Journal of the Nigerian Association of Mathematical Physics
Volume 25, No. 2 **(November, 2013), pp 337 – 344**
© J. of NAMP

The relationship between electron density and electron temperature near the magnetic equator

E. E. Iheonu and E.O. Somoye

Department of Physics
Lagos State University
Lagos, Nigeria

Abstract

The distribution profiles of electron velocity, electron density and electron temperature have been used to study the thermal structure of the ionospheric F-region over Ibadan (7.4^{0}N, 3.9^{0}E; magnetic dip 6.0^{0}S), Nigeria at an altitude of 300 km using a modified composition of the electron density continuity equation. Observations suggest that the magnitudes of daytime electron temperature at Ibadan are solar activity independent. The overall seasonal electron temperature in the neighbourhood of 3000K is in excess of the mean results reported by some workers but also in good agreement with other observations near the magnetic equator. The interplay between photoionisation and thermal conduction in the determination of the heat budget of the electron gas temperature has been discussed to a reasonable extent.

1.0 Introduction

The most important ionospheric parameter from the point of view of radiowave propagation and also of physics is the electron density. The behaviour of electrons in the ionosphere is largely governed by the Earth's magnetic field. The equatorial ionosphere and thermosphere constitute a coupled system, with its electrodynamical and plasma physical processes being responsible for a variety of ionospheric phenomena peculiar to the equatorial zone.

Diurnal and seasonal profile distributions with respect to the altitude of electron density and temperature are of particular interest in the study of the equatorial ionosphere

Over the last few decades, measurements of the diurnal and long-term behaviour of the exospheric temperature as observed by incoherent scatter sounding in the F_2- region [1] and the effects of geomagnetic disturbances on neutral winds and temperature in the thermosphere observed at locations near the mid-latitude [2], have been carried out.

Several authors have also studied the F-region of the ionosphere based on their conception of the relationship between the electron density, solar activity and ionospheric temperature. Some of the studies have concluded that the temperature of the ionosphere in relation to electron density N_e, may be represented by an equation of the form [3]

$$T_e - T_i = A - \beta N_e \qquad (1)$$

where T_e is the electron temperature, T_i is the ion temperature while A and B are constant parameters. Equation (1) was initially an empirical formulation based on measurements of temperature and electron density.

Variations of electron density with height h and time, t and with latitude and longitude may be better represented either by purely descriptive empirical models, in which mathematical formulae are constructed to match the observed variations of N_e, or else by physical models generated by solving the theoretical continuity equation for electron density represented in the form:

$$\left.\begin{aligned} \frac{\partial N}{\partial t} &= \text{(Production)} - \text{(Loss)} - \text{(Transport)} \\ &= q-L-M \end{aligned}\right\} \qquad (2)$$

Corresponding author: ***Ernest Emeagi Iheonu,*** E-mail: eeiheonu@yahoo.com, Tel.: +2348052329349

On the other hand, the heat equation of the form

$$\frac{\partial T}{\partial t} = \text{(Heating)} - \text{(Cooling)} - \text{(Conduction)} \qquad (3)$$

Is also relevant to the upper atmosphere, incorporating temperatures of ions and the electrons as well as corresponding equations for the neutral particles. Several other theoretical explanations have been suggested which assumed that local production and loss dominated over conduction which led to the expressions involving $T_e - T_i$. In reference to equation (1), we note that the parameters A and B may seem to vary with solar activity. Reasoning along this line, Jones [4] used data taken at mid-day to establish an empirical relationship of the form:

$$T_e - T_i = A - BN_0 + CS \qquad (4)$$

where C is a new constant and S is a parameter representing solar activity. Solar activity is often used to denote either the daily values of solar flux density at a wavelength of 10.7 cm ($S_{10.7}$) or the daily values of the Zurich sunspot number, R_Z.

It is appropriate at this moment to examine the effects of temperature on the equatorial ionosphere in relation to vertical drift and electron density with a view to addressing some of the phenomenological model of global ionospheric electron density and the thermospheric dynamics.

