

## Theoretical Investigation of Breakdown Voltage of Air at Low Pressure.

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### Abstract

*This paper deals with the Paschen's law in electrical breakdown of air at low pressures. Using the Paschen's law, numerical values of the breakdown voltages were deduced at different inter-electrode spacing of 2.5cm and 5.0cm. The values generated were compared to the measured values from the literature and are observed to agree well. With these spacing, it confirms that the theoretical form of the Paschen's law is a function of the product of pressure and electrode spacing and spacing i.e  $V_{br} = f(pd, d)$ . The plots of the breakdown voltages versus electrode-spacing and product of pressure and electrode spacing are presented. Their minimums also agreed well with the experimental results.*

**Keywords:** Dielectric Barrier Discharge (DBD), Townsend's ionization coefficients, breakdown voltage, Paschen's law

### 1.0 Introduction

One method of how plasma is produced is when a gas, liquid or solid dielectric is placed between electrodes of a capacitor breakdown as a result of high voltage application (in order of KV or MV). The gas, liquid or solid turns to plasma as a result of ionization of atoms of electron impact, charged particles moving in an electric field, charge multiplication in electron avalanches and secondary electron production at the cathode by ion impact [1]. In Solid dielectric breakdown, when the electrodes are arranged symmetrically so that one electrode is exposed to the surrounding air and the other is totally encapsulated in a solid dielectric material (eg. Glass, quartz etc) and an AC/DC voltage above the minimum breakdown is applied, a discharge (plasma) appears on the dielectric (or insulator) surface above the encapsulated electrode. The plasma formed has numerous applications such as the treatment of polymers [2, 3], Plasma coating [4], thin –film deposition

[5, 6], the cleaning and activation of substrate [7, 8], driving high-power Co<sub>2</sub> lasers [2] and excimer ultraviolet lamps [9], sterilization of surfaces of materials [10], pollution control [11], Production of large-area flat plasma display panels [12], Flow control to aircrafts to derive the body force effect [13], and purification of water [14]. Future applications may include their use in the greenhouse gas control technologies [15].

Townsend [16] was the first fellow to study the variation of current between parallel electrodes as a function of the applied electric field with air/gas between the electrodes.

### Theory and Method of the Study

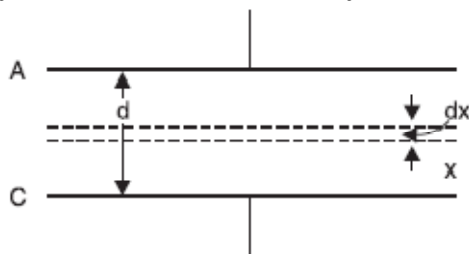


Fig. 1: Parallel plate capacitor

A pair of parallel electrodes with A being the anode and C the cathode having gas as an insulating medium and separated by a distance  $d$  as shown in Fig. 1.

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Let  $n_0$  = number of electrons leaving the cathode C.

$x$  = distance which these electrons have moved from the cathode C.

$n$  = is the present number of electrons at  $x$  (that is  $n_0$  is now  $n$  at  $x$ ).

Now, when  $n$  electrons move a distance  $dx$  further, additional  $dn$  electrons have been provided due to collisions.

Townsend [16] introduced a coefficient  $\alpha$  known as Townsend's first ionization coefficient which is defined as the number of electrons produced by an electron per unit length path in the direction of the field.

Therefore  $dn = \alpha n dx$

$$\frac{dn}{n} = \alpha dx$$

Integrating

$$\ln n = \alpha x + A \quad \text{At } x = 0, n = n_0$$

$$\ln n_0 = 0 + A$$

$$\therefore A = \ln n_0$$

$$\therefore \ln n = \alpha x + \ln n_0$$

$$\therefore \ln n - \ln n_0 = \alpha x$$

$$\ln \frac{n}{n_0} = \alpha x$$

$$\frac{n}{n_0} = e^{\alpha x}$$

$$n = n_0 e^{\alpha x} \quad \text{at } x = d$$

$$n = n_0 e^{\alpha d}$$

But flow of electrons is current flow, therefore in terms of current

$$I = I_0 e^{\alpha d} \tag{1}$$

The term  $e^{\alpha d}$  in (1) is called electron avalanche and is the number of electrons produced by one electron in travelling from cathode to anode. The first ionization coefficient  $\alpha$  divided by the pressure,  $\alpha/P$ , is function of the reduced electric field strength,  $E/P$  [16]. Furthermore, it depends on the type of gas. Generally, the first ionization coefficient  $\alpha$  can be described by the empirical formula [17] as:

$$\frac{\alpha}{p} = A \exp\left(-\frac{Bp}{E}\right) \tag{2}$$

Taking the log of equation (1) gives

$$\ln I = \alpha x + \ln I_0 \tag{3}$$

Equation (3) is a straight line with slope  $\alpha$  and intercept  $\ln I_0$  shown in Fig. 2.

