumbar vertebra (L_1-L_5) and the sacral vertebra. This is necessary that symmetry on the x axis can be assumed.
X
For mathematical convenience the above figure is represented as below;

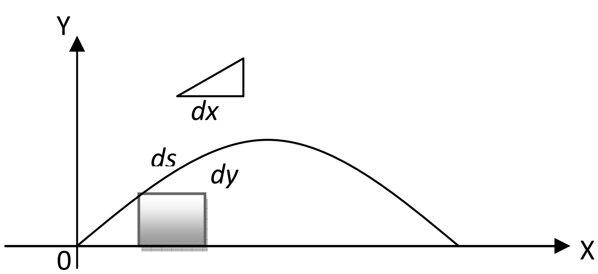

Figure 2.2: A SCHEMATIC REPRESENTATION OF THE LUMBAR

We are interested in finding an equation of the curve in Figure 2.2. To do this, we employ the method of calculus of variation i.e. we are to find the equation for the curve that encloses maximum area.

Assumptions; the following assumptions are, without loss of generality, utilised:

1. that the curve is symmetric on x axis
2. that the curve passes through the origin 0.
3. that the length of the curve is l

Now, using the distance s along the curve, measured from the origin, as the independent variable and y as the dependent one, we have the boundary conditions y(0) = y(l) = 0. The element of area shown in figure 2.2 is then given by

$$dA = ydx = y((ds)^2 - (dy)^2)^{\frac{1}{2}} \quad (2.01)$$

Therefore, total area enclosed is given by

$$A = \int_0^l ydx = \int_0^l y(1 - y'^2)^{\frac{1}{2}} ds \quad (2.02)$$

Here y' stands for $\frac{dy}{ds}$ rather than $\frac{dy}{dx}$. Since the integrand does not contain s explicitly, we can use (2.01) to obtain a first integral of the Euler's-Langrage's (EL) equation [9] for y, as

$$y(1 - y'^2)^{\frac{1}{2}} + yy'^2(1 - y'^2)^{-\frac{1}{2}} = k \quad (2.03)$$

Where k is a constant. On rearranging, we have

$$ky' = \pm(k^2 - y^2)^{\frac{1}{2}} \quad (2.04)$$

Which, using y (0) = 0, integrates to

$$y/_k = \sin\left(s/_k\right) \quad (2.05)$$

The other end-point, y(l) = 0, fixes the value of k as $l/_\pi$ to yield

$$y = \frac{l}{\pi}\sin\left(\frac{\pi s}{l}\right) \quad (2.06)$$

Also, it can be shown using calculus of variation that the shortest distance joining any two point is a straight line [9]. Hence s can be written in terms of x as

$$s = \mu x \quad (2.07)$$

Where μ is a dimensionless constant of proportionality. Equation (2.06) becomes

$$y = \frac{l}{\pi}\sin\left(\frac{\pi\mu x}{l}\right) \quad (2.08)$$

Equation (2.08) is the neutral posture equation of lumbar spine.

In order that equation (2.08) presents the behaviour of lumbar spine proper, we define the following two functions α and β called action functions which are responses of action that brings about the curvature change of the lumbar spine and have the following properties:

1. The norms of the function called action parameter are the same i.e. $|\alpha| = |\beta|$
2. The direction of α alternates at $|\alpha| = \theta$ and $|\alpha| = -(\frac{2l}{\pi} + \theta)$
3. The direction of β alternates at $|\alpha| = \theta$ and $|\alpha| = \frac{l}{\pi}$
4. $|\alpha|$ has the dimension of length per radian and $|\beta|$ has dimension of angle only

Where θ and $\frac{2l}{\pi} + \theta$ measured in radian are the maximum bending angles of dorso and flexion [10,11] actions respectively.

Equation (2.08) becomes

$$Y = \left(\frac{l}{\pi} \mp |\alpha|\right)\sin\frac{\pi\mu}{l}(x \pm |\beta|) \quad (2.09)$$

Equation (2.09) is the equation of dynamics of the lumbar spine.

3.0 Result and Discussion

Equation (2.09) is the dynamic equation of the lumbar spine when $|\beta|$ is zero the equation reduces to equation (2.08) which is the equation of neutral posture of the lumbar spine and produce a shape such as shown in the Figure 2.3 while at maximum dorso posture (backward bending) and flexion posture (forward bending) the various shapes are as represented in the Figure 2.3.

It should be noted that the effect of action parameter in dorso action (backward bending) reduces the length x of the cord formed by the curve of the lumbar spine while it increases the amplitude of the curve of the spine. In flexion action (forward bending), while the cord increases the amplitude decreases until it becomes zero and the cord begin to reduce but amplitude increases in the negative direction. And this is in good agreement with observation.