Since theoretical techniques have been used to attain good results as those using satellite and radar measurements in the determination of drifts [5,6], it appears logical thus, to attempt the determination of the thermal structure of the daytime equatorial ionosphere near Ibadan, comparing the results with similar studies carried out at Radio observatories elsewhere. Examining further, the relationship between electron density and electron temperature, in assessing the heat budget of the electron gas, subject to solar flux conditions, might add some innovations to thermal distribution study near the magnetic equator.

2.0 Theoretical Framework

2.1 Consideration of the electron density continuity equation

The major equation of interest in this study is the well-known continuity equation for electron density written mathematically in the form:

$$\frac{\partial N_e}{\partial t} = q(h,\chi) - \beta N_e(h,t) - \nabla \cdot \{N_e(h,t)\mathbf{v}\} = 0 \qquad (5)$$

where $q(h,\chi)$ is the rate of electron production per unit volume, a function of both height and solar zenith distance, χ. β is the electron loss coefficient, a function of height, **v** is the drift velocity vector. The velocity vector **v** may be that of either electron or ion (i.e., $\mathbf{v}_e$ or $\mathbf{v}_i$), without assuming these to be the same. The divergence term is a function representing electron transport process in the ionosphere.

While the height of transition from predominantly O^+ to He^+ will depend on local time and latitude, consideration of the order of magnitude indicates that at least up to 600 km, it can be safely assumed that O^+ predominates. Hence, the ionisable constituent may be taken to be O atoms only, and the simple theory of Chapman [7] may be applied. The rate of production of ions or electrons may therefore be assumed to be given by the Chapman formula

$$q = q_0 \exp[1 - z - e^{-z} Ch(R,\chi)] \qquad (6)$$

where q is the production rate, q_0 is the maximum production rate for overhead sun, defined by Ratcliffe and Weekes (1960) as:

$$q_0 = 250(1 + 16 \times 10^{-3} \overline{R}) \quad \text{cm}^{-3}\text{sec}^{-1}; \qquad (6a)$$

z is the height, measured from a datum level h_0 corresponding to q_0 in units of scale height of the ionisable constituent, and $Ch(R,\chi)$ is a function developed by Chapman to take account of the sphericity of the atmosphere. R is the radial distance in scale height, and is equal to the distance measured in terms of the scale height of the ionisable constituent as unit of length, from the earth's centre to the level at which, at noon at the equator, the absorption of radiation is a maxima.

For daytime analysis which is primarily of interest in the present study, we rearrange the continuity equation (5), as follows:

$$\frac{\partial N_e(h,t)}{\partial t} = q(h,\chi) - \beta N_e(h,t) + \frac{vN_e(h,t)}{H_e} \qquad (7)$$

where
$\beta Ne\ (h,t) is\ the\ \ electron\ loss\ term = L\ (N_e)$.

$\frac{\partial N_e(h,t)}{\partial t}$ is assumed negligible in the daytime near the equator [5,8].

The transport term

$$M = \frac{vN_e(h,t)}{H_e} \tag{8}$$

while the electron density scale height,

$$H_e = \frac{kT_e}{M_e g} \tag{8a}$$

Ordinarily, close to about 100km altitude in the ionosphere, the gases nitrogen (N_2) and Oxygen (O_2) are well mixed by wind and turbulence and their concentration ratio of 4:1 does not change with height. Above 100km however, oxygen largely becomes dissociated into atoms by the action of solar UV radiations. Secondly, turbulence ceases at the turbopause near 100km and the gases are diffusively separated by the action of gravity so that atomic oxygen progressively becomes more abundant than molecular nitrogen and oxygen. These factors assist in shaping the criteria used in determining electron temperature profiles at locations near the magnetic equator. In any plasma where electrical neutrality exists, atomic oxygen becomes equal to electron density [9, 10].

The simple equation for resolving the electron temperature profile can be written in the form:

$$T_e = 19.2464 \times 10^{-3} \left[\frac{Ve.Ne\ (h,t)}{\beta Ne - q}\right] \tag{9}$$

All the quantities in the equation have been well-defined.

