Journal of the Nigerian Association of Mathematical Physics
Volume35,(May, 2016), pp 49 – 56

On Soft Lattice Structure

A. O. Yusuf[1] and H.M. Balami[2]

Department of Mathematical Sciences and Information Technology Federal University, Dutsin-Ma, Katsina, Nigeria

Abstract

In this paper we focus our discussion on the development of soft seton lattice theory structure. Some concepts of lattices were also discussed in details. The main purpose of this paper is to study the generalization of the union and intersection operations between soft lattices. The concepts of isomorphic soft lattices and soft sublattices were also study. We also gave some useful remarks.

1.0 Introduction

Primarily the aim of soft set theory is to provide a tool with enough parameters to deal with uncertainty associated with the data, whereas on the other hand it has ability to represent the data in a useful manner. In this section, the definitions of some concepts in lattices are givenas follows [1-7]. We also define the notions of soft lattices, soft distributive lattices and soft modular lattices. We focus on the generalization of the union and intersection operations between soft lattices.

2.0 Some Concepts on Lattices

Definition 2.1

A nonempty set together with a partial order relation is called as a partially ordered set or a poset.

Definition 2.2

A partial ordered set (Poset) is a set in which a binary relation $\leq$ is defined, which satisfies the following conditions, for all x, y, z,

P1. $x \leq x$, for any x (Reflexive)

P2. $x \leq y$ and $y \leq x$ implies $x = y$ (Antisymmetry)

P3. $x \leq y$ and $y \leq z$ implies $x \leq z$ (Transitivity)

Definition 2.3

A lattice is a partial ordered set (Poset) any two of whose elements have a greatest lower bound ($g.l.b$), denoted by$x \wedge y$, and a least upper bound ($l.u.b$), denoted by $x \vee y$. A lattice L is complete when each of its subsets X has a $l.u.b$ and a $g.l.b$in L.

Definition 2.4

Let L be an arbitrary, and let there be given two binary operations on L, denoted by $\vee$and $\wedge$. Then the structure $(L,\vee,\wedge)$ is an algebraic structure with two binary operations. We call the structure $(L,\vee,\wedge)$ a lattice provided that it satisfies the following properties:

(1) for any $x, y, z \in L$, $x \wedge (y \wedge z) = (x \wedge y) \wedge z$ and $x \vee (y \vee z) = (x \vee y) \vee z$

(2) for any $x, y \in L$, $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$

(3) for any $x \in L$, $x \wedge x = x$ and $x \vee x = x$

(4) for any $x, y \in L$, $x \wedge (y \vee z) = x$ and $x \vee (y \wedge z) = x$.

Definition 2.5

A poset$\langle A, \leq \rangle$ is a join semilattice if $sup\{a, b\}$ exists for all $a, b \in A$. A poset $\langle A, \leq \rangle$ is a meet semilattice if $inf\{a, b\}$ exists for all $a, b \in A$.

Definition 2.6

If a lattice has a smallest element, denoted by O, and a greatest element, denoted by I, then it is called a bounded lattice.A nonempty subset R of L is said to be a sublattice of L if $a, b \in R$implies$a \vee b, a \wedge b \in R$.

Corresponding author:A.O. Yusuf, E-mail:locheman1@yahoo.com, Tel.: +2347039057669& 7033499076(HMB)

Definition 2.7

A nonempty subset I of L is said to be an ideal of L if (i) $a, b \in I$ implies $a \vee b \in I$ and (ii) for any $a, b \in L$such that $a \leq b$ and $b \in I$ implies $a \in I$.

Definition2.8

A lattice $(L,\vee,\wedge)$ is said to be distributive if $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$, for all x, y, z in L.

Definition 2.9

A lattice $(L,\vee,\wedge)$ is said to be modular if $x \leq z$ implies that $x \vee (y \wedge z) = y \wedge (x \vee z)$, for all x, y, zin L.

Definition 2.10

A mapping f of a lattice $(L_1, \vee_1, \wedge_1)$ into a lattice $(L_2, \vee_2, \wedge_2)$ is said to be a lattice homomorphism if $f(a \wedge_1 b) = f(a) \wedge_2 f(b)$, for all$a, b in L_1$. If such a mapping is one-one and onto, then it is called a lattice isomorphism.

3.0 Soft Lattices

In this section, the definition of soft sets, soft lattices and some properties of soft lattices are reviewed in [2, 4, 5, 6, 7]. Throughout this section, L is a lattice and A is any nonempty set. R will refer to arbitrary binary relation between elements of A and elements of L. That is, $R \subseteq A \times L$. A set valued function $F: A \rightarrow P(L)$ can be defined as $F(x) = \{y \in L / xRy, \forall x \in A\}$.The pair (F, A) is a soft set over L.

Definition 3.1[3]

Let (F, A) be a soft set over L. Then (F, A) is said to be a soft lattice over L if $F(x)$ is a sublattice of L, for all $x \in A$.The set of all soft lattices over L is denoted by$S\ell(L)$.

Remark 3.1

In the above *Definition 3.1[3]*, for the set valued-function $F(x)$ to be sublattice of L, it has to first satisfies the properties of lattice, i.e., for every pair of elements of $F(x)$ the least upper bound $(\vee)$ and the greatest lower bound $(\wedge)$ exists in $F(x)$.

