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Equation of State for Nuclear Matter Based on New One Boson (NOB) Interaction

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Abstract

Four different density dependent versions ($BDNOB_0$, $BDNOB_1$, $BDNOB_2$, and $BDNOB_3$) of newly developed one Boson interactionshave been used in numerical calculation of incompressibility of symmetric nuclear matter. The values obtained for the incompressibility from this calculation vary from 192 to 447 MeV. This result is in excellent agreement with the values of the incompressibility obtained in a similar calculation using the M3Y – Reid and M3Y – paris interactions in which the incompressibility vary from 190 to 454 MeV for the M3Y – Reid and 218 to 566 MeV for M3Y – paris interactions respectively. The NOB interaction by this result has proven to be effective for nuclear matter studies. Hence more calculations need to be done to further test the performance of the NOB interaction.

1.0 Introduction

Nuclear matter (NM) can be described as an idealized system of nucleons interacting without Coulomb force which is translationally invariant with fixed ratio of neutrons to protons [1]. Nuclear matter theoryis aimed at obtaining an equation of state (EOS) for nuclear matter starting from the underlying two – body nucleon – nucleon (NN) interaction. Bulk properties of nuclear matter such as the nuclear incompressibility, the energy density, the pressure, the velocity of sound in nuclear medium etc, can be obtained using nuclear EOS, which is the energy per nucleon $E/A = \epsilon$ of nuclear matter as a function of nucleonic density ρ.The EOS is also found to play a significant role in the nuclear ground state properties as, stability of neutron-rich nuclei, excitation energies of giant monopole resonances (GMR), dynamics of heavy-ion collisions, structure of neutron stars as well as the simulation core collapse supernova [2,3]. The existence of the relation between the EOS and the density dependence of the effective NN interaction is commonly accepted.

The most widely used interaction for nuclear matter studies is the effective M3Y- Reid or Paris interaction [4]. This interaction was derived by fitting its matrix elements in oscillator basis to those elements of the G-matrix obtained with the Reid - Elliot soft – core NN interaction [1].

Recently [5], a similarly motivated interaction was derived based on the lowest order constrained variation (LOCV) approach [5,6,7].This interaction has also proven to be effective for nuclear matter study in the sense that, it reproduces the binding energy of NM of -16MeV at a saturation nuclear density of 0.17fm^{-3}[8]. In [9], the interaction has been used to calculate the incompressibility K_0 of nuclear matter. The result obtained from this calculation is in close agreement to that of common version of interactions which are based on G – matrix calculation.

In the present work, we have used yet another newly developed one boson interaction [10] for nuclear matter studies to calculate incompressibility of NM.This interaction is also derived based on the LOCV techniques. This paper is organized as follows: In section 2, we describe the theoretical formulism of the method used. In section 3, the results of the present calculations and the discussion of the results are presented. Section 4 gives the conclusion.

2.0 Theoretical Formalism

In the standard Hartree – Fock (HF) calculations, the total ground-state energy of the cold NM is the sum of kinetic and the potential energy contribution [11]. In terms of the spin (σ, σ′) and isospin (τ, τ′) of the nucleons, the ground-state energy of nuclear matter in a standard mean-field approach, such as the Bruckner-Hartree-Fock calculation can be written in the form [11].

$$E = 4\sum_{k\le k_F}\frac{\hbar^2k^2}{2m} + \frac{1}{2}\sum_{k\sigma\tau}\sum_{k'\sigma'\tau'}\left(\langle k\sigma\tau, k'\sigma'\tau'|V_D|k\sigma\tau, k'\sigma'\tau'\rangle + \langle k\sigma\tau, k'\sigma'\tau'|V_{EX}|k\sigma\tau, k'\sigma'\tau'\rangle\right), \quad (1)$$

