Journal of the Nigerian Association of Mathematical Physics
Volume 35, (May, 2016), pp 39 – 44
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Nuclear Incompressibility and Nuclear Symmetry Energy Calculated using M3Y-type Interaction Derived from Variational Calculation

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Abstract

A generalized version of density dependence has been introduced into an M3Y-type effective nucleon-nucleon (NN) interaction that was derived from variational calculation using the Reid NN potential. The density dependent parameters have been chosen to reproduce the saturation binding energy and density of normal nuclear matter in standard Hartree-Fock scheme and generate different equations of state starting from a very soft one, $K_0 = 178$ ***MeV, upto the stiff one with*** $K_0 = 472.5$ ***MeV. The description of nuclear matter from the isoscalar and isovector components of the density dependent M3Y-type effective interaction provided a value of the symmetry energy that is consistent with the empirical value of the symmetry energy.***

1.0 Introduction

Nuclear matter (NM) is an idealized system of nucleons interacting with out Coulomb forces which is transnationally invariant with a fixed ratio of neutrons to protons. The goal of NM theory is to obtain an equation of state (EOS) for NM starting from the underlying two-body nucleon-nucleon (NN) interaction. In the past, researchers were accustomed to the use of many simple models in nuclear physics that usually give very little information concerning nuclear properties. At present, folding models are used for investigation of NM properties. Folding procedures allow us to incorporate many aspect of nuclear structure as well as theoretical ideas about the effective interaction of two nucleons within NM. It also provides us with a means of linking information obtained from nuclear (hadronic) interactions with that from other sources, as well as correlating that from the use of different hadronic probes.The nuclear EOS, which is the energy per nucleon $E/A = \in$ of NM as a function of nucleonic density, ρ can then be used to obtain the bulk properties of NM such as the nuclear incompressibility and the symmetry energy. The EOS is also of fundamental importance in the theories of nucleus-nucleus collisions at energies where the nuclear incompressibility K comes to play as well as in the theories of neutron stars and supernova explosion [1]. The basic inputs to a folding calculation are the nuclear densities of the colliding nuclei and the effective NN interaction. Once we have realistic nuclear densities, available from different nuclear models or directly from the electron-scattering data, it still remains necessary to have a realistic effective NN interaction before the success of the folding model can be reliably assessed[2]. Popular choices for this have frequently been based upon the M3Y interactions which were designed to reproduce the G-matrix element of Reid[3] and Paris [4] NN potentials in an oscillator basis. We refer to these as M3Y-Reid and M3Y-Paris interaction, respectively.

In a recent paper [5] we derived a similarly motivated effective interaction which was based on the lowest order constrained variation (LOCV) approach. [5,6,7]. The results of that paper compared very favourably with the results obtained from the G-matrix calculations. Motivated by the success of the results, an M3Y-type effective interaction was derived [8] from the results of the work in Ref [5], and was used to reproduce the binding energy of nuclear matter of -16MeV at the normal nuclear matter density of $\rho = 0.17\ fm^{-3}$

when a density dependent factor $F(\rho)$ [9] was introduced. In a further development, when the choice of this density dependence was used to calculate values of the nuclear incompressibility with the exchange term of the interaction supplement by zero-range pseudo potential, a hard equation of state was obtained with K values ranging from 301 MeV to 307 MeV. To have a wide range of k values which define a soft EOS as well as a hard one, both the direct and exchange terms of the M3Y-type effective interaction have to be considered. In addition to this, a

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more flexible power-law density dependence has to be used [10,11]. The used of the intermediate set of K values allows one to trace in finer detail the sensitivity of refractive scattering data to K values and thus, hopefully, its determination with more precision [2].
In the present work, a power-law density dependence is used on an M3Y-type interaction [8] derived from LOCV to generate equation of states ranging from a very soft to a very stiff one. The nuclear symmetry energy is also calculated theoretically using the isoscalar and isovector components of the M3Y-type interaction of Ref. [8] supplemented by a zero-range pseudo potential. A brief description of our density dependent (DD) M3Y-type interaction based on the LOCV results, as well as nuclear incompressibility are present in section 2. The nuclear symmetry energy is derived in section 3, while section 4 is devoted to results and discussion. Summary and conclusion are finally given in section 5.