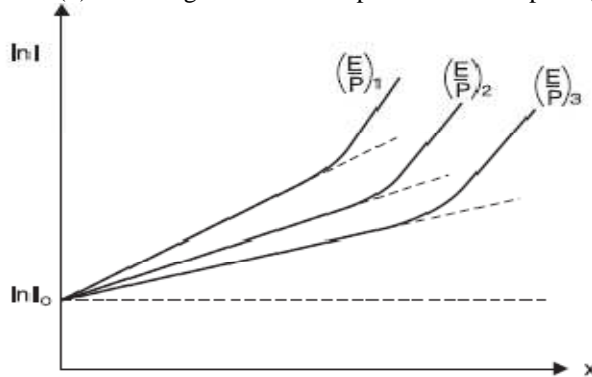


Fig.2. Variation of gap current with electrode spacing in uniform E.

Townsend [16] initially observed that the current in parallel plate gap increased more rapidly with increased in voltage as compared to the one given in (3). This departure from linearity made Townsend suggested that a second mechanism is responsible for the rapid rise in the current and postulated that the sudden rise in the current must be due to the positive ions and photons. The positive ions liberate electrons by collision with the gas molecules and by bombardment against the cathode. Similarly, the photons will liberate electrons after collision with gas molecules and on impact with the cathode.

From his postulation of the second ionization coefficient,

If  $n_+$  = number of electrons released by photon impact with gas molecules and cathode.

$n_0$  = Number of electrons from cathode due to positive ion bombardment

$n$  = Number of electrons reaching the anode.

$\gamma$  = Townsend's second ionization coefficient defined as the number of electrons released from cathode per incident positive ion.

Equation (1) becomes

$$n = (n_0 + n_+)e^{\alpha d} \tag{4}$$

$n_0 + n_+$  = Total number of electrons released from the cathode.

$n$  = Number of electrons reaching the cathode.

$\therefore [n - (n_0 + n_+)]$  = number of electrons released from the gas and corresponding to each electron released from the gas, there will be one positive ion releases  $\gamma$  effective electrons from the cathode, then

$$n_+ = \gamma [n - (n_0 + n_+)]$$

Solving for  $n_+$

$$n_+ = \frac{\gamma(n + n_0)}{1 + \gamma} \tag{5}$$

Putting (5) into (4)

$$n = [n_0 + \frac{\gamma(n - n_+)}{1 + \gamma}]e^{\alpha d}$$

$$n = [\frac{n_0(1 + \gamma) + \gamma(n - n_+)}{1 + \gamma}]e^{\alpha d}$$

$$n = [\frac{n_0 + \gamma n}{1 + \gamma}]e^{\alpha d}$$

Solving for  $n$

$$n = \frac{n_0 e^{\alpha d}}{1 + \gamma(1 - e^{\alpha d})}$$

$$n = \frac{n_0 e^{\alpha d}}{1 - \gamma(e^{\alpha d} - 1)}$$

In terms of current

$$I = \frac{I_0 e^{\alpha d}}{1 - \gamma(e^{\alpha d} - 1)} \tag{6}$$

Applying breakdown criterion in (6) that is for infinite current we equate the denominator to zero:

$$1 - \gamma(e^{\alpha d} - 1) = 0$$

$$\gamma(e^{\alpha d} - 1) = 1 \tag{7}$$

$$e^{\alpha d} = \frac{1}{\gamma} + 1 \tag{8}$$

Taking the  $\ln$  of both sides of (8) leads to:

$$\alpha d = \ln\left(\frac{1}{\gamma} + 1\right) \tag{9}$$

Substituting for  $\alpha$  in (9) from (2) and  $E = V_{br}/d$  gives:

$$PdAe^{-\frac{BPd}{V_{br}}} = \ln\left(1 + \frac{1}{\gamma}\right) \tag{10}$$

$V_{br}$  is the breakdown voltage.

Equation (2) can be arranged to give:

$$d = \frac{\exp(Bpd/V_{br})}{Ap} \ln\left[\frac{1}{\gamma} + 1\right] \tag{11}$$

Taking the  $\ln$  of both sides of (10) gives:

$$\ln(PdA) - \frac{BPd}{V_{br}} = \ln\left[\ln\left(\frac{1}{\gamma} + 1\right)\right] \tag{12}$$

Solving for  $V_{br}$  gives:

$$V_{br} = \frac{BPd}{\ln(PdA) - \ln\left[\ln\left(\frac{1}{\gamma} + 1\right)\right]} \tag{13}$$

Let  $K = \ln\left(\frac{1}{\gamma} + 1\right)$  (14)

$$V_{br} = \frac{BPd}{\ln(PdA) - \ln[K]} \tag{15}$$

Equation (15) is known as Paschen's law for gases.