H_e = electron density scale height

$N_e(h,t)$ = electron density in electron m^{-3}; a function of the ionospheric height and time

k = Boltzmann's constant = 1.38×10^{-23} JK^{-1}

g = gravitational acceleration; taken numerically as $10ms^{-2}$ in this study.

M_e = electron mass (= 1.66×10^{-27}kg) expressed in unified atomic mass unit (amu):

The atomic mass for oxygen is ~ 15.9994 (amu)

Chapman's expression for q was used with peak production $q_0 = 620$ $cm^{-3}sec^{-1}$ (for a typical year of high solar activity.

According to Ratcliffe and Weekes [11] the loss term, $L(N_e)$ has various forms at different ionospheric altitudes, which is slightly modified in this study as follows:

$$L(N_e) = \begin{cases} \beta Ne & F_2 - region & (10) \\ \frac{\propto \beta N_2}{\beta + \propto N_e} & F_1 - region & (10a) \\ \propto N_{e^2} & E - region & (10b) \end{cases}$$

Here, α is the effective recombination coefficient, β is the attachment coefficient which for both daytime and/or nighttime analysis, may take the form:

$$\beta = 10^{-4} \exp\left(\frac{300 - h[km]}{50}\right) sec^{-1} \tag{11}$$

In the intermediate F_1-region, with both quadratic and linear recombination, however, the general loss term (10a) has to be used. The transition from the region of recombination (10b) to the region of attachment (10) is at the altitude of about 200 km. Details of the loss processes may be found in the literature: [5, 11, 12].

The analytical form of the transport term will be considered in due course.

3.0 Data

Ground-based ionosonde observations acquired at Ibadan, Nigeria were used. Data acquired were good enough to be scaled while avoiding magnetic storm days. The records were used to obtain electron density profiles between 06:00 and 18:00 LT. The theoretical model calculations obtained by simplifying the continuity equation for electrons were used to estimate the vertical **E x B** drifts and hence electron temperature variability during the period. The data were grouped into 4-month segment as follows:

December solstice (November, December, January and February)
Equinox (March-April; September -October)

June solstice (May, June, July and August)

Generally, electron density, N_e (in cm^{-3}) is calculated using the expression

$$N_e = 1.24 \times 10^4 (f_N{}^2) \tag{12}$$

where f_N is the plasma critical frequency (in MHz).

To solve equation (9) for T_e at Ibadan, data for 300km were used.

4.0 Results and discussion

4.1 Equatorial electrodynamics vertical drifts

In Fig. 1, observations show that the mean drift velocities at June Solstice, Equinox and December Solstice are of the order of 5 ms^{-1}, 7 ms^{-1} and 6 ms^{-1} respectively at Ibadan, during the period of high solar activity. The corresponding drift results (not shown) for the period of low solar activity are of the order of 4 ms^{-1}, 5 ms^{-1} and 4 ms^{-1}, respectively. The drift results suggest that drift velocity is independent of solar activity, since the emergence of classed distinction is not really visible.

Similar studies carried out at Jicamarca (magnetic dip 2.0 ^{0}N), Peru, at a period of high solar activity (inferred results from the work of Huang [13]), show that the mean drift velocities of the order of 13 ms^{-1}, 13 ms^{-1} and 9 ms^{-1}, were obtained for June Solstice, Equinox and December Solstice, respectively. In another development, Fejer et al. [14] also working at the same Jicamarca observatory, reported drift results of the order of 14 ms^{-1}, 16 ms^{-1} and 11 ms^{-1}, for June Solstice, Equinox and December Solstice, respectively.

The above results are also in good agreement with the findings of Scherliess and Fejer [15] and thus give a firm indication of an equation of continuity for electrons that could probably provide a veritable means of determining electron temperature – an envisaged intrinsic component of the transport processes in the equatorial ionosphere.

