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Stability analysis for a system of interacting populations with a similar carrying capacity

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Abstract

The purpose of this present contribution is to illustrate the use of a computational method to determine the stability for a system of two interacting legumes such as cowpea and groundnut for a limited resource in a Niger Delta Region agricultural setting. The optimal regions of stability and degeneracy for several co-existence steady-state solutions were obtained. The results which we have obtained and have not seen elsewhere are presented here and discussed.

Keywords: co-existence steady-state solution, stability, degeneracy, interacting legumes

1.0 Introduction

The concept of stability as it applies to a system of interacting populations without a delay term has a long standing history within the mathematical literatures [1- 6]. In this context, the analytical aspect of using the process of linearization in the neighbourhood of an arbitrary steady-state solution to determine the stability of this steady-state solution is not a new mathematical formulation. Despite the application of the analytical method of studying the stability of a steady-state solution, this method can have some setbacks when it comes to its use in studying the stability for interacting populations. This limitation which does not look very obvious concerns the weakness of the analytical method in handling agricultural data involving changes in the intrinsic growth rate. It is against this background that we propose a computational method which is capable to handle bigger data sets with a minimum computational time and its computational efficiency. The data which this analysis has used were based on the research contribution of Ekpo and Nkanang [7].

2.0 Mathematical Formulation

Following Ekaka-a [5], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a_c - b_c C(t) - c_c G(t)] \quad (1)$$

$$\frac{dG(t)}{dt} = G(t)[d_g - f_g G(t) - e_g C(t)] \quad (2)$$

where C(0) > 0 and G(0) > 0 define the starting biomasses of cowpea and groundnut at the start of the growing season otherwise called the initial conditions when t = 0. The duration of growth is in the unit of days hereby denoted by the independent variable t. For the purpose of this simulation study, thebest-fit model parameters such as a_c and d_g that define the intrinsic growth rates for cowpea and groundnut were selected. The next best-fit parameters such as b_c and f_g define the intra-species competition parameters which measure the inhibiting factors on the growth of cowpea and groundnut due to self-interaction whereas the parameters c_c and e_g define the inter-species competition parameters which also measure other inhibiting factors on the growth of cowpea and groundnut due to interspecific interaction between cowpea and groundnut. In this study, we have considered the following parameter values: $a_c = 0.0225$, $d_g = 0.0446$, $b_c = 0.006902$, $f_g = 0.0133$, $c_c = 0.0005$, $e_g = 0.01$.

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3.0 Method of Solution

In this present study, we have used a computational method to determine each type of stability for a system of two interacting populations in the absence of a time delay. First of all, the two carrying capacities for the interacting legumes were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of C_e and G_e which were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of C_e and $G_e$with respect to C_e and $G_e$were derived and evaluated at the arbitrary co-existence steady-state solution or point (C_e, G_e). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated computationally and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the other hand, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. It should also be noted that if any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting legumes can go into the ecological risk of extinction escaping survival while the other legume can tend to survive.

4.0 Results and Discussions

The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Tables below: the notation a_c stands for the intrinsic growth rate parameter value for cowpea, the notation css stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 1

Example	a_c	css	λ_1	λ_2	Type of stability
1	0.0225	3.1908: 0.9543	-0.0234	-0.0113	stable
2	0.0011	-0.0845:3.4169	0.0006	-0.0454	degenerate
3	0.0022	0.0878:3.2873	-0.0006	-0.0438	stable
4	0.0034	0.2602:3.1577	-0.0017	-0.0421	stable
5	0.0045	0.4326:3.0281	-0.0028	-0.0404	stable
6	0.0056	0.6050:2.8985	-0.0039	-0.0388	stable
7	0.0067	0.7774:2.7689	-0.0050	-0.0372	stable
8	0.0079	0.9498:2.6393	-0.0061	-0.0355	stable
9	0.0090	1.1222:2.5097	-0.0072	-0.0339	stable
10	0.0101	1.2945:2.3800	-0.0083	-0.0323	stable

