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Effect of the inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing populations

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Abstract

The differing effects of the intra-species and inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing yeast species can be clearly differentiated. In this context, we have systematically investigated the effect of the inter-species competition parameters on the onset of stability and degeneracy of co-existence steady-state solutions between competing yeast species which is quite distinct from the analysis of the intra-species competition parameters. A mathematical approach has been proposed to tackle this challenging problem. The implications of this contribution for the dominant survival of the interacting yeast species are discussed quantitatively. The results which we have obtained have not been seen elsewhere, they are presented here and discussed.

Keywords: inter-species competition parameters, stability, degeneracy, competing populations

1.0 Introduction

The relationship between the carrying capacity of the interacting system, stability, instability and degeneracy of the co-existence steady-state solution can play significant role in ecosystem functioning analysis [1- 6]. While the concept of the carrying capacity concerns the maximum population which can sustain the growth of a population and the concept of stability theory has a long standing history in the study of ordinary differential equations, we propose an application of a biological idea and a mathematical idea using a computational approach in order to understand the effect of the inter-species competition parameters on the onset of stability and degeneracy of the co-existence steady-state solution in the context of two competing yeast populations [7].

2.0 Model formulation of interacting yeast populations

Following Pielou [7], we have used the following parameters which define the dynamics of two interacting yeast populations: intrinsic growth rate parameters a and d having the values of 0.1 and 0.08, the intra-species competition parameters b and f having the values of 0.0014 and 0.001, the inter-species competition parameters c and e having the values of 0.0012 and 0.0009. The dynamics of the proposed interaction between two yeast species have also been described recently in the work of Ford, Lumb and Ekaka-a [2].These model equations take the following mathematical structure of the Lotka-Volterra type

$$\frac{dy_1(t)}{dt} = y_1(t)[a - by_1(t) - cy_2(t) \tag{1}$$

$$\frac{dy_2(t)}{dt} = y_2(t)[d - ey_1(t) - fy_2(t)] \tag{2}$$

Here, the notations $y_1(t)$ and $y_2(t)$specify the limiting biomasses of the yeast species 1 and yeast species 2 under the simplifying assumption that the positive starting biomass for the yeast species 1 is $y_1(0)$while the positive starting biomass for the yeast species 2 is$y_2(0)$.

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3.0 Method of Solution

In this study, we have used a computational method to determine each type of stability for a system of two interacting yeast species. Firstly, the two carrying capacities for the interacting yeast species were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of y_{1e} and y_{2e} which were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of y_{1e} and y_{2e} with respect to y_{1e} and y_{2e} were derived and evaluated at the arbitrary co-existence steady-state solution or point (y_{1e}, y_{2e}). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the contrary, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. If any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting yeast species can go into the ecological risk of extinction escaping survival while the other yeast species can tend to survive.

4.0 Results and Discussions

In this present study, we have used a computational method to determine each type of stability for a system of two interacting yeast populations. The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Table 1 and Table 2 below: the notations aand d stand for the intrinsic growth rate parameter values for yeast species, the notations c and e stand for the inter-species competition parameters, the notation css stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution. This presentation concerns when the model parameters a, d, b and f are fixed. Here, we are interested to vary only the precise values of the inter-species competition parameters c and e.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the inter-species competition parameters c and e: summary of results 1

Example	c	e	css	λ_1	λ_2	Type of stability
1	0.0012	0.0009	12.5:68.75	-0.0033	-0.0829	stable
2	0.00006	0.000045	68.13:76.93	-0.0961	-0.0762	stable
3	0.00012	0.000090	65.07:74.14	-0.0938	-0.0715	stable
4	0.00018	0.0001350	62.22:71.60	-0.0923	-0.0664	stable
5	0.00024	0.0001800	59.55:69.28	-0.0914	-0.0612	stable
6	0.00030	0.0002250	57.04:67.17	-0.0908	-0.0562	stable
7	0.00036	0.0002700	54.65:65.24	-0.0903	-0.0514	stable
8	0.00042	0.0003150	52.38:63.50	-0.0900	-0.0469	stable
9	0.00048	0.0003600	50.20:61.93	-0.0897	-0.0426	stable
10	0.00054	0.0004050	48.08:60.53	-0.0894	-0.0385	stable
11	0.00060	0.0004500	46.02:59.29	-0.0891	-0.0346	stable
12	0.00066	0.0004950	43.98:58.23	-0.0889	-0.0309	stable
13	0.00072	0.0005400	41.93:57.36	-0.0886	-0.0274	stable
14	0.00078	0.0005850	39.84:56.70	-0.0883	-0.0241	stable
15	0.00084	0.0006300	37.67:56.27	-0.0880	-0.0210	stable

