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Stability analysis for a system of interacting populations with a dis-similar carrying capacity

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Abstract

The aim of this present contribution is to illustrate the use of a computational method to determine the type of stability for a system of two interacting populations when the carrying capacities values are 3.26 and 3.76 respectively provided the intra-species and inter-species inhibiting factors on the growth of cowpea and groundnut within the Niger Delta agricultural setting are fixed. For the purpose of this pioneering study, the intrinsic growth rate parameter value for cowpea is 0.0225 grams while the intrinsic growth rate parameter value for groundnut is 0.05 grams. Within the range of variations for the intrinsic growth of cowpea, fewer optimal instances of degeneracy for a fewer co-existence steady-state solutions were obtained. The possibility when the degeneracy characteristic of the co-existence steady-state solution will be lost remains to be empirically verified.The novel results which we have obtained and have not seen elsewhere are presented here and discussed.

Keywords: co-existence steady-state solution, stability, degeneracy, interacting legumes

1.0 Introduction

The concept of carrying capacity in both ecological and mathematical ecology studies has diverse applications [1- 4]. However, the link between the carrying capacity and stability can present a challenging problem to ecologists and mathematicians. By using the standard techniques of stability [5, 6], we propose to study the scenario when the carrying capacity for the growth of cowpea is 3.26 grams per area of cultivable land space and the carrying capacity for the growth of groundnut is 3.76 grams per area of cultivable land space.Following Ekpo and Nkanang [7], in this second study, we have considered the following parameter values: $a_c = 0.0225$, $d_g = 0.05$, $b_c = 0.006902$, $f_g = 0.0133$, $c_c = 0.0005$, $e_g = 0.01$. When the carrying capacity of the groundnut in these new parameter modifications slightly outweighs the previous carrying capacity of the groundnut ecologically, how would these changes in the carrying capacity of groundnut affect the stability of the co-existence steady-state solution mathematically especially when the two inhibiting parameter values $c_c = 0.0005$ and $e_g = 0.01$ do not change? This scientific question is beyond the application of the traditional analytical method of mathematics. Therefore, we propose to explore the technique of numerical simulation in order to tackle this question.

2.0 Mathematical Formulation

Following Ekaka-a [4], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a_c-b_cC(t)-c_cG(t)] \tag{1}$$

$$\frac{dG(t)}{dt} = G(t)[d_g-f_g\,G(t)-e_gC(t)] \tag{2}$$

where $C(0) > 0$ and $G(0) > 0$ define the starting biomasses of cowpea and groundnut at the start of the growing season otherwise called the initial conditions when $t = 0$. The duration of growth is in the unit of days hereby denoted by the

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independent variable t. For the purpose of this simulation study, the best-fit model parameters such as a_c and d_g that define the intrinsic growth rates for cowpea and groundnut were selected. The next best-fit parameters such as b_c and f_g define the intra-species competition parameters which measure the inhibiting factors on the growth of cowpea and groundnut due to self-interaction whereas the parameters c_c and e_g define the inter-species competition parameters which also measure other inhibiting factors on the growth of cowpea and groundnut due to interspecific interaction between cowpea and groundnut. In this study, we have considered the following parameter values: $a_c = 0.0225$, $d_g = 0.05$, $b_c = 0.006902$, $f_g = 0.0133$, $c_c = 0.0005$, $e_g = 0.01$.

3.0 Method of Solution

Following a similar explanation as in our earlier analysis, we have used a computational method to determine each type of stability for a system of two interacting populations in the absence of a time delay. First of all, the two carrying capacities for the interacting legumes were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of C_e and G_e which were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of C_e and $G_e$with respect to C_e and G_e were derived and evaluated at the arbitrary co-existence steady-state solution or point (C_e, G_e). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated computationally and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the other hand, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. It should also be noted that if any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting legumes can go into the ecological risk of extinction escaping survival while the other legume can tend to survive.

4.0 Results and Discussions

In the scenario when the two inhibiting parameter values do not change and the carrying capacity of groundnut slightly changes, crop scientists are interested to know how these changes will affect the co-existence theory as an ecological-agricultural idea and its subsequent effect on the qualitative behaviour of stability which is a mathematical idea. The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Tables below: the notation a_c stands for the intrinsic growth rate parameter value for cowpea, the notation css stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 1

Example	a_c	css	λ_1	λ_2	Type of stability
1	0.0225	3.1597: 1.3837	-0.0251	-0.0151	stable
2	0.0011	-0.1156:3.8463	0.0008	-0.0511	degenerate
3	0.0022	0.0567:3.7167	-0.0004	-0.0495	stable
4	0.0034	0.2291:3.5871	-0.0015	-0.0478	stable
5	0.0045	0.4015:3.4575	-0.0026	-0.0461	stable
6	0.0056	0.5739:3.3279	-0.0037	-0.0445	stable
7	0.0067	0.7463:3.1983	-0.0048	-0.0429	stable
8	0.0079	0.9187:3.0687	-0.0059	-0.0412	stable
9	0.0090	1.0911:2.9391	-0.0070	-0.0396	stable
10	0.0101	1.2634:2.8094	-0.0081	-0.0380	stable