• 2.0 Density-Dependent Effective Interaction

In this section, we present our density-dependent effective NN potential suitable for calculation of NM properties. Here, the direct and exchange potentials in the singlet-even (SE), singlet-odd (SO), and the triplet odd (TO) channels can be recast into the spin-isospin formalism as [12]:

$$V^{D} = \frac{1}{16}\left(3V^{SE} + 3V^{TE} + V^{SO} + 9V^{TO}\right) \tag{1}$$

with

$$V^{EX} = V^{D} P^{\sigma} P^{\tau} \tag{2}$$

where P^{σ} and P^{τ} are projection operators in the spin and isospin channels. Hence

$$V^{EX} = \frac{1}{16}\left(3V^{SE} + 3V^{TE} - V^{SO} - 9V^{TO}\right) \tag{3}$$

Using the various ranges as defined in table (V) of Ref. [5], we obtained

$$V^{D}(r) = 11012\frac{\exp(-4r)}{4r} - 2359\frac{\exp(-2.5r)}{2.5r}$$

$$V^{EX}(r) = 1039.25\frac{\exp(-4r)}{4r} - 1503.94\frac{\exp(-2.5r)}{2.5r} - 7.847\frac{\exp(-0.7072r)}{0.7072r} \tag{4}$$

These results may be compared with the very popular M3Y interaction derived from the G-matrix approach given in Ref. [11]:

$$V^D(r) = 7999.0\frac{\exp(-4r)}{4r} - 2134.85\frac{\exp(-2.5r)}{2.5r}$$
$$V^{EX}(r) = 4631.38\frac{\exp(-4r)}{4r} - 1787.13\frac{\exp(-2.5r)}{2.5r} - 7.847\frac{\exp(-0.7072r)}{0.7072r} \quad (5)$$

As is well-known in NM calculations, the presence of the direct and exchange terms alone cannot reproduce the binding energy of NM, except a density dependence is included. We include the density dependence in the form similar to that of Refs. [2,13] in the form:

$$V(r,\rho) = V^{D(EX)}(r)F(\rho) \quad (6)$$

with

$$F(\rho) = C[1 + \alpha\exp(-\beta\rho)], \quad (7)$$

where C, α and β are constants.With this form of the density dependence, the binding energy of nuclear matter per nucleon can be written as [14]:

$$\frac{E}{A} = \frac{3\hbar^2 K_F^2}{10m} + \frac{F(\rho)\rho}{2}\left\{J_v + \int [j_B(K_F r)]^2 V^{EX}(r)d^3r\right\}, \quad (8)$$

where $J_v = \int V^D(r)d^3r$ and $j_B(x) = 3j_1(x)/x$, while $j_K(x)$ is the K^{th}-order spherical Bessel function.

The nuclear incompressibility, K for the spin and isospin symmetric could infinite nuclear matter (INM) is defined as [10]:

$$K = 9\rho^2 \frac{\partial^2}{\partial\rho^2}\left(\frac{E}{A}\right)\Bigg|_{\rho=\rho_0}$$

$$= -\frac{3\hbar^2 K_F^2}{5m} + b(\rho)J_D + \int V_{EX}(r)\left\{\begin{matrix} b(\rho)[j_1(K_F r)]^2 - c(\rho)\hat{j}_1(K_F r)j_2(K_F r) \\ +9\rho F(\rho)\{[j_2(K_F r)]^2 + j_1(K_F r)j_3(K_F r)\} \end{matrix}\right\} d^3r \Bigg|_{\rho=\rho_0} \quad (9)$$

where the parameters $b(\rho)$ and $c(\rho)$ are given in Ref. [11].Now due to the exponential form of Eq. (7), other solutions of Eq. (9) are not possible, so a more flexible power-law density dependence is introduced [10,11],

$$F(\rho) = C(1 - \alpha\rho^\beta). \quad (10)$$

A population version of this form of density dependence uses β= 2/3 [11].

• 3.0 The Nuclear Symmetry Energy

The nuclear matter EOS is calculated using the isoscalar and isovector components [15] of M3Y interaction along with density dependence. The density dependence of the effective interaction, DDM3Y, is completely determined from nuclear matter calculations. The equilibrium density of the NM is determined by minimizing the energy per nucleon. The energy variation of the zero-range potential is treated accurately by allowing it to vary freely with the kinetic energy part of the energy per nucleon E over the entire range of $\in$

[16]. This is not only more plausible, but yields excellent result for the incompressibility K of the symmetry NM which does not suffer from the super luminosity problem.

Assuming interaction Fermi gas of neutrons of protons, we define isospin asymmetry X, in terms of the neutron density, ρ_n, proton density, ρ_p, and nucleonic density, $\rho = \rho_n + \rho_p$, as

$$X = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \quad (11)$$

The nuclear EOS can be expanded in terms of isospin asymmetry X as [17]:

$$\varepsilon(\rho, X) = \varepsilon(\rho, 0) + S(\rho)X^2 + S_1(\rho)X^4 + S(\rho)X^6 + \ldots \quad (12)$$

which provides the standard definition of the nuclear symmetry energy $S(\rho)$ where $\varepsilon(\rho, X)$ is the energy per nucleon of NM. The isoscalar and isovector components of the DDM3Y interaction supplemented by zero range pseudo-potential are given in the form [17]:

$$t_{00}^{M3Y}(r,\varepsilon) = V^D(r) - 276(1-\alpha\varepsilon)\delta(r) \quad ,(13)$$

and

$$t_{01}^{M3Y}(r,\varepsilon) = -6197.625\frac{\exp(-4r)}{4r} + 1277.4\frac{\exp(-2.5r)}{2.5r} + 288(1-\alpha\varepsilon)\delta(r) \quad (14)$$

respectively, where the energy dependence parameter $\alpha = 0.005$ MeV. Based on the mean field assumption and using the DDM3Y interaction, the EOS can be derived as [17]:

$$\varepsilon(\rho, X) = \left[\frac{3\hbar^2 K_F^2}{10m}\right]F(X) + \left(\frac{\rho J_{vi} C}{2}\right)(1-\beta\rho^{2/3}), \quad (15)$$