We observe from Table 1 that example 1 illustrates that for the fixed values of the intra-species competition parameters c and e, the co-existence steady-state solution is said to be stable because of its two negative eigenvalues based on the mathematical theory of stability. Every other varied values of these two competition parameters clearly shows that its corresponding co-existence steady-state solution is stable for the intervals of variations such as [0.00006, 0.001212] and [0.000045, 0.0009090] for the inter-species competition parameters c and e. It is also worth noting that subsequent co-existence steady-state solutions are considered as degenerate for the intervals of variations [0.001260, 0.001560] and [0.0009450, 0.001170]. Part of these deductions can be observed from Table 2. The other implications of these results will be discussed next in terms of the survival of these two interacting yeast species.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the inter-species competition parameters c and e: summary of results 2

Example	c	e	css	λ_1	λ_2	Type of stability
16	0.00090	0.0006750	35.33:56.15	-0.0877	-0.0179	stable
17	0.00096	0.0007200	32.73:56.43	-0.0873	-0.0150	stable
18	0.00102	0.0007650	29.70:57.30	-0.0867	-0.0122	stable
19	0.00108	0.0008100	25.90:59.03	-0.0859	-0.0093	stable
20	0.00114	0.0008550	20.70:62.31	-0.0848	-0.0065	stable
21	0.001152	0.0008640	19.37:63.26	-0.0845	-0.0059	stable
22	0.001164	0.0008730	17.93:64.35	-0.0842	-0.0053	stable
23	0.001176	0.0008820	16.32:65.61	-0.0838	-0.0046	stable
24	0.001188	0.0008910	14.53:67.06	-0.0834	-0.0040	stable
25	0.001212	0.0009090	10.19:70.74	-0.0824	-0.0026	stable
26	0.001260	0.0009450	-3.82:83.61	0.0008	-0.0791	degenerate
27	0.001320	0.0009900	-60.1:139.50	0.0117	-0.0670	degenerate
28	0.001380	0.001035	367.5:-300.35	-0.1984	-0.0157	degenerate
29	0.001440	0.001080	97.94:-25.77	-0.1077	-0.0036	degenerate
30	0.001500	0.001125	69.57:1.74	-0.0995	0.0003	unstable
31	0.001560	0.001170	58.33:11.76	-0.0964	0.003	unstable

For the inter-species competition parameters' analysis, a dominant stable co-existence steady-state solution is observed with fewer instances for the degenerate characteristic and instability of the co-existence steady-state solution. Since the inhibiting effect of the yeast species 2 on the growth of the yeast species 1 is often calculated by using the formula $\alpha_{12} = \frac{c}{b}$ and the inhibiting effect of the yeast species 1 on the growth of the yeast species 2 is often calculated by using the formula $\alpha_{21} = \frac{e}{f}$ such that α_{12} is smaller than the ratio of the carrying capacity of yeast species 1 to the carrying capacity of yeast species 2 and α_{21} is smaller than the ratio of the carrying capacity of yeast species 2 to the carrying capacity of yeast species 1 to satisfy the survival criteria [5], it is very clear that these two yeast species will dominantly tend to survive together in addition to their being stable. On the basis of this analysis, it is interesting to comment that there are only two instances when the yeast species 1 will go into extinction while the yeast species 2 will survive alone [see example 26 and example 27 of Table 2]. In this scenario, our present analysis shows only two instances when the yeast species 1 will survive alone while the yeast species 2 will go into extinction [see example 28 and example 29 of Table 2]. Thereafter, example 30 and example 31 clearly illustrate the onset of the instability of the co-existence steady-state solution.

5.0 Conclusion

In summary, a variation of the inter-species competition parameters has clearly illustrated a dominant stable co-existence steady-state solution with fewer instances of degeneracy and instability which is a distinct result from the variation of the intra-species competition parameters.For the purpose of ecosystem functioning and planning, our present inter-species competition parameters analysis is more likely preferred which is consistent with a dominant ecological thought. The loss of degeneracy will require a sophisticated numerical application of the D-bifurcation stochastic method which is beyond the scope of this present contribution. It may also be possible to observe a dominant instance of instability due to some further variations of the inter-species competition parameters. The method of solving the proposed problem as reflected in this article can be extended to the scenario of a similar interaction between the two yeast species and also to the scenario when the inter-species competition parameters have slightly relatively the same precise values.

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