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Where the first, second and the third terms represent the kinetic energy term, the Hartree term and the Fock term respectively. The summation in equation (1) is taken over all single particle wave functions $(k \le k_F)$. The Fermi momentum is related to the nuclear matter density by $k_F = (1.5\pi^2\rho)^{1/3}$. The factor 4 multiplying the kinetic energy term comes from the two-folded degeneracy in spin σ and isospin τ. In the Hartree term, V_D represents the direct term in the NN interaction, and is determined from the singlet and triplet even (V_{SE}, V_{TE}) components, whereas in Fock term, V_{EX} accounts for the exchange term and is determined from the singlet and triplet odd (V_{SO}, V_{TO}) components. Our new one boson interaction henceforth referred to as new one boson – Fetal (NOB - Fetal) has both thedirect and exchange parts expressed respectively as:

$$V_D(r) = -2272.37\frac{e^{-5r}}{5r} + 10208.88\frac{e^{-3r}}{3r} - 1896.94\frac{e^{-2r}}{2r}, \quad (2)$$

and

$$V_{EX}(r) = 24124.32\frac{e^{-5r}}{5r} - 6372.59\frac{e^{-3r}}{3r} + 178.66\frac{e^{-2r}}{2r} - 7.8474\frac{e^{-0.7072r}}{0.7072r}. \quad (3)$$

Here $V_D(r)$ is the direct part of the NOB interaction which receives no contribution from the long-range one pion exchange potential (OPEP) component. The $V_{EX}(r)$ is the exchange part of the interaction in which the OPEP contribution plays a role by making the correct treatment of the antisymmetrization effects very essential [12].
The total nuclear matter energy per particle is taken to be of the form

$$\frac{E}{A} = \frac{3\hbar^2 k_F^2}{10m} + \frac{\rho}{2}\left\{J_D + \int [j_B(k_F r)]^2 V_{EX}(r) d^3 r\right\}, \quad (4)$$

where $J_D = 4\pi \int V_D . r^2\, dr$ is the volume integral of the direct part of the interaction, and $j_B(k_F r)$ is the Second Order Spherical Bessel function.
In nuclear matter calculations, the direct and exchange terms of an interaction alone cannot reproduce the binding energy and the saturation condition of nuclear matter except a density dependence factor is included. Here, we have adopted density dependence in the form [13] as:

$$V_{D(EX)}(\rho, r) = F(\rho) V_{D(EX)}(r), \quad (5)$$

where $V_{D(EX)}$ is the NOB interaction which is both radial and density- dependent(DD), the explicit form of the DD factor can be expressed as

$$F(\rho) = c(1 - \beta\rho^{\alpha}), \qquad BDNOB - Fetal \quad (6)$$

where c and β are constants. This DD – factor provided a very flexible power-law DD, where only the integer values of α are used since they allow for simple separation of variables when the overlapping density in $F(\rho)$ is taken as the sum of the ground state densities at the position of each nucleus [14, 12]. This type of interaction provides higher values of the nuclear matter incompressibility and seemed to give the best overall description of elastic nucleus-nucleus scattering. Thus, the nuclear matter energy and the saturation condition are respectively written as [13]:

$$\frac{E}{A} = \frac{3\hbar^2 k_F^2}{10m} + \frac{F(\rho)\rho}{2}\left\{J_D + \int [j_B(k_F r)]^2 V_{EX}(r) d^3 r\right\} \quad (7)$$

and

$$\frac{\partial}{\partial\rho}\left(\frac{E}{A}\right)\bigg|_{\rho=\rho_0} = \frac{\hbar^2 K_F^2}{5m\rho} + \frac{1}{2} J_D\, a(\rho)$$

$$+\frac{1}{2}\int V_{EX}(r)\left\{a(\rho)[j_1(K_F r)]^2 - 2f(\rho) j_1(K_F r) j_2(K_F r)\right\} d^3 r\bigg|_{\rho=\rho_0} = 0, \quad (8)$$

Where, $a(\rho) = \rho\frac{dF(\rho)}{d\rho} + F(\rho)$.

The nuclear incompressibility K is also takes the form [13]

$$K = k_F{}^2 \frac{d^2\left(\frac{E}{A}\right)}{d^2 k_F}\bigg|_{\rho=\rho_0} = 9\rho^2 \frac{\partial^2\left(\frac{E}{A}\right)}{\partial^2\rho}\bigg|_{\rho=\rho_0}$$

From eq. (7), the nuclear incompressibility is simplified to take the form.

