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Adjacent Rhotrix of a Complete, Simple and Undirected Graph

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Abstract

The adjacent matrix of a graph is a well-known concept in the combinatorial matrix theory. On the other hand nothing seems to be known on the adjacent rhotrix of a graph. In this paper we introduce the concept of an adjacent rhotrix of a graph and discuss its properties. An n-dimensional rhotrix is known to be an object that lies in some way between $n\times n$ dimensional matrix and $(2n-1)\times(2n-1)$ dimensional matrix and representation of vectors in rhotrix is different from the representation of vectors in matrix.

Keywords: adjacency, graph, complete, simple, undirected, rhotrix, eigenvalue
AMS Subject Classifications [2010]: 05C50.

1.0 Introduction

Mathematical arrays that are in some way between two-dimensional vectors and 2×2 dimensional matrices were suggested by Atanassov and Shannon [1]. As an extension to this idea, Ajibade [2] introduced an object that lies between 2×2 dimensional matrices and 3×3 dimensional matrices called 'rhotrix' presently referred to as base rhotrix. The initial algebra and analysis of rhotrices were presented in [2]. Let *R* and *Q* be two base rhotrices such that

$$R=\left\langle \begin{matrix} & a & \\ b & h(R) & d \\ & e & \end{matrix} \right\rangle \text{ and } Q=\left\langle \begin{matrix} & f & \\ g & h(Q) & j \\ & k & \end{matrix} \right\rangle . \qquad (1)$$

The addition and multiplication of rhotrices R and Q defined by Ajibade [2] are as follows:

$$R+Q=\left\langle \begin{matrix} & a+f & \\ b+g & h(R)+h(Q) & d+j \\ & e+k & \end{matrix} \right\rangle ,$$

$$R\circ Q=\left\langle \begin{matrix} & ah(Q)+fh(R) & \\ bh(Q)+gh(R) & h(R)h(Q) & dh(Q)+jh(R) \\ & eh(Q)+kh(R) & \end{matrix} \right\rangle .$$

Another multiplication method for rhotrices called *row-column multiplication* was introduced by Sani [3] in an effort to answer some questions raised in [2]. The row-column multiplication method is in a similar way as that of multiplication of matrices and is illustrated using the rhotrices *R* and *Q* defined in (1) as follows:

$$R\circ Q=\left\langle \begin{matrix} & af+dg & \\ bf+eg & h(R)h(Q) & aj+dk \\ & bj+ek & \end{matrix} \right\rangle .$$

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A generalization of the row-column multiplication method for *n*-dimensional rhotrices was given by Sani [4]. That is: given *n*-dimensional rhotrices $R_n = \langle a_{ij}, c_{lk} \rangle$ and $Q_n = \langle b_{ij}, d_{lk} \rangle$ the multiplication of R_n and Q_n is as follows:

$$R_n \circ Q_n = \langle a_{i_1 j_1}, c_{l_1 k_1} \rangle \circ \langle b_{i_2 j_2}, d_{l_2 k_2} \rangle = \left\langle \sum_{i_2 j_1 = 1}^{t} (a_{i_1 j_1} b_{i_2 j_2}), \sum_{l_2 k_1 = 1}^{t-1} (c_{l_1 k_1} d_{l_2 k_2}) \right\rangle, \; t = (n+1)/2 .$$

The method of converting a rhotrix to a special matrix called 'coupled matrix' was suggested by Sani [5] which is used to solve systems of $n \times n$ and $(n-1)\times(n-1)$ matrix problems simultaneously. This method was proved to be a linear transformation by Aminu [6]. The concept of vectors, one-sided system of equations and eigenvector eigenvalue problem in rhotrices were introduced by Aminu [7]. A necessary and sufficient condition for the solvability of one sided system of rhotrix was also presented in [7]. It was shown in the paper how a solution can be found provided the system is solvable. Rhotrix vector spaces and their properties were presented by Aminu [8]. Square root of a rhotrix, Cayley-Hamilton theorem in rhotrix, determinant method for solving system of equations in rhotrices and minimal polynomial of a rhotix were discussed in [9, 10, 11] and [12].

To the best of our knowledge the concept of adjacent rhotrix of a graph was not discussed in the literature, however an article that treats the concept of graph in rhotrix seem to only be Abdul [13]. It is the primary aim of this paper to discuss the concept of the adjacent rhotrix of a graph and present some of its properties.