4.2 Deductions of electron temperature

Following similar procedures used in estimating vertical drift velocity at Ibadan, the distribution profiles of electron temperature have also been deduced using equation (9) and other relevant, inclusive relationships. The temporal variations of temperature at Ibadan (see Fig. 2) have been compared with the variations at altitudes near 300 and 400 km, inferred from the work of Zhou and Sulzer [16] at Arecibo (magnetic latitude 30 ^{0}N), Puerto Rico.

The overall mean daytime electron temperature at 300 km and 400 km from the results of Zhou and Sulzer are of the order of 1700 K and 2200 K, respectively.

The result in the present study at the height of 300 km shows that the mean electron temperature is of the order of 3000 K. Although values of the electron temperature in this study seem to be slightly off-mark when compared with the temperature variation from the work of Zhou and Sulzer the magnitudes of the observed results of these workers near 600 km are in good agreement with the Ibadan results at 300 km. Using data obtained from the Pioneer Venus Instrument Complement, Knudsen et al. [17] showed that electron temperatures within the height regime of 200 – 500 km ranged from about 2600K to 4000K.

The discrepancies observed in the results above may be accounted for, by the different techniques employed. The results of Zhou and Sulzer [16] are particularly of interest because it is possible that the records made during the "disturbed days" as indicated in their report, may have adversely affected the overall electron density and/or the temperature profiles. It is also possible that the database used for the study was sparse for the kind of comparison required in studies of this nature. However, the observations by Abe et al. [18] and Schunk et al. [19] are in excellent conformity with the present Ibadan daytime results.

At the sunlit trough regions of the ionosphere, Schunk et al for instance, observed that the electron temperature hotspots showed a marked UT variation by as much as 2500 K. The dominant parameter controlling T_e variations according to these workers, is the magnetospheric heat flux into the ionosphere.

Generally, temperature enhancements are regarded as being due to higher electron density increasing energy dissipation from electrons to ions and neutral gases [20, 21, 22], but the enhancement of electron temperature reported by Sharma et al [23] over low-latitudes was attributed to the production of UHF gamma radiation during a lightning sprite.

4.3 Discussions on the relation between T_e and N_e

In this section, we examine the relationship between the electron temperature and electron density with a view to establishing their individual roles and/or overall contribution to the heat budget of the electron gas temperature.

Fundamentally, N_e and T_e relationships are such that photoionisation will dominate thermal conduction as a heating source for electrons if increasing N_e corresponds to diminishing T_e, Secondly, if heat conduction is the main heating source for electron gas, then increasing N_e will correspond to increasing T_e [17]. The daytime variations of N_e and T_e (Fig.3) do not strictly exhibit any of these distinct correlations under the prevailing sunspot condition as demonstrated in another research report during the sunrise period (research reports not yet published). The lack of absolute correlation during the daytime may imply that near 300 km at Ibadan, the distinguishing characteristics of the interplay between heat conduction and photoionisation in the determination of heat budget of the electron gas temperature, may be resolved through some other analytical measures.

In an attempt to establish the source of heating of the electron gas in the daytime somewhat, we employed the electron energy balance equation which is written in the general form [16,22]:

$$\frac{3}{2} N_e K \frac{\partial T_e}{\partial t} = Q_e + Q_c - \alpha C_1 N_e^2 \frac{(T_e - T_i)}{T^{3/2}} - \alpha_{cn} N_e (T_e - T_i) \qquad (13)$$

where Q_e and Q_c represent local electron heating and thermal conduction rate respectively, and T_n is the neutral atmospheric temperature. The third term on the right hand side of equation (13) represents the energy loss to ambient ions and the last term is the energy loss to the neutrals.

The electron temperature in the F-region can thus be regarded as being in a thermal quasi-steady state so that its derivative can be neglected [22]. Furthermore, electrons mainly lose their energy to ions above 220 km and to neutrals below 220 km [24]. In this case, the electron energy balance equation above 220 km becomes:

$$\alpha_c T_e^{3/2} + Q_e - \alpha C_1 N_e^2 \frac{(T_e - T_i)}{T^{3/2}} = 0 \qquad (14)$$

where α_c is a positive constant.