It is observed that  $V_{br}$  (that is breakdown voltage) is a function of the pressure-distance (gap) product. That is  $V_{br} = f(Pd)$ . For air,  $A = 14.6$ ,  $B = 365$  and  $\gamma = 0.036$  [18, 19].

Differentiating  $V_b$  w.r.t  $Pd$  and equating the derivative to zero to get  $V_{b\ min}$ .

$$\frac{dV_b}{d(Pd)} = \frac{\ln\left(\frac{APd}{K}\right) \cdot B - BPd \cdot \frac{K}{APd} \cdot \frac{A}{K}}{\left[\ln\left(\frac{APd}{K}\right)\right]^2} = 0$$

$$\frac{B \ln\left(\frac{APd}{K}\right)}{\left[\ln\left(\frac{APd}{K}\right)\right]^2} - \frac{B}{\left[\ln\left(\frac{APd}{K}\right)\right]^2} = 0$$

$$\ln\left(\frac{APd}{K}\right) = 1 \tag{16}$$

Taking exponential of both sides of equation (16) yields

$$\frac{APd}{K} = e$$

$$Pd = \frac{e}{A}K$$

Or  $(Pd)_{min} = \frac{e}{A}K = \frac{2.718}{A} \ln\left(\frac{1}{\gamma} + 1\right)$  (17)

And  $A = \frac{eK}{Pd}$  (18)

Putting (16) into (15), we find that

$$V_{br} = \frac{BPd}{1}$$

$$B = \frac{V_{br}}{Pd} \quad \text{But } E = \frac{V_{br}}{d}$$

Therefore,  $B = \frac{E}{p}$  (19)

The ratio of (19) to (18) gives

$$\frac{B}{A} = \frac{Ed}{eK}$$

But  $Ed = V_{br}$

$$\frac{B}{A} = \frac{V_{br}}{eK}$$

$$V_{b\ min} = eK \frac{B}{A}$$

But  $e = 2.718$  and  $K = \ln\left(\frac{1}{\gamma} + 1\right)$

$$V_{b\ min} = 2.718 \frac{B}{A} \ln\left(\frac{1}{\gamma} + 1\right) \tag{20}$$

### Theoretical Results

Equation (15), the objective of this paper, describes the breakdown voltage of the air. It is observed that the breakdown voltage is a function of the product of pressure and electrode spacing. Table 1 show the data generated using Microsoft Excel with electrode spacing of 5 cm. Table 2 shows the data generated when with electrode spacing of 2.5cm. Figures 3 and 4 show the plots of Table 1, and Figs. 5 and 6 show the plots of Table 2.

**Table 1: Calculated Values of Breakdown Voltages and Product of Pressure and Breakdown Voltage at given pressures with the Electrode Spacing  $d = 5\text{cm}$**

Pressure (Torr)	Breakdown Voltage(V)	P*d(Torr.cm)
0.0533431	659.4476	0.266715
0.06666	328.2278	0.333382
0.1333	228.755	0.666713
0.20009	248.4383	1.000045
0.26665	277.01	1.333377
0.33332	307.2391	1.666708
0.400008	337.6	2.00004
0.46664	367.6563	2.333372
0.53331	397.2776	2.666703
0.60007	426.4363	3.000035
0.66663	455.1426	3.333367
0.7334	483.4205	3.666698
0.80006	511.2978	4.00003
0.86662	538.8023	4.333362
0.93339	565.9606	4.666693
1.00005	592.7969	5.000025
1.066671	619.3331	5.333357
1.13338	645.5893	5.666688
1.20004	671.5833	6.00002
1.26667	697.3314	6.333352
1.33337	722.8483	6.666683
1.40003	748.1472	7.000015
1.46669	773.2401	7.333347
1.53336	798.1381	7.666678
1.60002	822.8512	8.00001
1.66668	847.3885	8.333342
1.733335	871.7585	8.666673
1.800001	895.969	9.000005
1.866667	920.027	9.333337
1.933334	943.9393	9.666668