2.0 Some basic definitions

Graph: A graph is made up of vertices or nodes and lines called edges that connect them. Throughout this work, we shall denote a graph by G.

Undirected Graph: a graph is undirected if there is no distinction between any two vertices associated with each edge

Vertex: the vertex set of a graph G is usually denoted by V(G) or V, which is the set containing all the vertices in the graph G.

Edge: edge is a drawn line connecting two vertices called endpoints. We shall denote the edge set of G by E(G). Also the size of a graph is the number of edges in the graph G.

A loop: a loop is an edge for which its starting point and endpoint is the same vertex

Simple graph: a graph is simple if it has no loops

Compliment of a graph G: the complement of a graph G is denoted by $\bar{G}$. It is a graph with the same vertex set as G but with an edge set such that xy is an edge in $\bar{G}$ if and only if xy is not an edge in G.

Edgeless graph or Null graph: this is a graph with zero or more vertices but no edges. Empty or null graph is graph with no edge, no vertex.

Adjacent Graph: a graph is adjacent if there is an edge between any two vertices.

Complete graph: a complete graph is a simple graph with n vertices in which every vertex is adjacent to every other. It is denoted by K_t. It has $\frac{t(t-1)}{2}$ edges.

Line graph: it is denoted by L(G). The line graph of an undirected graph G is another graph L(G) that represents the adjacent between edges of G.

Hoffman polynomial of a rhotrix: is defined as the minimum polynomial H(x) such that $H(R_n)$= J; such that R_n is an n-dimensional rhotrix and J is a rhotrix whose entries are ones.

3.0 Adjacent Rhotrix of a Graph

Suppose G is a simple, complete and undirected graph with vertices V= {1,2,3,…,t}, then the adjacent rhotrix of G is an n-dimensional rhotrix:

$$R_n(G)=\left\langle a_{ij},c_{lk}\right\rangle=\left\langle\begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{21} & c_{11} & a_{12} & & & \\ & & a_{31} & c_{21} & a_{22} & c_{12} & a_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ a_{t1} & c_{(t-1)1} & \cdots & \cdots & \cdots & \cdots & \cdots & c_{1(t-1)} & a_{1t} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & a_{t(t-2)} & c_{(t-1)(t-2)} & a_{(t-1)(t-1)} & c_{(t-2)(t-1)} & a_{(t-2)t} & & \\ & & & a_{t(t-1)} & c_{(t-1)(t-1)} & a_{(t-1)t} & & & \\ & & & & a_{tt} & & & & \end{array}\right\rangle, \qquad (2)$$

where $a_{ij}, c_{lk} \in \Re$(set of real numbers) for i,j = 1,2,3,...,t, k,l = 1,2,3,...,t-1 and n=2t-1.

The rhotrix $R_n(G)=\left\langle a_{ij},c_{lk}\right\rangle$ given in (2) can be converted into a coupled matrix [5] in which one of the matrices $A=(a_{ij})$ is a $t\times t$ dimensional while the other $C=(c_{lk})$ is a $(t-1)\times(t-1)$ dimensional. The matrix $A=(a_{ij})$ embedded in $R_n(G)=\left\langle a_{ij},c_{lk}\right\rangle$ will be called major matrix while the other $C=(c_{lk})$ will be called minor matrix. It can easily be verified that the major matrix in adjacent matrix (2) is the adjacent matrix of the graph G, while the minor matrix is a null matrix. This implies that for any simple, complete and undirected graph G, the adjacent rhotrix $R_n(G)$is such that$n=(2t-1)$ where t is the number of vertices in the graph. Therefore (2) becomes;

$$R_n(G)=\left\langle\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & 0 & 1 & & & \\ & & 1 & 0 & 0 & 0 & 1 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ 1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1 \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & 1 & 0 & 0 & 0 & 1 & & \\ & & & 1 & 0 & 1 & & & \\ & & & & 0 & & & & \end{array}\right\rangle.$$

A matrix is said to be *null* if all its entries are zero. It can therefore be observed that the adjacent rhotrix has 1 as entries except those in its main diagonal which are zeros and the minor matrix which is null.

A rhotrix $R_n=(a_{ij},c_{lk})$ is *symmetric* if it is equal to its transpose. That is a rhotrix is symmetric if $R_n=(a_{ij},c_{lk})=R_n^T=(a_{ji},c_{kl})=(A^T,C^T)$.