Equation (14) can be rewritten as:

$$\alpha_c T_e^{3/2} + \beta N_e - \alpha C_1 N_e^2 \frac{(T_e - T_i)}{T^{3/2}} = 0 \qquad (15)$$

where the photoionisation rate is linearly proportional to N_e. We can also assume that $\frac{\partial T_e}{\partial N_e} >> \frac{\partial T_i}{\partial N_i}$,

since the temporal variation of $T_i << T_e$.

If photoionisation (the second term) in equation (15) dominates thermal conduction (first term), we have:

$$\frac{\partial T_e}{\partial N_e} = \frac{2T_e (T_e - T_i)}{N_e (3T_i - T_e)} \qquad (16)$$

On the other hand, if thermal conduction dominates photoionisation, we have:

$$\frac{\partial T_e}{\partial N_e} = \frac{2T_e (T_e - T_i)}{N_e (2T_e - 3T_i)} \qquad (17)$$

At great heights above 250 km, $T_e >> T_i$ [16, 17], so that the right-hand side of equation (16) is always negative and that of equation (17) is always positive.

In our efforts to further analyse and distinguish the main heating source of the electron gas during daytime, we developed a simple equation relating T_e and N_e using data for Ibadan under high solar flux conditions. We obtained a correlation coefficient less than 0.1, signifying that the exclusion of the intrinsic parameters such as electron velocity, the electron production rate and the electron loss coefficient rates may lead to some results that might negate the entire processes of determining electron temperature profiles across the seasons.

The variations of T_e and N_e from the plots [Figure 3 (a) and (b)], were also not very distinct. The determination of the most dominant factor controlling the source of heating of the electron gas via either photoionization or thermal conduction during daytime at 300km, may thus require some additional verifications through extensive treatment of the dynamical processes near the magnetic equator.

5.0 Conclusion

We have reported a comprehensive study on the thermal structure of the equatorial ionosphere over Ibadan, Nigeria, using a modified version of the electron density continuity equation.

On a preliminary note, we found out that the daytime electron velocities are comparable with drift results obtained at Jicamarca, Puerto Rico, and other stations as reported by some workers.

The average daytime electron temperature at 300 km ionospheric height over Ibadan has a value of the order of 3000 K, contrary to the observations of Knudsen et al. [17] and Zhou and Sulzer [16]. Our sunrise results in a preliminary work show that photoionisation and thermal conduction were both distinct and hence essential in determining the heat budget of the electron gas temperature. In the daytime however, our findings which tend to suggest that heating of the daytime equatorial ionosphere may probably be due to photoionisation of the electron gas were somewhat cumbersome and hence inconclusive. Further studies may be carried out to resolve this issue.

Several studies[5, 6 25], including the present study have erased the impression given by Awojobi [26]that the transport term may be excluded in the resolution of the continuity equation in toto, citing complexities that might arise due to the magnitudes of "certain parameters" that are not well known; such that conclusions drawn from the solution of the equation are likely to be unrealistic.

On the contrary, the beauty of solving the full continuity equation for electron density and in particular, the contribution of the transport term in the analysis leading to the deduction of electron temperature, has revealed a new innovation in the study of the dynamics of the equatorial ionosphere.

Acknowledgment

The authors wish to thank the Physics Department, University of Ibadan, Nigeria, for providing the data used for this study.