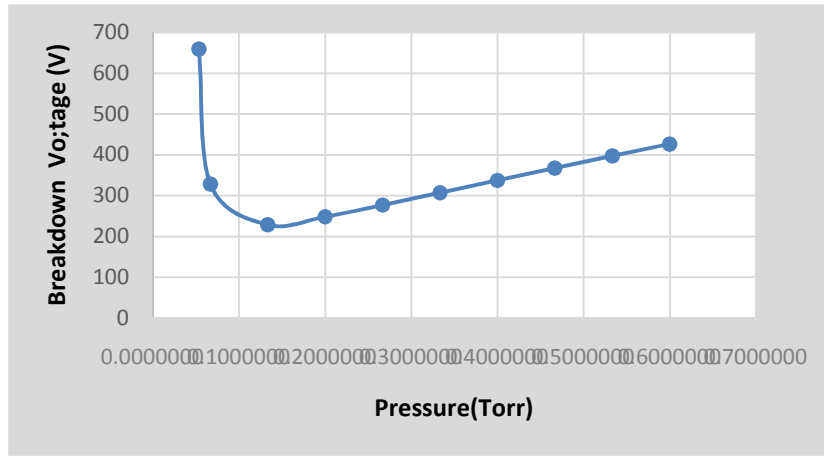


Fig 3: Breakdown Voltage Against Pressure At Electrode Spacing= 5cm

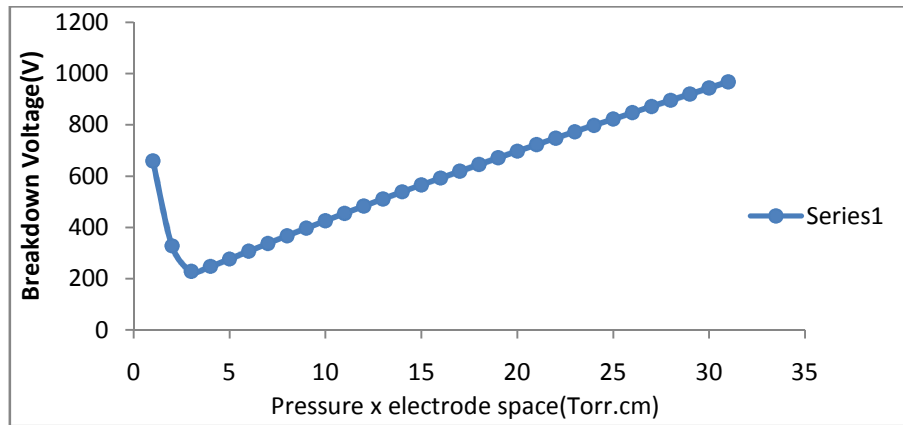


Fig 4: Breakdown Voltage Against Pressure x Electrode Spacing (d) = 5cm

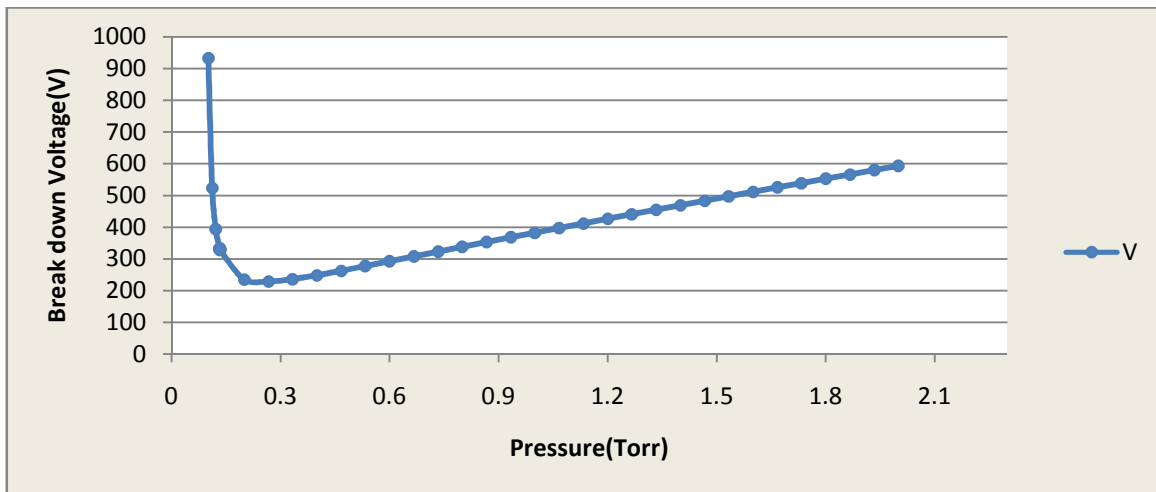


Fig 5: Breakdown Voltage against Pressure at Electrode Spacing = 2.5cm

Table 2: Calculated Values of Breakdown Voltages and Product of Pressure and Breakdown Voltage at given pressures with the Electrode Spacing  $d = 2.5\text{cm}$

