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Effect of the intra-species competition parameters on the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing populations

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Abstract

Experts in mathematical ecology have observed the role of a lower carrying capacity and a higher carrying capacity in the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing yeast species populations. To solve this complex problem, a mathematical approach has been proposed in this study with the expectation of providing further insight into ecosystem functioning and stability. Our novel results are clearly presented and discussed quantitatively.

Keywords: intra-species competition parameters, stability, instability, degeneracy, competing populations

1.0 Introduction

Within the mathematical literatures of mathematical biology or mathematical ecology, the relationship between the carrying capacity of the interacting system, stability, instability and degeneracy of the co-existence steady-state solution can play significant role in ecosystem functioning analysis [1- 6] . However, it remains a challenging scientific task to provide a further insight to a clearer understanding of this intrinsic relationship between interacting populations. While the concept of the carrying capacity concerns the maximum population which can sustain the growth of a population and the concept of stability theory has a long standing history in the study of ordinary differential equations, we propose an application of a biological idea and a mathematical idea using a computational approach in order to understand the effect of the intraspecific coefficients on the onset of stability, instability and degeneracy of the co-existence steady-state solution in the context of two competing yeast populations [7].

2.0 Model formulation of interacting yeast populations

Following Pielou [7], we have considered the following parameters which define the dynamics of two interacting yeast populations: intrinsic growth rate parameters a and d having the values of 0.1 and 0.08, the intra-species competition parameters b and f having the values of 0.0014 and 0.001, the inter-species competition parameters c and e having the values of 0.0012 and 0.0009. The dynamics of the proposed interaction between two yeast species have also been described recently in the work of Ford, Lumb and Ekaka-a [2].These model equations take the following mathematical structure of the Lotka-Volterra type

$$\frac{dy_1(t)}{dt} = y_1(t)[a - by_1(t) - cy_2(t)] \tag{1}$$

$$\frac{dy_2(t)}{dt} = y_2(t)[d - ey_1(t) - fy_2(t)] \tag{2}$$

Here, the notations $y_1(t)$ and $y_2(t)$ specify the limiting biomasses of the yeast species 1 and yeast species 2 under the simplifying assumption that the positive starting biomass for the yeast species 1 is $y_1(0)$ while the positive starting biomass for the yeast species 2 is $y_2(0)$.

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3.0 Method of Solution

Following a similar explanation as in our earlier analyses, we have used a computational method to determine each type of stability for a system of two interacting yeast species. First of all, the two carrying capacities for the interacting yeast species were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of y_{1e} and y_{2e} which were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of$y_{1e}(t)$and $y_{2e}(t)$with respect to$y_{1e}$and y_{2e} were derived and evaluated at the arbitrary co-existence steady-state solution or point (y_{1e}, y_{2e}). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated computationally and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the contrary, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. If any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting yeast species can go into the ecological risk of extinction escaping survival while the other yeast species can tend to survive.

4.0 Results and Discussions

In this present study, we have used a computational method to determine each type of stability for a system of two interacting yeast populations. The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Table 1 and Table 2below: the notations aand d stand for the intrinsic growth rate parameter values for yeast species, the notations b and f stand for the intra-species competition parameters,the notation css stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution. This presentation concerns when the model parameters a, d, c and e are fixed. Here, we are interested to vary only the precise values of the intra-species competition parameters b and f.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-species competition parameters b and f: summary of results 1

Example	b	f	css	λ_1	λ_2	Type of stability
1	0.0014	0.001	12.5: 68.75	-0.0033	-0.0829	stable
2	0.00007	0.00005	84.53:78.40	-0.0895	0.0797	unstable
3	0.00014	0.00010	80.68:73.92	-0.0896	0.0709	unstable
4	0.00021	0.00015	77.25:69.81	-0.0897	0.0630	unstable
5	0.00028	0.00020	74.22:66.02	-0.0898	0.0558	unstable
6	0.00035	0.00025	71.54:62.47	-0.0900	0.0493	unstable
7	0.00042	0.00030	69.18:59.12	-0.0901	0.0433	unstable
8	0.00049	0.00035	67.14:55.92	-0.0903	0.0378	unstable
9	0.00056	0.00040	65.42:52.80	-0.0904	0.0327	unstable
10	0.00063	0.00045	64.03:49.72	-0.0907	0.0280	unstable