$$k = 9\rho^2 \frac{\partial^2}{\partial\rho^2}\left(\frac{E}{A}\right) = -\frac{3\hbar^2 K_F^2}{5m} + b(\rho)J_D + \int V_{EX}(r)\left\{\begin{matrix} b(\rho)[j_1(K_F r)]^2 - c(\rho)\hat{j}_1(K_F r) j_2(K_F r) \\ +9\rho F(\rho)\{[j_2(K_F r)]^2 + j_1(K_F r) j_3(K_F r)\} \end{matrix}\right\} d^3 r \Bigg|_{\rho=\rho_0} \quad (9)$$

where

$$b(\rho =) 4.5\rho^3 \frac{d^2 F(\rho)}{d\rho^2} + 9\rho^2 \frac{dF(\rho)}{d\rho} \quad (10)$$

and

$$c(\rho) = 18\rho^2 \frac{dF(\rho)}{d\rho} + 15\rho F(\rho). \quad (11)$$

3.0 Results and Discussion

The theoretical estimate of the incompressibility (K_0) of infinite symmetric nuclear matter (SNM) obtained from the present work is presented in Table 1. In obtaining the parameterization of the density dependentNOB - Fetal, α was varied in the order of α = 2/3, 1, 2, 3. The other two parameters, c and β of the density dependent $F(\rho)$were determined to reproduce the saturation properties of the symmetric NMand give the nuclear incompressibility K_0 ranging from 192 to 447 MeV with the $BDNOB_0$, $BDNOB_1$, $BDNOB_2$, and $BDNOB_3$ versions of the interaction. The present result of K_0 is found to be in a perfect agreement with the work of Dao etal. [15], in which the value of K_0 ranges from 150 to 454MeV.The present estimate of K_0 especially that of the $BDNOB_2$ is in good agreement with the experimental determination of K_0 based on the production of hard photons in heavy ion collision which led to experimental estimate of $K_0 = 290 \pm 50$ MeV [16].

Table 2 shows a comparison of three sets of the density dependent parameters c, β and α as well as a wide range of EOS for the isoscalars part of M3Y-Reid, M3Y-Paris and NOB -Fetal NN interactions. As can be seen from Table 2, the best candidates for our interaction are both $BDNOB_0$ and $BDNOB_1$ – Fetal corresponding to $BDM3Y_0$ and $BDM3Y_1$because the corresponding K_0 are within the experimental range of 210 $\pm$30 MeV. The incompressibilities corresponding to $BDNOB_2$, $BDNOB_3$ and those of $BDM3Y_2$ and $BDM3Y_3$ respectively are rather too large [13].

Table1: Parameters of different density dependences of the NOB interaction equation (6), alongside the nuclear matter incompressibilities k_0 obtained using equation (9)

DD Factor	c	B	α	k (MeV)
BD NOB0	1.2638	1.0354fm^2	2/3	192
BD NOB1	1.1287	1.3937fm^3	1	229
BD NOB2	0.9950	4.6500 fm^6	2	338
BD NOB3	0.9504	19.0902fm^9	3	447

Table 2: Parameters of different density dependent versions of the M3Y interactions alongside the computed nuclear matter incompressibility taken from ref. [13, 17] compared.

Interaction	c	B	α	k (MeV)
BDM3Y0-Reid	1.3827	1.1135fm^2	2/3	190.0
BDM3Y0-Paris	1.4377	1.2627fm^2	2/3	218.0
BD NOB0-Fiase	2.4658	1.035fm^2	2/3	192.0
BDM3Y1-Reid	1.2253	1.5124fm^3	1.0	232.0
BDM3Y1-Paris	1.2521	1.7452fm^3	1.0	270.0
BD NOB1-Fiase	1.1287	1.3937fm^3	1.0	229.0
BDM3Y2-Reid	1.0678	5.1069fm^6	2.0	332.0
BDM3Y2-Paris	1.0664	6.0296fm^6	2.0	418.0
BD NOB2-Fiase	0.9950	4.6500fm^6	2.0	338.0
BDM3Y3-Reid	1.0153	21.073fm^9	3.0	454.0
BDM3Y3-Paris	1.0045	25.115fm^9	3.0	566.0
BD NOB3-Fiase	0.9504	19.0902fm^9	3.0	447.0

4.0 Conclusion

We have studied nuclear matter incompressibility using different versions of newly developed density-dependent one boson interaction (NOB -Fiase) derived from LOCV method. The results of our calculation reasonably agree with results obtained from similar calculations using M3Y-Reid and M3Y-Paris interactions which are derived from the G-matrix calculation. Different equations of state(EOS) ranging from a soft one with k_0 of 192*MeV* to a harder one with $k_0$of 447*MeV* have been obtained with the new interaction. The result obtained from this work also agrees with those of other researchers [12, 13, 17].

5.0 References

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