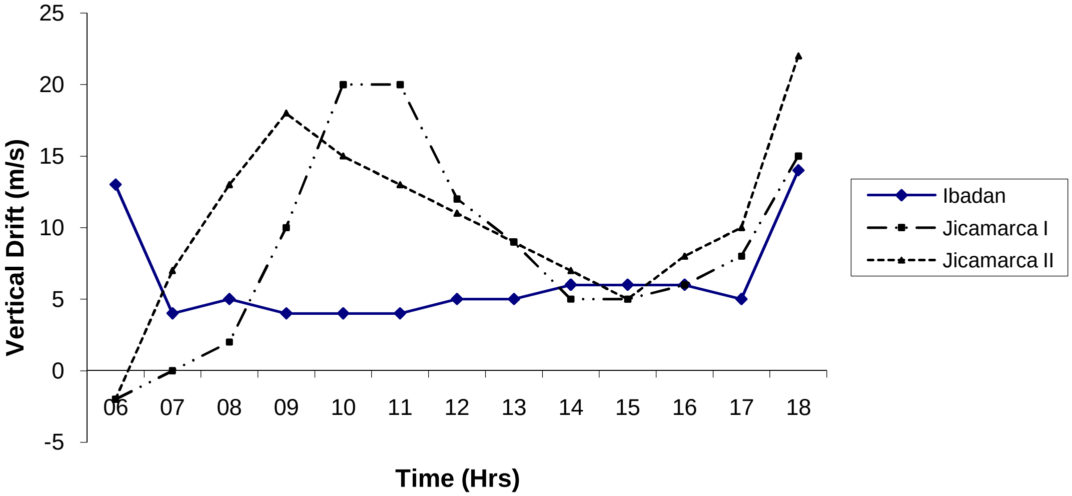

Fig. 1: Comparison of averaged Ibadan and Jicamarca F-region vertical drifts during the periods of high solar flux index under magnetically quiet conditions. The Jicamarca results were inferred from discrete points from the diurnal variations of drifts [after Chun-Ming Huang (1974) – Jicamarca I and Fejer et al., 1995 – Jicamarca II]

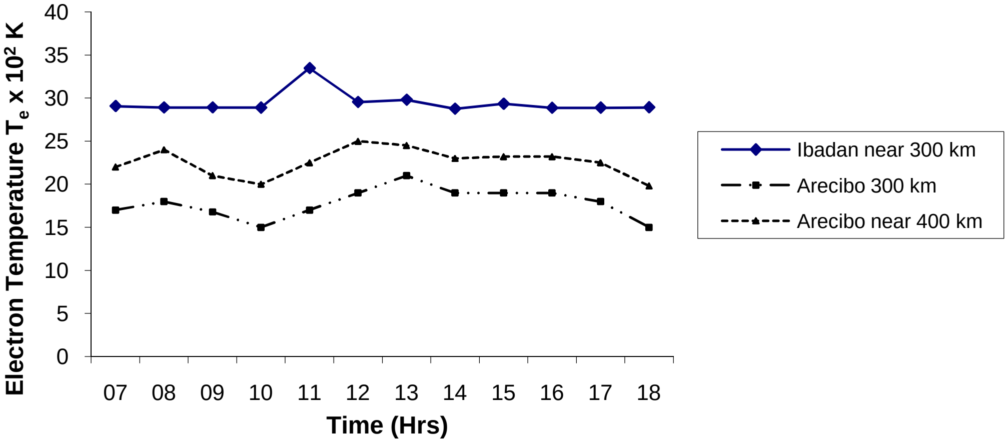

Fig. 2: Overall average variations of electron temperature at two low-latitude stations