One unique advantage of the equation is that it directly depend on the length l of lumbar spine and length x of the cord formed by the curve of the lumbar spine which can easily be generated, and at low cost, from an X-ray image of the lumbar spine and many other information such as displacement of centre of mass with variation of shape, effect of variation of shape on the inter-vertebral disc, etc can be generated and analyzed

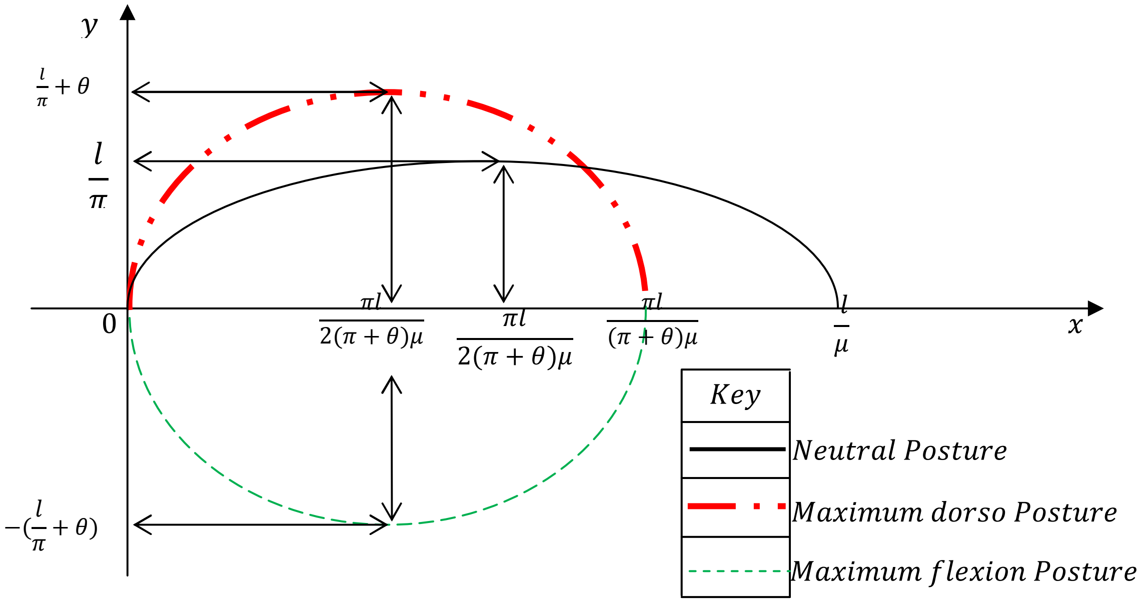

Figure 2.3: Sketch Of Lumbar Spine Under The Influences Of The Action Functions

At neutral posture of the lumbar spine, the action parameter is zero (0) i.e. no action has been generated at the lumbar spine and at maximum dorso, $|\alpha|\ is\ \theta$ while at maximum flexion $|\alpha|\ is - (\frac{2l}{\pi} + \theta)$

References

[1] https://en.wikipedia.org/wiki/Lumbar_vertebrae#cite_note-4. Accessed July 24, 2013

[2] Lumbar-spine-lower-back-structure- Function;http://healthpages.org/anatomy-function/lumbar-spine-lower-back-structure-function. Accessed July 24, 2013

[3] Michelle L. Gatton (2000), Mathematical modeling of muscles recruitment and function in the lumbar spine, *PhD Thesis submitted to Center for Rehabilitation Science, School of Mechanical, Manufacturing and Medical Engineering, Queensland University of Technology*

[4] Jensen M, Brant-Zawadzki M, Obuchowski N, et al. Magnetic Resonance Imaging of the Lumbar Spine in People Without Back Pain. N Engl J Med 1994; 331: 69-116.

[5] Back pain national health priority Area in Australia; https://www.mja.com.au/journal/2009/190/7/back-pain-national-health-priority-area-australia Accessed July24, 2013.

[6] Back pain national health priority Area in Australia; https://www.mja.com.au/journal/2009/190/8/back-pain-national-health-priority-area-australia Accessed July24, 2013

[7] Hoy DG, Bain C, Williams G, March L, Brooks P, Blyth F, Woolf A, Vos T, Buchbinder R. “A systematic review of the global prevalence of low back pain.” Arthritis Rheum. 2012 Jan 9. Accessed from; http://www.ncbi.nlm.nih.gov/pubmed/22231424

[8] Global burden of diseases; http://www.thelancet.com/themed/global-burden-of-disease Accessed July24, 2013

[9] K.F. Riley, M.P. Hobson and S. J. Bence (2006); Mathematical Methods for Physics and Engineering,Third Edition, *Published in the United States of America by Cambridge University Press, New York* , ISBN-13 978-0-511-16842-0. P 775

[10] Lone Hansen, Mark de Zee, John Rasmussen, *et al* (2006), *Anatomy and Biomechanics of the Back Muscles in the Lumbar Spine With Reference to Biomechanical Modeling* .Retrieved from http://www.medscape.com/viewarticle/542466 Accessed July 28, 2013

[11] Bogduk N, Macintosh JE, Pearcy MJ. *A universal model of the lumbar back muscles in the upright position*. Spine 1992;17:897–913. Retrieved from; http://www.ncbi.nlm.nih.gov/pubmed/1523493 Accessed July 28, 2013