Pressure (Torr.)	Breakdown Voltage(V)	P x d(Torr.cm)
0.132258	332.9358	0.330645
0.132366	332.457	0.330916
0.132475	331.9809	0.331187
0.132583	331.5076	0.331458
0.132692	331.0371	0.33173
0.1328	330.5692	0.332001
0.132909	330.104	0.332272
0.133017	329.6414	0.332543
0.133126	329.1815	0.332814
0.133234	328.7243	0.333086
0.133343	328.2696	0.333357
0.200009	235.1618	0.500023
0.266675	228.7545	0.666688
0.333342	236.3616	0.833354
0.400008	248.4364	1.00002
0.466674	262.3194	1.166686
0.533341	277.0077	1.333352
0.600007	292.054	1.500018
0.666673	307.2368	1.666683
0.73334	322.4394	1.833349
0.800006	337.5977	2.000015
0.866672	352.6758	2.166681
0.933339	367.6541	2.333347
1.000005	382.5222	2.500013
1.066671	397.2754	2.666678
1.133338	411.9125	2.833344
1.200004	426.4341	3.00001
1.26667	440.8425	3.166676
1.333337	455.1405	3.333342
1.400003	469.3313	3.500008
1.466669	483.4184	3.666673
1.533336	497.4053	3.833339
1.600002	511.2957	4.000005
1.666668	525.0929	4.166671
1.733335	538.8003	4.333337
1.800001	552.4212	4.500003
1.866667	565.9586	4.666668
1.933334	579.4156	4.833334
2	592.7949	5

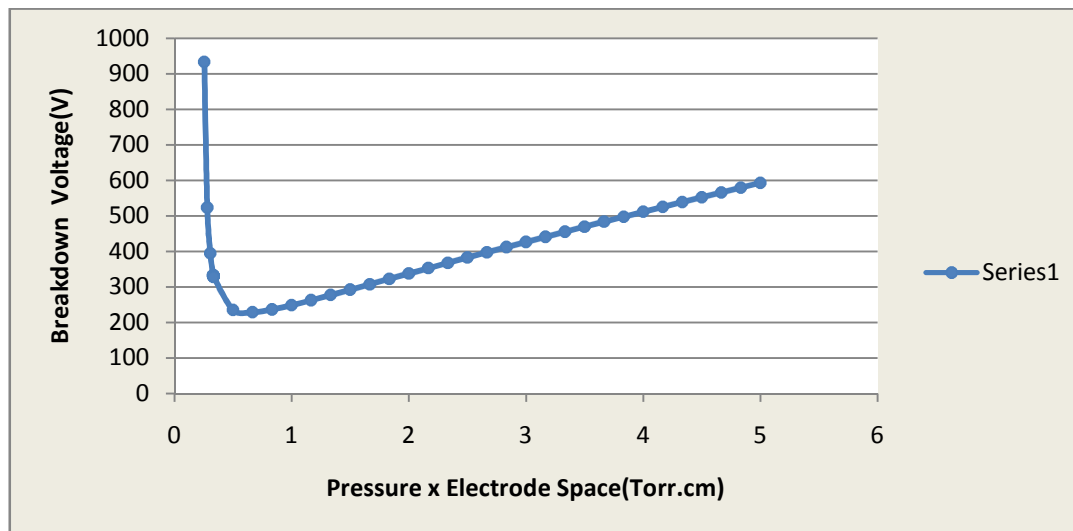


Fig 6: Breakdown Voltage Against Product of Pressure and Electrode Spacing ( $d = 2.5\text{cm}$ )  
*Journal of the Nigerian Association of Mathematical Physics Volume 25, No. 2 (November, 2013), 257 – 264*

## Discussion and Conclusion

We observed that the plots of the breakdown voltage  $V_b$  versus the product of the pressure  $P$  and electrode spacing  $d$  exhibit the same shape as shown in Figs. 3, 4, 5, and 6. On the other hand we note the minimum voltage  $V_{min}$  of the Paschen's curves for air when the electrode spacing is 5cm is 228.755V as shown in Figs. 3 and 4, while the product of the pressure and electrode spacing are 0.13334 Torr-cm and 3 Torr-cm respectively. In Figs. 5 and 6 in which the electrode spacing was reduced to 2.5 cm, the  $V_{min}$  of the Paschen's curves increased to 235.16V, while the product of the pressure and electrode spacing are respectively 0.200 and 0.500 Torr-cm. The values of  $V_{min}$  in the two electrode spacing agree well with the experimental values in the available literatures.

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