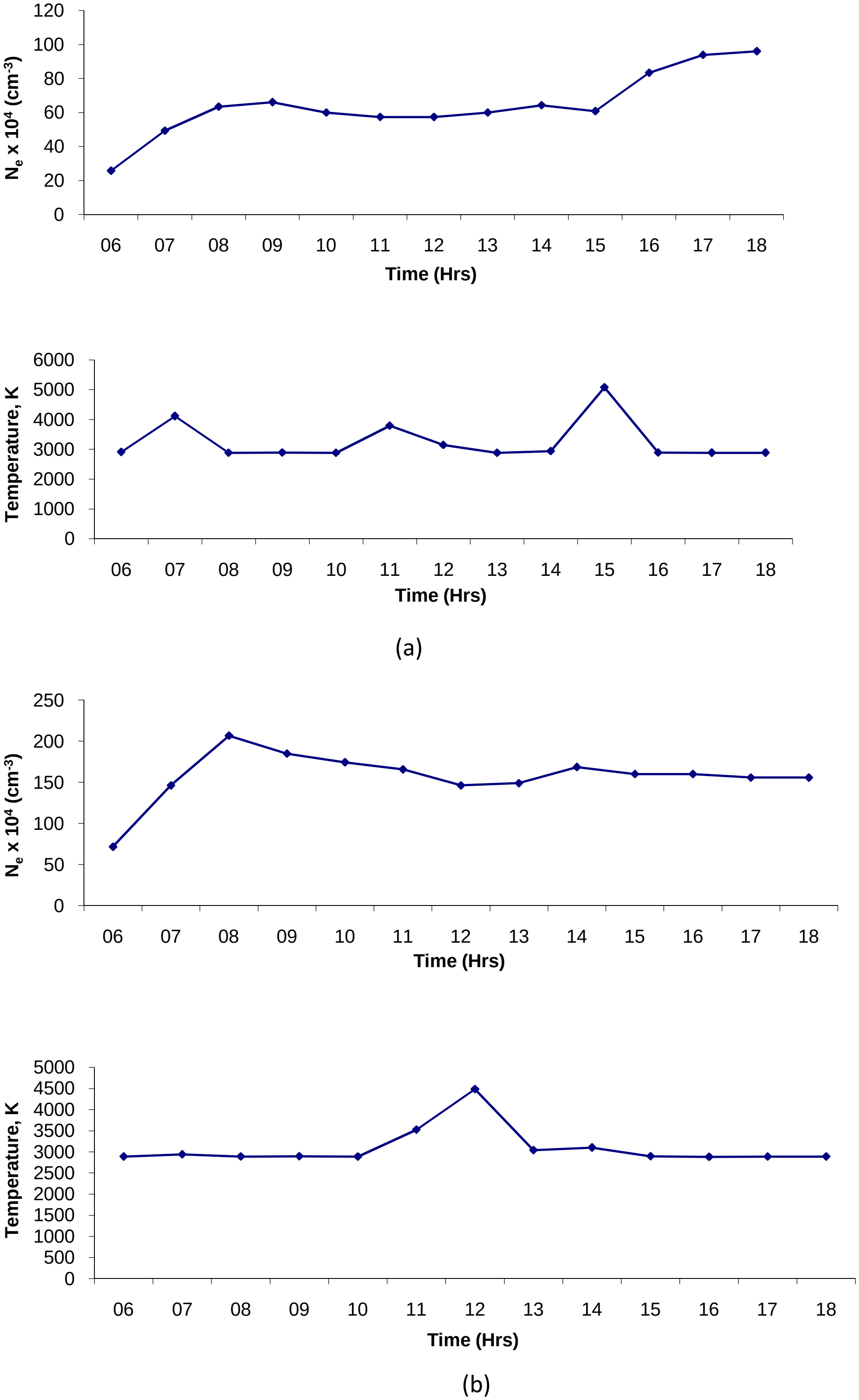

Fig. 3: Variations of N_e and T_e at June Solstice

(a) Period of low solar activity
(b) Period of high solar activity

References

[1] Alcayde, D. (1974). Decemal and Long-term behaviour of the exospheric temperature as observed by incoherent scatter sounding in the F2 – region, *Radio Sci*., 9, 239 – 246.

[2] Babcock, R. R. and Evans, J. V. (1979). Effects of geomagnetic disturbances on neutral winds and temperatures in the thermospheres observed over Millstone Hill. *J. Geophys. Res*., 84, 5347 – 5354.

[3] McDonald, J. N. and. Williams, P. J. S (1980), The relationship between ionospheric temperature, electron density and solar activity, *J. Atmos. Terr. Phys*. 42, 41-44.

[4] Jones, H. W. W. (1976). MSc Thesis, U. C. W. Aberystwyth

[5] Iheonu, E. E. and Oyekola, O. S. (2006). Vertical drift velocity in the daytime F-region at Ibadan estimated from ionosonde data, *Indian J. Radio Space Phys*., 35, 9-13.

