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Riemanns Geodesic Equations Of Motion For Particles Of Non Zero Rest Mass In Gravitational Fields In Rotational Spherical Polar Coordinates

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Abstract

In this paper we formulate the Riemann's Geodesic equations of motion for particle of non zero rest mass in Rotational Spherical Polar Coordinates.

Key Words: Riemann's Geodesic Equations of motion, Particles of non zero rest mass, Gravitational fields, Rotational Spherical Polar Coordinates

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Theory

The Cartesian coordinates are related to the Spherical polar coordinates as in equations (1),(2),(3):

$$x = uw(1-v^2)^{\frac{1}{2}} \tag{1}$$

$$y = u(1-v^2)^{\frac{1}{2}}(1-w^2)^{\frac{1}{2}} \tag{2}$$

$$z = uv \tag{3}$$

From the well known Transformation equation given by the Covariant $tensor$[1]

$$\overline{g_{\mu\nu}} = \frac{\partial x^q}{\partial \bar{x}^\mu}\ \frac{\partial x^s}{\partial \bar{x}^\nu} g_{qs} \tag{4}$$

Subsequently upon transformation of (1), (2), (3) using (4) we obtain the metric tensor for all gravitational fields in the rotational spherical coordinates as:

$$g_{11} = \left[1 + \frac{\varepsilon}{c^2} f\right]^{-1} \tag{5}$$

$$g_{22} = \left[\frac{u^2}{1-v^2}\right] \tag{6}$$

$$g_{33} = \left[\frac{u^2[1-v^2]}{1-w^2}\right] \tag{7}$$

$$g_{00} = \left[1 + \frac{\varepsilon}{c^2} f\right] \tag{8}$$

It is easy to see that the contra variant metric tensor for the gravitational field obtained by using the Quotient Theorem of tensor analysis becomes:

$$g^{11} = \left[1 + \frac{\varepsilon}{c^2} f\right] \tag{9}$$

$$g^{22} = \frac{1-v^2}{u^2} \tag{10}$$

$$g^{33} = \frac{1-w^2}{u^2[1-v^2]} \tag{11}$$

$$g^{00} = \left[1 + \frac{\varepsilon}{c^2} f\right]^{-1} \tag{12}$$

If we accept the well known Schwarzschild's metric tensor for the gravitational fields exterior to all static homogeneous spherical distributions of mass, then, from the well known tensorial Riemann's Geodesic Equations of Motion for a particles of nonzero rest mass in gravitational fields is given by [2] :

$$\frac{\partial}{\partial \tau}(m_0 u^\alpha) = m_o \frac{\partial}{\partial \tau}(u^\alpha) = m_o a^\alpha = 0 \tag{13}$$

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where $\frac{\partial}{\partial\tau}$ is the covariant differentiation with respect to the proper time τ , m_0 is the rest mass , u^α is the linear velocity vector and a^α is the four dimensional linear acceleration tensor given by:

$$a^\alpha = \ddot{x}^\alpha + \Gamma^\alpha_{\mu\upsilon}\dot{x}^\mu\dot{x}^\nu \tag{14}$$

where $\ddot{x}^\alpha$ is the space time coordinate tensor, $\Gamma^\alpha_{\mu\upsilon}$ is the Christoffel symbol of the second kind and a dot denotes one differentiation with respect to proper time. It may be noted that from tensor analysis that tensors are generally pure abstract mathematical quantities. Secondly it is well known that to every tensor there exist a corresponding vector whose components are all of the exact same physical unit or dimension and are physically measureable and comparable. Now we define the vector $\underline{a}$ corresponding to the linear acceleration tensor as:

$$\underline{a} = (a_{x^0}, a_u, a_v, a_w) \tag{15}$$

where

$$a_{x^o} = (g_{00})^{\frac{1}{2}}a^0 \tag{16}$$

$$a_u = (g_{11})^{\frac{1}{2}}a^1 \tag{17}$$

$$a_v = (g_{22})^{\frac{1}{2}}a^2 \tag{18}$$

$$a_W = (g_{33})^{\frac{1}{2}}a^3 \tag{19}$$

Equations (16 - 19) are called the linear acceleration vector in the gravitational fields. It therefore follows that the tensorial Riemann's Geodesic Equations of Motion (13) can be written equivalently as a vector equation given by:

$$m_o\underline{a} = \underline{0} \tag{20}$$

It now follows from equations (5),(6),(7),(8) and (14) that the Riemann's Geodesic Equations of Motion for particles of non zero rest mass in gravitational fields in rotational spherical polar coordinates can be written as:

$$\left[1+\frac{2}{c^2}f\right]^{\frac{1}{2}}\Big(m_o\ddot{x}^0 + \frac{1}{2}\left[1+\frac{2}{c^2}f\right]^{-1}\left[1+\frac{2}{c^2}f\right]_{,0}\dot{x}^0\,\dot{x}^0 + \left[1+\frac{2}{c^2}f\right]^{-1}\left[1+\frac{2}{c^2}f\right]_{,u}\dot{x}^0\dot{u}\,\left[1+\frac{2}{c^2}f\right]^{-1}\left[1+\frac{2}{c^2}f\right]_{,v}\dot{x}^0\dot{v} + \left[1+\frac{2}{c^2}f\right]^{-1}\left[1+\frac{2}{c^2}f\right]_{,w}\dot{x}^0\dot{w} - \frac{1}{2}\left[1+\frac{2}{c^2}f\right]^{-1}\left[1+\frac{2}{c^2}f\right]^{-1}_{,0}\dot{u}\dot{u}\Big) = 0 \tag{21}$$

$$\left[\left[1+\frac{2}{c^2}f\right]^{-1}\right]^{\frac{1}{2}}\Big(m_o\ddot{x}^1 - \frac{1}{c^2}\left[1+\frac{2}{c^2}f\right]_{,u} f\dot{x}^0\ \dot{x}^0 - \frac{2}{c^2}\left[1+\frac{2}{c^2}f\right]^{-1}_{,0} f\dot{x}^0\ \dot{u} + \frac{1}{2}\left[1+\frac{2}{c^2}f\right]\left[1+\frac{2}{c^2}f\right]^{-1}_{,u}\dot{u}\dot{u} + \left[1+\frac{2}{c^2}f\right]\left[1+\frac{2}{c^2}f\right]^{-1}_{,v}\dot{u}\dot{v} + \left[1+\frac{2}{c^2}f\right]\left[1+\frac{2}{c^2}f\right]^{-1}_{,w}\dot{u}\dot{w} - \frac{1}{2}\left[1+\frac{2}{c^2}f\right]^{-1}\left[\frac{v}{1-v^2}\right]_{,u}\dot{v}\dot{v} - \frac{1}{2}\left[1+\frac{2}{c^2}f\right]\left[\frac{u^2[1-v^2]}{1-w^2}\right]_{,u}\dot{w}\dot{w}\Big) = 0 \tag{22}$$

$$\left[\left[\frac{u^2}{1-v^2}\right]\right]^{\frac{1}{2}}\Big(m_o\ddot{x}^2 - \frac{1}{2}\left[\frac{1-v^2}{u^2}\right]\left[1+\frac{2}{c^2}f\right]_{,v}\dot{x}^0\dot{x}^0 + \left[\frac{1-v^2}{u^2}\right]\left[\frac{u^2}{1-v^2}\right]_{,v}\dot{u}\dot{v} - \frac{1}{2}\left[\frac{1-v^2}{u^2}\right]\left[1+\frac{2}{c^2}f\right]^{-1}_{,v}\dot{u}\dot{u} + \frac{1}{2}\left[\frac{1-v^2}{u^2}\right]\left[\frac{u^2}{1-v^2}\right]_{,v}\dot{v}\dot{v}\Big) = 0 \tag{23}$$

$$\left[\frac{u^2[1-v^2]}{1-w^2}\right]^{\frac{1}{2}}\Big(m_o\ddot{x}^3 - \frac{1}{2}\left[\frac{1-w^2}{u^2[1-v^2]}\right]\left[1+\frac{2}{c^2}f\right]_{,w}\dot{x}^0\dot{x}^0 + \left[\frac{w}{1-w^2}\right]\dot{w}\dot{w} + \frac{2}{u}\dot{w}\dot{u} - \left[\frac{2v}{1-v^2}\right]\dot{w}\dot{v} - \frac{1}{2}\left[\frac{1-w^2}{u^2[1-v^2]}\right]\left[1+\frac{2}{c^2}f\right]^{-1}_{,w}\dot{u}\dot{u} = 0 \tag{24}$$

Summary and Conclusion

The Riemann's geodesic equations obtained [(21), (22), (23), (24)] paves a way for application for the motion of a particle in the gravitational field of any spherical distribution of mass.

References

[1] M.R. Spiegel, Theory and Problems of Vector Analysis and Introduction to Tensor Analysis (Mc Graw - Hill, New York, 1974) 168

[2] Howusu S.X.K. Riemannian Revolutions in Physics and Mathematics, pp.3-4 (Jos University press: Jos,2013)