In Eq. (15), J_{vi} represents the volume integral while $F(X)$ is given as

$$F(X) = \left[\frac{(1+X)^{5/3} + (1-X)^{5/3}}{2}\right] \quad (16)$$

The nucleonic mass is equal to 938.91897 MeV/c^2. An alternative physical definition of the nuclear symmetry energy is the energy required per nucleon to change the symmetric nuclear matter (SNM) to pure neutron matter (PNM) which is given by

$$S(\rho) = \varepsilon_{PNM} - \varepsilon_{SNM} = (2^{2/3}-1)\frac{3}{5}E_F^0\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C}{2}\rho(1-\beta\rho^{2/3})\iiint t_{01}^{M3Y}(r,\varepsilon)d^3r \quad (17)$$

In Eq. (16), ρ_0 is the saturation nucleonic density, E_F^0 is the Fermi energy for the SNM in the ground state. The density dependence C and β of the effective interaction are obtained by reproducing the saturation energy per nucleon and the saturation density of SNM [18]. The first term of the right hand side of Eq (15) is the kinetic energy contribution whereas the second term is the potential energy contribution and accounts for the nuclear interaction.

• 4.0 Results and Discussion

We have shown in an earlier study [8] that a density department M3Y-type effective NN interaction derived from lowest order variational calculation can be used for investigation of NM properties such as incompressibility of cold nuclear matter. The previous work [8] used, as a first approximation, the zero-range pseudo-potential which represents the single-nucleon exchange term while the density dependence accounted for the higher order exchange effects and the Pauli blocking effectives. Due to the exponential dependence on density, Eq. [7], chosen largely for its simplicity in folding calculations [2], other solution were not possible.In the present calculation, we introduce a power-law density dependence in the form of Eq. (10) [10,11]. A population version of the density dependence uses β =2/3 [15], which yields value of K = 181.3 MeV when tested on our interaction. However, we used integer values of β =1,2 and 3 to explore the values of K using our interaction.These give K =231.8, 351.0 and 472.5MeV respectively.
The results of the folding analysis investigated by Dao and co-workers [2, 10] showed unambiguously, that integer values of β = 1, 2 and 3 when used on M3Y interaction derived from G-matrix calculationsproduced values of K = 270, 418 and 566 MeV respectively. The first choice of β =2/3 corresponds to a nuclear equation of state that is quit soft. At one time a very soft EOS was thought to be sufficient to allow a prompt explosion in supernovas[19 -20] but more recent numerical studies indicate that this is not the case and current thought about supernovas explosions places less emphasizes on the value of K [21]. It was reported in Ref. [2] that the determination of K based upon the production of hard photons in heavy ion (HI) collision led to the estimate $K \approx 290 \pm 50$ MeV, this valuelies within the range of the results obtained in our calculation. The K values found by our present calculations indicate that they are generally compatible with the values obtained from other sources.
A mean field calculation with the DDM3Y-type effective interaction is performed using the usual values of density dependence parameter $\alpha = 0.005$ MeV [18], the saturation density $\rho_0 = 0.17\, fm^{-3}$ [8] and the saturation energy per nucleon ε_0 = -16 MeV. The value obtained for the NSE at the saturation density $S(\rho_0)$ is found to be 28.7 MeV using definition of Eq. (17). The theoretical estimate of $S(\rho_0)$ using DDM3Y effective interaction from G-matrix calculation gave a value of 31.68 $\pm$ 0.29 MeV [18], while the value of $S(\rho_0)$ extracted from experimental mass excesses is 30.048 $\pm$ 0.004 MeV. The value obtained from the liquid drop model calculation [22] was 27.3 MeV.

• 5.0 Summary and Conclusion

We have introduced some generalized and realistic explicit density dependences into the original M3Y effective NN interactions that were based upon LOCV calculations [8]. The values of the parameters that described the density dependence have been chosen so as to reproduce the saturation properties (binding energy and saturation density) of normal nuclear matter, within the Hartee-Fock scheme, and which predict the nuclear incompressibility K to have values ranging from181.3 to 472.5 MeV. The theoretical description of nuclear matter based on mean field calculation using density dependent M3Y interaction [8] gives a value of symmetry energy that is consistent with the empirical value extracted by fitting the droplet model and extrapolated atomic mass excesses using the maximum likelihood estimator method [18].

6.0 References

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Journal of the Nigerian Association of Mathematical Physics Volume 35, (May, 2016), 39 – 44

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