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A Standard Generator/Parity Check Matrix for Codes from The Cayley Tables Due To The Non-Associative (123)-Avoiding Patterns of Aunu Numbers

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Abstract

In this paper, we aim at utilizing the Cayley tables demonstrated by the Author in the construction of a Generator/Parity check Matrix in standard form for a Code say Our goal is achieved first by converting the Cayley tables using $Mod\,2$ ***arithmetic into a Matrix with entries from the binary field. Echelon Row operations are then carried out on the matrix in line with existing algorithms and propositions to obtain a matrix say whose rows spans and a matrix say whose rows spans , the dual code of , where*** $G\ and\ H$ ***are given as*** $G=(I_k\left|X\right.)\ and\ H=(-X^T\left|I_{n-k}\right.)$***. The report by Williem Haemers, that the adjacency Matrix of a graph can be interpreted as the generator matrix of a Code is in this context extended to the Cayley table which generates matrices from the permutations of points of the AUNU numbers of prime cardinality.***

Keywords: Cayley tables, AUNU permutation patterns, Generator matrix, Parity check Matrix, Standard form, Pattern avoidance, Echelon row operation, Non-associative, Non-commutative

1.0 Introduction

In Coding theory, the generator matrix of a Code plays an important role. Once the generator matrix of a code is known, such a code can easily be encoded and decoded. Since Procedures for obtaining the parity check matrix say of the code from the generator matrix is obvious through existing algorithms and theorems. As such, we shall not be out of place to mention here that once we have the generator matrix for a particular code, then a message can be encoded, decoded and analyzed by the matrix.

The special class of the (132) and (123)-avoiding class of permutation patterns which were first reported in [1], where some group and Graph theoretic properties were identified, had enjoyed a wide range of applications in various areas of applied Mathematics since then. The Authors in [2] described how the Non-associative and Non-commutative properties of the patterns can be established using their Cayley Tables where a binary operation was defined to act

on the (132) and (123)-avoiding patterns of the AUNU numbers using a Pairing scheme. The generation and analysis of some small classes of linear and cyclic codes from the adjacency matrices of Eulerian graphs due to AUNU patterns had been reported in [4] and [5]. The (132)-avoiding class of AUNU Patterns had also found applications in the construction of Eulerian graphs[6]. The reports in [4] and [5] were based on the assertion by Williem H.H as in [7].

In this paper, we utilize the Cayley tables generated, first as Matrices, then as matrices over the binary field and lastly transform such Matrices into generator/parity check matrices for some codes in standard form $G = (I_k \left| X_{n-k \times k} \right.) / \ H = (-X^T \left| I_{n-k} \right.)$ [8].

We review some basic concepts and propositions for the easy understanding of this paper.

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2.0 Permutation Patterns:

An arrangement of the objects $1, 2, \ldots, n$ is a sequence consisting of these objects arranged in any order. When in addition, a particular order of arrangement is desired, such an arrangement becomes an ordered arrangement governed by a pattern σ and such a permutation $\sigma \in S_n$ naturally results into a certain arrangement of $1, 2, \ldots, n$ given by $\sigma(1)\ \sigma(2). \ . \ .\sigma(n) \ . \ . \ . \ . \ . \ . \ . \ . \ (1)$
which is called the arrangement associated with a permutation pattern σ of points of a non empty set,

$\Omega = \{1, 2, . \ . \ . \ n\} \ . \ . \ . \ . \ . \ . \ . \ .(2)$

Given a sequence π consisting of n elements arranged in a given pattern and another sequence σ having m elements such that $m < n$ then; σ is said to be contained as a pattern in π provided π has a sub word which its order is isomorphic to σ. If π does not contain σ, it is said to avoid it. The set of all σ-avoiding permutation is denoted by $S_n(\sigma)$.

It is useful to differentiate between a subsequence and a sub word. For instance, if $\sigma = 4132 \in S_4$ *, then* $\pi = 78364521 \in S_8$ Contains σ as a sub word since; $\rho(8364) = 4132.$ however, $\pi = 54321 \in S_5$ does not contain σ as a sub word although it does contain it as a subsequence. Occurrences of sub words can be overlapped. As an example, the sequence $5716243 \in S_7$ contains two occurrences of $\sigma = 7162$ *and* 6243. Determination of $\left| S_n(\sigma) \right|$ has remained a hard and intractable problem for a given σ containing more than three elements. This is among the reasons that motivate the Author to construct some class of pattern-avoiding permutations using some special sub word governed some succession scheme as in [3]. Thus $A_n(132)$ in this context, represents the sub words of length $n \geq 3$ that are (132)-avoiding in A_n being the set of strictly consecutive succession scheme containing pairs i , j such that

$j = i + 1 \quad i, j \in N \ . \ . \ . \ . \ . \ . \ . \ .(3)$

3.0 Methodology

We State an important theorem for the enumeration of these permutation patterns, whose proof can be found in [1] and [2].

Theorem 1: *The number of sub words for the permutation patterns under study is Enumerated as: 2, 3, 5, 5, 8, . . . corresponding to the length (cardinality) of the special (123)-Avoiding sequences 5, 7, 11, 13, 17, . . . [1].*

We now define a mapping $\Theta : W = \{1, 2, . . . , n\}$ of $(132)-$ avoiding patterns of AUNU numbers of cardinality n, where n is necessarily prime. $\Theta : W = \{1, 2, . . . , n\}(R)A_{n(123)}$. Where $A_{n(123)}$ represents the $(123)-$ avoiding patterns of AUNU permutation reported in ***theorem 1*** and n is a prime greater than or equal to five. Then for $W = \{1, 2, 3, 4, 5\}$,

$\Theta((123) - avoiding)$ can be generated using the relation $\Theta(i) = i + i^{th}\ permutations$.