Theorem 3.1. *Let G be simple, complete and undirected graph, then the adjacent rhotrix of G is symmetric.*

Proof: Follows directly from the definitions of an adjacent rhotrix $R_n(G)$ of a graph G and that of a transpose of a rhotrix.

Let $R_n=\left\langle a_{ij},c_{lk}\right\rangle$ be an *n*-dimensional rhotrix, the determinant of $R_n=\left\langle a_{ij},c_{lk}\right\rangle$ as defined in [4] is

$$\det(R_n)=\det\left(\left\langle a_{ij},c_{lk}\right\rangle\right)=\det(A)\det(C). \qquad (3)$$

Also, $R_n^{-1}=\left\langle a_{ij}^{-1},c_{lk}^{-1}\right\rangle=\left\langle A^{-1},C^{-1}\right\rangle$.

It follows that the determinant of a rhotrix is the product of the determinants of its two embedded matrices. Therefore, we have the following proposition.

Proposition 3.1. *Let G be simple, complete and undirected graph, then* $\det(R_n(G)) = 0.$

Proof: Suppose that $R_n(G) = \langle a_{ij}, c_{lk} \rangle$, where G is a simple graph. Since G is simple it implies that all the entries in the leading diagonal of the major matrix in $R_n(G)$ are zero, also the minor matrix $C = (c_{lk}), l, k = 1, 2, ..., t-1$ is a null matrix with $\det(C) = 0.$ It follows from (3) that $\det(R_n(G)) = \det(A)\det(C) = 0$.

Corollary 3.1. *The inverse of adjacent Rhotrix is not defined*

Proof: The proof follows from Proposition 3.1 and the fact that every matrix whose determinant is zero is not invertible.

The *main diagonal* of a rhotrix $R_n = \left\langle a_{ij}, c_{lk} \right\rangle$ is a collection of cells a_{ii} , c_{ll} where $i = 1,...,t$, $l = 1,...,t-1$ and $t = (n+1)/2$. The other diagonal (which is not main diagonal) is a collection of the cells $a_{t1}, c_{t-11}, a_{t-12}, c_{t-22}, ..., c_{1t-1}, a_{1t}$ where $t = (n+1)/2$. A *diagonal rhotrix* is a rhotrix in which all entries outside the main diagonal are zero. That is, diagonal rhotrix is of the form

$$\left\langle \begin{array}{ccccccc} & & & a_{11} & & & \\ & & 0 & c_{11} & 0 & & \\ & 0 & 0 & a_{22} & 0 & 0 & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & 0 & 0 & a_{t-1t-1} & 0 & 0 & \\ & & 0 & c_{t-1t-1} & 0 & & \\ & & & a_{tt} & & & \end{array} \right\rangle$$

The *identity* rhotrix is a diagonal rhotrix with ones on the main diagonal and is usually denoted by *I*.

The *rhotrix eigenvector-eigenvalue problem* or just *rhotrix eigenproblem* (REP) is the following: Given $R_n = \left\langle a_{ij}, c_{lk} \right\rangle$, find all $\lambda \in \Re$ *(eigenvalue)* and an *n*-dimensional rhotrix column vector $\left\langle x^{nj} \right\rangle$, $\left\langle x^{nj} \right\rangle \neq 0$ *(eigenvectors)* such that

$$R_n \left\langle x^{nj} \right\rangle = \lambda \left\langle x^{nj} \right\rangle.$$

Theorem 3.2 [7] *Let* $R_n = \left\langle a_{ij}, c_{lk} \right\rangle$ *be an n-dimensional rhotrix. Then* $\lambda \in \Re$ *is a rhotrix eigenvalue of* R_n *if and only if* $\det(A - \lambda I) = 0$ *, where* $A = (a_{ij}) \in \Re^{t \times t}$ *, I is a* $t \times t$ identity matrix *and* $t = (n+1)/2$.

Theorem 3.3 *Let G be an undirected graph, then the eigenvalue of G is the eigenvalue of its adjacent Rhotrix.*

Proof: Let R_G be the adjacent rhotrix of a graph G then the eigenvalue of R_G is given as the solution of det $\left(R_G - \lambda I\right) = 0$, where $\lambda \in \Re$ and *I* is an identity rhotrix having the order with R_G .

The eigenvalue of a graph G is the eigenvalue of its adjacent matrix [15], also

$R_G = \langle a_{ij}, c_{lk} \rangle$, where a_{ij} is an adjacent matrix(major matrix) and c_{lk} is a null matrix(minor matrix)

$\Rightarrow$ The adjacent rhotrix R_G contain both the adjacent matrix A and the null matrix C

$\Rightarrow$ The eigenvalue of the adjacent matrix, A is the eigenvalue of R_G (Theorem 3.1).