[6] Iheonu E. E. and Somoye, E. O. (2007). The relationship between electron density and electron temperature near the magnetic equator. A paper presented at the 30th Annual Conference of the Nigerian Institute of Physics held at the Lagos State University, Lagos 15th – 18th August, 2007. Abstract No. NIP-LASU-B30.

[7] Chapman, S. (1931). "The absorption and dissociative or ionising effect of rotating monochromatic radiation in an atmosphere on a rotating earth." *Proceedings of the Physical Society*, 43, 26-45; 484.

[8] Picquenard, A. (1974). Radio wave propagation (Macmillan Press Ltd., London)

[9] Olatunji, E. O. (1964). PhD Thesis, University of London, UK.

[10] Iheonu E. E. (1992). Equatorial ionospheric F. Region at sunrise. M.Sc Thesis, University of Lagos, Akoka-Nigeria.

[11] Ratcliffe J. A. and Weekes, K. (1966). Physics of the Upper Atmosphere, J. A. Ratcliffe, ed. (Academic Press, New York).

[12] Vellinov, P. I. and Teniv, P. T. (1994). Penetration of multipole thundercloud electric fields into the ionosphere. *J. Atmos. Terr. Phys*. 56, 349.

[13] Huang, C. (1974). On the Ionisation Transport in the Low-latitude Ionosphere: J. Atmos. Terr. & Phys. (UK), 36, 43.

[14] Fejer, B. G., de Paula, E. R. Heelis, R. A. and Hanson, W. B (1995). Global equatorial ionosphric vertical plasma drifts measured by AE-E satellites, *J. Gephys. Res*. 100, 5769-5776.

[15] Scherliess, L., and. Fejer, B. G (1999), Radar and satellite global equatorial F-region vertical drift model, *J. Geophys. Res*. 104, 6829-6842.

[16] Zhou, Q. H. and Sulzer, M. P. (1997), Incoherent scatter radar observations of the F-region ionosphere at Arecibo during January, 1993. *J. Atmos. Solar – Terr. Phys*. 59 (17), 2213-2229.

[17] Knudsen, W. C., Spenner, K., Whitten, R. C., Spreiter, J. R.. Miller, K. L. and Novalc, V. (1979). Thermal structure and Energy Influx to the Day – and Nightside Venus Ionosphere, *Science*, 205, 105 – 107.

[18] Abe, T. Balan, N. Oyama, K.I and Bailey, G.J. (1997). Plasmasphere electron temperature: observations and theory. Adv. Space Res. 20(3), 401-405.

[19] Schunk, R. W., Sojka, J. J. and Bowline, M. D. (1986), Theoretical study of the electron temperature in the high latitude ionosphere for solar maximum and winter conditions. J. Geophys. Res., 91, 12041-12054.

[20] Rishbeth, H. and Garriot, O. K. (1969). *Introduction to Ionospheric Physics*. Academic Press, 138-139.

[21] Banks, P. M. and Kockcarts, G. (1973). *Aeronomy*, Academic Press, New York; part B.

[22] Schunk, R. W. and Nagy, A.F. (1978). Electron temperatures in the F-region of the ionosphere: Theory and observations, Rev. Geophys. And space Phys.-16, 355-399.

[23] Sharma, D.K. Rai, J., Israip, M., Subrahmanyan, P., Chopra, P. and Garg, S.C. (2003). Sunrise effect on ionospheric temperature as measured by SROSS-C2 satellite. J. Indian Geophys. Union 7, 117-123.

[24] Perkins, F.W. and Roble, R.G. (1978). Ionospheric heating by radio waves: Predictions for Arecibo and the satellite power station.. J. Geophys. Res. 83(A4), 1611-1624.

[25] Oyekola O. S. and Iheonu, E. E. (2006), The F-Region Equational Ionospheric Electrodynamics Drifts, *J. Nig. Assoc. Maths. Phys*. 10, 27-34.

[26] Awojobi, O. A. (1966). Recombination coefficient and the pre-noon maximum of electron density in ionospheric F_2-region. *J. Atmos. Terr. Phys*., 28, 375-384