Illustration for $n = 5$.

Thus for $i = 1$, we have,

$$\Theta(1) = 1 + 0 = 1, \Theta(2) = 2 + 1 = 3, \Theta(3) = 3 + 2 = 5, \Theta(4) = (4 + 3) Mod 5 = 2, \Theta(5) = (5 + 4)$$

And the result follows. It follows that Θ provides a shift in points of W corresponding to the succeeding elements (points) of W as a pairing scheme. Similarly, for $i = 2, 3, ..., (n-1) Mod\ n$, the subsequent sub words can be generated and the following table is constructed using $A_{n(123)}$ for $n = 5$.

We now consider the Cayley Table 1, which is constructed using $A_n(132)$ for $n = 5$ as in [2]

Table 1: Cayley Table for n = 5 showing generated points of Ω as permutations of (132) and (123)-avoiding patterns of AUNU scheme under the action of Θ.

Θ	1	2	3	4	5
1	1	3	5	2	4
2	1	4	2	5	3
3	1	5	4	3	2

We now convert the entries of the Cayley Table 1 to the binary system using $Modulus\ 2\ \ arithmetic$. The Table 1 thus becomes;

Table 2: Entries of Table 1 converted to Mod 2.

Θ	1	2	3	4	5
1	1	1	1	0	0
2	1	0	0	1	1
3	1	1	0	1	0

The Table 2 becomes the matrix A below which is,

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Proposition:

The Matrix $A=\begin{bmatrix}1 & 1 & 1 & 0 & 0\\ 1 & 0 & 0 & 1 & 1\\ 1 & 1 & 0 & 1 & 0\end{bmatrix}$ *from the Cayley Table 1 is equivalent to a matrix* $G^I=\left[I_k \middle| X_{k\times n-k}\right]$*, the generator matrix in standard form of a code* C *spanned by the rows of A.*

Proof: Suppose A above is a generator matrix for some linear Code C*. Then* C *has dimension* $k=3$ *and length* $n=5$*. According to an established result, the rows of a generator matrix G are independent, which is obvious for our G=A above. Now, since every generator matrix can either be put in standard form or is equivalent to a generator matrix in standard form* $i.e\left[I_k \middle| X_{k\times n-k}\right]$*. Apply the following Reduced Row Echelon form (RREF) operations on A;* $i.\ R_2=R_3\ \ and\ ii.\ R_3=R_2+R_3$ *clearly gives us* $G^*=\left[X_{k\times n-k}\middle| I_k\right]=X_{3\times 5-3}\left|I_3\right.=X_{3\times 2}\left|I_3\right.$

where $X_{k\times n-k}=X_{3\times 2}\begin{bmatrix}1 & 1\\ 1 & 1\\ 0 & 1\end{bmatrix}\ and\ I_k=I_3$ *the identity matrix of order 3. On juxtaposition of* G^**, we obtain* $G^I=\left[I_k\middle| X_{k\times n-k}\right]$ *which is our required result.* □

Next, we transform the matrix A to a matrix G in standard form using the above proposition. Now, applying the following row operations;

$$i.\ R_2=R_3\ on\ A\rightarrow A=A^I=\begin{bmatrix}1 & 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 1 & 0\\ 1 & 0 & 0 & 1 & 1\end{bmatrix}$$

And

$$ii.\ R_3=R_2+R_3\ on\ A^I\rightarrow A^I=A^{II}=\begin{bmatrix}1 & 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 1\end{bmatrix}$$

Clearly, A^{II} can be written as $A^{II}=$ $G^I=\left[\begin{array}{cc|ccc}1 & 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 1\end{array}\right]=\left[X_{k\times n-k}\middle| I_k\right]$ and on juxtaposition of G^I, we obtain $G=\left[I_k\middle| X_{k\times n-k}\right]$ which is the required generator matrix in standard form for a Code of length $n=5\,and$ *dimension* $k=3$.

For the dual code $C^{\perp}$ of the Code C whose generator matrix in standard form is H (also the parity check matrix of C). We define a matrix $H^I=\left[-X^T\middle| I_{n-k}\right]$ from G^I above. $i.e\ H^I=\left[\begin{array}{ccc|cc}1 & 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 0 & 1\end{array}\right]$ and on juxtaposing H^I, we obtain

$H = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right]$. And we conclude that H is the parity check matrix of C and the generator matrix of the orthogonal complement of C ($C^{\perp}$).

4.0 Findings

The Cayley Table 1, when interpreted as a binary Matrix using $Modulus\,2\ \ arithmetic$, has been transformed into a generator matrix in standard form for a code of length $n = 5$ and *dimension* $k = 3$. Moreover, using existing algorithms and results, the parity check matrix H of the code which is also the generator matrix of the dual code $C^{\perp}$ has also been obtained.

5.0 Conclusion

The Cayley tables due to the non-associative property of the (123) - avoiding class of AUNU permutation pattern has found suitable application in determining the generator /parity check matrices in standard form for some classes of codes.

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