Hence, the eigenvalue of any graph G is the eigenvalue of its adjacent rhotrix.

Corollary 3.2 *The spectrum of the adjacent rhotrix is of order t, where t is the order of the graph.*

Proof: It follows from Theorem 3.2 that the eigenvalue of a graph is the eigenvalue of its adjacent rhotrix R_G. The spectrum = $\{\lambda_i : i = 1, 2, 3, ..., t\}$ of the graph G is of order t

Since the eigenvalue of the graph G is equal to the eigenvalue of its adjacent Rhotrix,

Then the spectrum of R_G = $\{\lambda_i : i = 1, 2, 3, ..., t\}$ is also of order t

Theorem 3.4: *The eigenvalue of the adjacent Rhotrix of a complete and undirected graph G is -1 of multiplicity (t-1) and (t-1) of multiplicity 1.*

Proof: G is an undirected graph, implies its adjacent rhotrix is symmetric (Theorem 3.1). R_G is symmetric, implies all its eigenvalues are real [16]. Also, G is a simple and complete graph, implies that all the entries in the major axis of the rhotrix is zero. We know that the eigenvalue of the adjacent matrix is the eigenvalue of the rhotrix R_G.

In graph theory, the eigenvalue of a complete, undirected graphis of -1 of multiplicity (t-1) and (t-1) of multiplicity 1, hence same on Rhotrix.

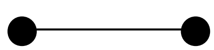
Theorem 3.5: *The Hoffman polynomial of a complete and undirected graph G is a linear polynomial*

Proof: By definition, $H(x)$ is the minimum polynomial such that $H(x) = J$ where $J = \langle A, C \rangle$ where *A* is a matrix of ones and *C* identity. Since *G* is undirected and complete, then its minimum eigenvalue is -1. Therefore, the Hoffman polynomial reduces to $H(x) = x + 1$ and $H(R_G) = R_G + I$ where R_G is the adjacent rhotrix of *G and I* and identity rhotrix. Now that $H(R_G) = \langle a_{ij}, c_{lk} \rangle = \langle A, C \rangle$ where *A* is a matrix of ones and *C* identity matrix. It follows that $H(R_G)$ is a linear polynomial since the highest power of R_G is one.

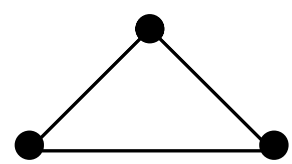
1. **Some Examples**

1. Consider the following simple, complete and undirected graphs G_1, G_2 and G_3 :

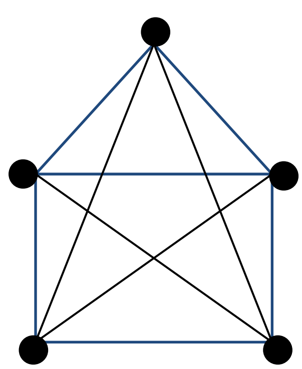
$G_1 =$



$G_2 =$



$G_3 =$



Considering G_1 we have $V = \{1,2\}$ $E = \{(12), (21)\}$, so that the rhotrix generated is of order $n = 2t - 1$, where t=2 hence $n = (2 \times 2) - 1 = 3$. Thus G_1 generates the adjacent rhotrix R_3:

$$R_3 = \left\langle \begin{matrix} & 0 & \\ 1 & 0 & 1 \\ & 0 & \end{matrix} \right\rangle$$

The major matrix embedded in the adjacent rhotrix is:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Now we shall determine the eigenvalue and their corresponding eigenvectors of G_1. The eigenvalues of a rhotrix is the eigenvalues of its major matrix, also the eigenvalue of any graph G is the eigenvalue of its adjacent matrix.
The characteristic equation of G is

$$\det(A - \lambda I) = 0,$$

$$= \det\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0,$$

$$= \det\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda^2 = 1$$

$$\Rightarrow \lambda = \pm 1$$

$$\Rightarrow \lambda = -1, +1.$$

The spectrum = {-1,1},

the corresponding eigenvectors are;