What do we learn from Table 1? When the value of the intrinsic growth rate parameter a is 0.1 and the value of the intra-species competition parameter b is 0.0014, the value of the expected carrying capacity [or the maximum population that can sustain the growth of yeast species 1] is 71.43 grams while when the value of the intrinsic growth rate parameter d is 0.08 and the value of the intra-species competition parameter f is 0.001, the value of the expected carrying capacity [or the maximum population that can sustain the growth of yeast species 2 is 80.00 grams. In these two carrying capacity values of 71.43 grams and 80.00 grams for the yeast species 1 and yeast species 2, the corresponding steady-state solution (12.5, 68.75) is said to be stable having two eigenvalues as clearly calculated and displaced on the first row of Table 1. On the other hand, for the intervals of intra-species competition parameters b and f defined by [0.00007, 0.00063] and [0.0005, 0.00045], their corresponding co-existence steady-state solutions are dominantly unstable. On the basis of this detailed analysis, it is interesting to observe that a minimum carrying capacity value of 71.43 grams for the yeast species 1 and a minimum carrying capacity value of 80.00 grams for the yeast species 2 will guarantee the stability of the co-existence steady-state solution. However, for the higher values of the carrying capacity values of 1428.6, 714.3, 476.2, 357.14, 285.7, 238.09, 204.08, 178.57 and 158.73 for the yeast species 1 while for the higher values of the carrying capacity values of 1600, 800, 533.3, 400.0, 320.0, 266.7, 228.6, 200.0 and 177.8 for the yeast species 2, their corresponding co-existence steady-state solutions as presented in Table 1 clearly support instability of each co-existence steady-state solution. Ecologically, the minimum carrying capacity tends to support the stability of the co-existence steady-state solution while a relatively higher carrying capacity values tend to support the instability of the co-existence steady-state solution. This present analysis seems to suggest that a lower carrying capacity value which promote some sort of a mild competition between these two yeast species can support the stability of the co-existence steady-state solution in contrast to a higher value of the carrying capacity which promote some sort of severe competition between these two yeast species for limited resources in the environment do support the instability of the co-existence steady-state solution.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intra-species competition parameters b and f: summary of results 2

Example	b	f	css	λ_1	λ_2	Type of stability
11	0.00070	0.00050	63.01:46.58	-0.0910	0.0236	unstable
12	0.00077	0.00055	62.45:43.26	-0.0913	0.0194	unstable
13	0.00084	0.00060	62.50:39.58	-0.0918	0.0155	unstable
14	0.00091	0.00065	63.46:35.21	-0.0924	0.0118	unstable
15	0.00098	0.00070	65.99:29.44	-0.0935	0.0082	unstable
16	0.00105	0.00075	71.80:20.51	-0.0953	0.0045	unstable
17	0.00112	0.00080	86.96:2.17	-0.0995	0.0003	unstable
18	0.00119	0.00085	160.58:-75.91	-0.1196	-0.0070	degenerate
19	0.00126	0.00090	-111.11:200.00	0.0200	-0.0600	degenerate
20	0.00133	0.00095	-5.45:89.37	0.0011	-0.0788	degenerate

The same pattern of observation is maintained for other varied values of the intra-species competition parameters which also promote the instability of the co-existence steady-state solutions [see Table 2 from example 11 to example 17]. This dominant instability behaviour changes to a degeneracy of the co-existence steady-state solution which is a counter-intuitive biological observation since the biomass of yeast species does not have a negative sign.

5.0 Conclusion

In this inter-disciplinary study, we have successfully used a mathematical simulation approach to calculate the effect of the intra-species competition parameters on the onset of stability, instability and degeneracy of co-existence steady-state solutions between competing yeast species. Our present analysis has clearly shown that a minimum carrying capacity can support only the stability of the co-existence steady-state solution while a relatively higher carrying capacity value does support a dominant instability of the co-existence steady-state solution. Over 20 repeated simulations, there is a probability of 0.05 of obtaining a stable co-existence steady-state solution followed with a probability of 0.80 of obtaining an unstable co-existence steady-state solution and a probability of 0.15 of obtaining a degenerate co-existence steady-state solution. The key achievement of this study is that we have used a mathematical technique to solve an ecological problem. We would expect this contribution to provide further insight to the ecological issues of ecosystem functioning and stability. This mathematical technique can be extended to tackle other types of yeast species interactions such as mutualism, commensalism and predation in our future contributions with respect to the effects of intra-species and inter-species competition parameters which we did not attempt to analyse.

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