The first row of Table 1 shows that the co-existence steady-state solution (3.1908, 0.9543) is stable having two negative eigenvalues -0.0234 and -0.0113 when the intrinsic growth rate of cowpea is 0.0225 in the unit of grams. Since the negative biomass of cowpea does not provide any biological meaning, the co-existence steady-state solution (-0.0845, 3.4169) is said to be degenerate when the intrinsic growth rate of cowpea is 0.0011 in the unit of grams. Afterwards, it is obvious that the stability of the co-existence steady-state solution is sustained when the closed interval of the intrinsic growth rate of cowpea is [0.0022, 0.0101]. Similarly, for the closed interval [0.0113, 0.0214] of the intrinsic growth rate of cowpea, their corresponding co-existence steady-state solutions are stable. This second set of results is displayed in Table 2.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 2

Example	a_c	css	λ_1	λ_2	Type of stability
11	0.0113	1.4669:2.2504	-0.0093	-0.0307	stable
12	0.0124	1.6393:2.1208	-0.0103	-0.0292	stable
13	0.0135	1.8117:1.9912	-0.0113	-0.0277	stable
14	0.0146	1.9841:1.8616	-0.0122	-0.0262	stable
15	0.0158	2.1565:1.7320	-0.0130	-0.0249	stable
16	0.0169	2.3289:1.6024	-0.0136	-0.0237	stable
17	0.0180	2.5013:1.4727	-0.0140	-0.0229	stable
18	0.0191	2.6736:1.3431	-0.0224	-0.0139	stable
19	0.0203	2.8460:1.2135	-0.0224	-0.0134	stable
20	0.0214	3.0184:1.0839	-0.0228	-0.0125	stable

For the closed interval [0.0227, 0.0317] of the intrinsic growth rate of cowpea, their corresponding co-existence steady-state solutions are stable. This third set of results is displayed in Table 3.

Table 3: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 3

Example	a_c	css	λ_1	λ_2	Type of stability
21	0.0227	3.2253:0.9284	-0.0236	-0.0110	stable
22	0.0236	3.3632:0.8247	-0.0243	-0.0099	stable
23	0.0248	3.5356:0.6951	-0.0252	-0.0085	stable
24	0.0259	3.7080:0.5655	-0.0262	-0.0070	stable
25	0.0270	3.8803:0.4358	-0.0272	-0.0054	stable
26	0.0281	4.0527:0.3062	-0.0282	-0.0038	stable
27	0.0292	4.2251:0.1766	-0.0293	-0.0022	stable
28	0.0304	4.3975:0.0470	-0.0304	-0.0006	stable
29	0.0315	4.5699:-0.02826	-0.0315	0.0010	degenerate
30	0.0317	4.6044:-0.1085	-0.0317	0.0014	degenerate

For the closed interval [0.0319, 0.0340] of the intrinsic growth rate of cowpea, their corresponding co-existence steady-state solutions are said to be dominantly degenerate. This fourth set of results is displayed in Table 4.

Table 4: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate of cowpea: summary of results 4

Example	a_c	css	λ_1	λ_2	Type of stability
31	0.0319	4.6388:-0.1345	-0.0319	0.0017	degenerate
32	0.0322	4.6733:-0.1604	-0.0321	0.0020	degenerate
33	0.0324	4.7078:-0.1863	-0.0324	0.0024	degenerate
34	0.0326	4.7423:-0.2122	-0.0326	0.0027	degenerate
35	0.0328	4.7767:-0.2382	-0.0328	0.0030	degenerate
36	0.0331	4.8112:-0.2641	-0.0330	0.0033	degenerate
37	0.0333	4.8457:-0.2900	-0.0333	0.0037	degenerate
38	0.0335	4.8802:-0.3159	-0.0335	0.0040	degenerate
39	0.0338	4.9147:-0.3418	-0.0337	0.0043	degenerate
40	0.0340	4.9491:-0.3678	-0.0339	0.0047	degenerate

5.0 Conclusion

We observe from this study that the degeneracy of the co-existence steady-state solution has occurred when the intrinsic growth rate parameter value is 0.0011. The degeneracy of the co-existence steady-state solution is one of the primary characteristics of an ecological system which presents some sort of warning signs for ecologists to avoid in the bid to plan and manage ecosystem functioning and stability. In this computational study, we have found that out of fourty empirical examples, degeneracy of a co-existence steady-state solution has occurred in thirteen instances while valid stable co-existence steady-state solution has occurred in twenty seven instances for this similar type of interaction. Therefore, the proper management of the co-existence steady-state solution should be put in place to avoid the inherent degeneracy of this vital steady-state solution. In our next study, we intend to study the implications of this present contribution in terms of a dis-similar interaction between cowpea and groundnut.

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