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{respectively.}$$

Now, the Hoffman polynomial $H(x) = (x - \lambda_i) = J$, where λ_i is the minimum eigenvalue, and J is the rhotrix $\langle a_{ij}, c_{lk} \rangle$ such that a_{ij} is a matrix of ones and c_{lk} identity matrix. Therefore,

$$H(R_n) = (R_n - \lambda I) = (R_3 + I)$$

$$\Rightarrow \left\langle \begin{matrix} & 0 & \\ 1 & 0 & 1 \\ & 0 & \end{matrix} \right\rangle + \left\langle \begin{matrix} & 1 & \\ 0 & 1 & 0 \\ & 1 & \end{matrix} \right\rangle = \left\langle \begin{matrix} & 1 & \\ 1 & 1 & 1 \\ & 1 & \end{matrix} \right\rangle$$

Now considering G_2;
$V = \{1,2,3\}$ $E = \{(12), (21), (23)(32), (13)(31)\}$, so that the Rhotrix generated is order R_5.
The adjacent rhotrix is:

$$R_5 = \left\langle \begin{matrix} & & 0 & & \\ & 1 & 0 & 1 & \\ 1 & 0 & 0 & 0 & 1 \\ & 1 & 0 & 1 & \\ & & 0 & & \end{matrix} \right\rangle.$$

Spectrum of the graph is $\{-1, -1, 2\}$, and the corresponding eigenvectors are

$$x_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \text{ and } x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ respectively.}$$

$H(R_n) = (R_n - \lambda I) = (R_5 + I)$ Since the minimum eigenvalue is -1

$$H(R_5) = \left\langle \begin{array}{ccccc} & & 0 & & \\ & 1 & 0 & 1 & \\ 1 & 0 & 0 & 0 & 1 \\ & 1 & 0 & 1 & \\ & & 0 & & \end{array} \right\rangle + \left\langle \begin{array}{ccccc} & & 1 & & \\ & 0 & 1 & 0 & \\ 0 & 0 & 1 & 0 & 0 \\ & 0 & 1 & 0 & \\ & & 1 & & \end{array} \right\rangle = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 1 & 1 & 1 & \\ 1 & 0 & 1 & 0 & 1 \\ & 1 & 1 & 1 & \\ & & 1 & & \end{array} \right\rangle$$

Considering G_3;

$$V = \{1,2,3,4,5\}$$

E={(12),(21),(15),(51),(25),(52),(14),(41),(13),(31),(24),(42),(35),(53),(54),(45),(34),(43)(23),(32)} So that the Rhotrix generated is order R_9.

The adjacent Rhotrix of the graph $\boldsymbol{G_4}$ becomes;

$$R_9 = \left\langle \begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & 0 & 1 & & & \\ & & 1 & 0 & 0 & 0 & 1 & & \\ & 1 & 0 & 1 & 0 & 1 & 0 & 1 & \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ & 1 & 0 & 1 & 0 & 1 & 0 & 1 & \\ & & 1 & 0 & 0 & 0 & 1 & & \\ & & & 1 & 0 & 1 & & & \\ & & & & 0 & & & & \end{array} \right\rangle.$$

The spectrum of the graph is $\{-1,-1,-1,-1,4\}$, and the corresponding eigenvectors are

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}, x_4 \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \\ 1 \end{bmatrix}, x_5 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ respectively}$$

$H(R_n) = (R_n - \lambda I) = (R_5 + I)$ Since the minimum eigenvalue is -1

$$H(R_9)=\left\langle\begin{array}{ccccccccc} &&&&0&&&& \\ &&&1&0&1&&& \\ &&1&0&0&0&1&& \\ &1&0&1&0&1&0&1& \\ 1&0&1&0&0&0&1&0&1 \\ &1&0&1&0&1&0&1& \\ &&1&0&0&0&1&& \\ &&&1&0&1&&& \\ &&&&0&&&& \end{array}\right\rangle+\left\langle\begin{array}{ccccccccc} &&&&1&&&& \\ &&&0&1&0&&& \\ &&0&0&1&0&0&& \\ &0&0&0&1&0&0&0& \\ 0&0&0&0&1&0&0&0&0 \\ &0&0&0&1&0&0&0& \\ &&0&0&1&0&0&& \\ &&&0&1&0&&& \\ &&&&1&&&& \end{array}\right\rangle=\left\langle\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&1&1&&& \\ &&1&0&1&0&1&& \\ &1&0&1&1&1&0&1& \\ 1&0&1&0&1&0&1&0&1 \\ &1&0&1&1&1&0&1& \\ &&1&0&1&0&1&& \\ &&&1&1&1&&& \\ &&&&1&&&& \end{array}\right\rangle$$

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