The first row of Table 1 shows that the co-existence steady-state solution (3.1597, 1.3837) is stable having two negative eigenvalues -0.0251 and -0.0151 when the intrinsic growth rate of cowpea is 0.0225 in the unit of grams. Since the negative biomass of cowpea does not provide any biological meaning, the co-existence steady-state solution (-0.1156, 3.8463) is said to be degenerate when the intrinsic growth rate of cowpea is 0.0011 in the unit of grams. Afterwards, it is obvious that the stability of the co-existence steady-state solution is sustained when the closed interval of the intrinsic growth rate of cowpea is [0.0022, 0.0101]. Similarly, for the closed interval [0.0113, 0.0214] of the intrinsic growth rate of cowpea, their corresponding co-existence steady-state solutions are stable. This second set of results is displayed in Table 2.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 2

Example	a_c	css	λ_1	λ_2	Type of stability
11	0.0113	1.4358:2.6798	-0.0092	-0.0364	stable
12	0.0124	1.6082:2.5502	-0.0102	-0.0348	stable
13	0.0135	1.7806:2.4206	-0.0113	-0.0332	stable
14	0.0146	1.9530:2.2910	-0.0123	-0.0317	stable
15	0.0158	2.1254:2.1614	-0.0132	-0.0302	stable
16	0.0169	2.2978:2.0318	-0.0141	-0.0288	stable
17	0.0180	2.4701:1.9021	-0.0148	-0.0275	stable
18	0.0191	2.6425:1.7725	-0.0154	-0.0264	stable
19	0.0203	2.8149:1.6429	-0.0157	-0.0256	stable
20	0.0214	2.9873:1.5133	-0.0251	-0.0156	stable

For the closed interval [0.0227, 0.0317] of the intrinsic growth rate of cowpea, their corresponding co-existence steady-state solutions are stable. This third set of results is displayed in Table 3.

Table 3: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 3

Example	a_c	css	λ_1	λ_2	Type of stability
21	0.0227	3.1942:1.3578	-0.0251	-0.0150	stable
22	0.0236	3.3321:1.2541	-0.0254	-0.0143	stable
23	0.0248	3.5045:1.1245	-0.0260	-0.0132	stable
24	0.0259	3.6768:0.9949	-0.0267	-0.0119	stable
25	0.0270	3.8492:0.8652	-0.0276	-0.0105	stable
26	0.0281	4.0216:0.7356	-0.0285	-0.0090	stable
27	0.0292	4.1940:0.6060	-0.0295	-0.0075	stable
28	0.0304	4.3664:0.4764	-0.0306	-0.0059	stable
29	0.0315	4.5388:0.3464	-0.0316	-0.0043	stable
30	0.0317	4.5732:0.3209	-0.0318	-0.0040	stable

For the closed interval [0.0319, 0.0344] of the intrinsic growth rate of cowpea, their corresponding co-existence steady-state solutions are said to be dominantly stable. This fourth set of results is displayed in Table 4.

Table 4: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 4

Example	a_c	css	λ_1	λ_2	Type of stability
31	0.0319	4.6077:0.2949	-0.0320	-0.0037	stable
32	0.0322	4.6422:0.2690	-0.0323	-0.0034	stable
33	0.0324	4.6767:0.2431	-0.0325	-0.0030	stable
34	0.0326	4.7112:0.2172	-0.0327	-0.0027	stable
35	0.0328	4.7456:0.1913	-0.0329	-0.0024	stable
36	0.0331	4.7801:0.1653	-0.0331	-0.0021	stable
37	0.0333	4.8146:0.1394	-0.0333	-0.0017	stable
38	0.0335	4.8491:0.1135	-0.0336	-0.0014	stable
39	0.0338	4.8835:0.0876	-0.0338	-0.0011	stable
40	0.0340	4.9180:0.0616	-0.0340	-0.0008	stable
41	0.0342	4.9525:0.0357	-0.0342	-0.0004	stable
42	0.0344	4.9870:0.0098	-0.0344	-0.0001	stable
43	0.035	5.0215:-0.0161	-0.0346	0.0002	degenerate

In this study, we have observed that for the intrinsic growth interval [0.035, 0.0360], the co-existence steady-state solution is dominantly degenerate. This set of results is displayed in Table 5.

Table 5: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 5

Example	a_c	css	λ_1	λ_2	Type of stability
44	0.0350	5.0559:-0.0421	-0.0349	0.0005	degenerate
45	0.0351	5.0904:-0.0680	-0.0351	0.0009	degenerate
46	0.0353	5.1249:-0.0939	-0.0353	0.0012	degenerate
47	0.0356	5.1594:-0.1198	-0.0355	0.0015	degenerate
48	0.0358	5.1938:-0.1457	-0.0357	0.0018	degenerate
49	0.0360	5.2283:-0.1717	-0.0360	0.0022	degenerate

5.0 Conclusion

As in the case of a similar interaction between cowpea and groundnut, we have also observed from this study that the degeneracy of the co-existence steady-state solution has occurred when the intrinsic growth rate parameter value is 0.0011. It is interesting to mention that out of 49 empirical examples, this present carrying capacity gap degeneracy of a co-existence steady-state solution has occurred in only 7 instances while the stability of a co-existence steady-state solution is dominantly sustained 42 instances. The present carrying capacity gap does favour a key ecological theory in the sense that this analysis supports the survival of cowpea and groundnut. Since this carrying capacity supports some sort of competition between cowpea and groundnut for a limited resource in the Niger Delta agricultural setting, it is highly likely that the chance of having a degeneracy of a co-existence steady-state solution is minimised.

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