Example3.1[3]

Consider the lattice L as shown in figure 1. Let $A = \{a, b, d\}$. define the set-valued function F by $F(x) = \{y \in L: xRy \Leftrightarrow x \vee y = 1\}$. Then $F(a) = \{1, f\}$, $F(b) = \{1, e\}$, $F(d) = \{1, c, e, f\}$. Therefore, $F(x)$ is a sublattice of L, for all $x \in A$. Hence $(F, A) \in S\ell(L)$.

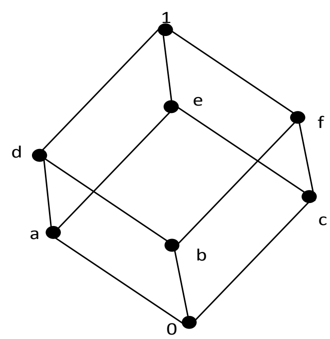

Figure 1

Remark 3.2

From the above *Example3.1[3]*, the set valued-functions of the given soft lattices are given base on the definition of the function and the nature of the lattices structures.

Example 3.2

Considered the lattice L as shown in figure 1. Let $A = \{a, b, d\}$. Define the set-valued function F by $F(x) = \{y \in L: xRy \Leftrightarrow x \wedge y = x\}$. Then $F(a) = \{1, a, d, e\}$, $F(b) = \{1, b, d, f\}$, $F(d) = \{1, d\}$. Therefore, $F(x)$ is a sublattice of L, for all $x \in A$. Hence $(F, A) \in S\ell(L)$.

Example 3.3[3]

Considered the lattice L as shown in figure 2. Let $A = \{0, a, b, c, d, 1\}$. Define the set-valued function F by $F(x) = \{y \in L: xRy \Leftrightarrow x \vee y = 1\}$. Then $F(0) = \{1\}$, $F(a) = \{1, b, c, d\}$, $F(b) = \{1, a, c, d\}$, , $F(c) = \{1, a, b, d\}$, $F(d) = \{1, a, b, c\}$, $F(1) = \{0, a, b, c, d, 1\}$. In $F(a)$, $b, c \in F(a)$. But $b \wedge c = 0 \notin F(a)$. Therefore, $F(a)$ is not a sublattice of L. Similarly, $F(b), F(c), F(d)$ are not sublattices of L. Hence$F(F, A) \notin S\ell(L)$.

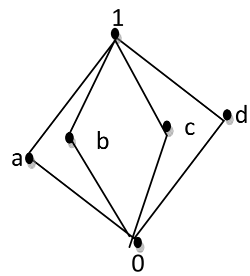

Figure 2

The following example illustrate that, if (F, A) is not a soft lattice over L then there may exists a nonempty subset $B \subset A$ such that $(F/B\ ,\ B)$ is a soft lattice over L.

Example3.4[3]

Considerer the lattice L as shown in Figure 3. Let $A = \{0\ ,\ a,\ b,\ c,\ d,\ 1\}$. Define the set-valued function F by $F(x) = \{y \in L: xRy \Leftrightarrow x \wedge y = 0\}$. Then $F(0) = \{0,\ a,\ b, c,\ d,\ 1\}, F(a) = \{0,\ b, c,\ d\}$, $F(b) = \{0,\ a,\ c,\ \}$, $F(c) = \{0,\ a,\ b\}, F(d) = \{0,\ a\}, F(1) = \{0\}$. Here $F(b) = \{0,\ a,\ c\},\ F(c) = \{0,\ a,\ b\}$ are not sublatices of L.

Therefore, $(F, A) \notin S\ell(L)$. Take $B = \{0,\ a,\ d,\ 1\} \subset A$. Then $F_B(0) = \{0\ ,\ a,\ b,\ c,\ d,\ 1\}, F_B(a) = \{0\ ,\ b,\ c,\ d\}$, $F_B(d) = \{0\ ,\ a\}, F_B(1) = \{0\}$. Therefore, $(F/B\ , B) \in S\ell(L)$.

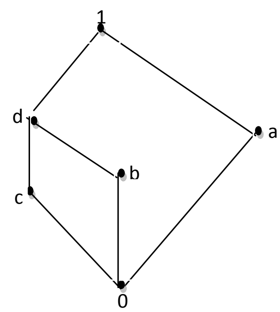

Figure 3

Proposition [3]3.1

Every lattice can be considered as a soft lattice.

Proposition[3] 3.2

Let $(F, A), (H, B) \in S\ell(L)$ be such that $A \cap B \neq \phi$ and $F(e) \cap H(e) \neq \phi forall e \in A \cap B$. Then their intersection$(F, A) \cap (H, B) \in S\ell(L)$.

The general case of the above proposition is another proposition, given as below.

Proposition 3.3

Let $(F_1, A_1), (F_2, A_2), \ldots (F_n, A_n) \in S\ell(L)$ be such that $A_1 \cap A_2 \cap \cdots \cap A_n \neq \phi$ and$F_1(e) \cap F_2(e) \cap \cdots \cap F_n(e) \neq \phi fore \in A_1 \cap A_2 \cap \cdots \cap A_n$. Then their intersection$(F_1, A_1) \cap (F_2, A_2) \cap \cdots \cap (F_n,\ A_n) \in S\ell(L)$.

Proof:

The intersection of n-numbers of soft set $(F_1, A_1), (F_2, A_2), \ldots (F_n, A_n)$ is given by $(F_1, A_1) \cap (F_2, A_2) \cap \cdots \cap (F_n,\ A_n) = (K, C)$ where $C = A_1 \cap A_2 \cap \cdots \cap A_n \neq \phi$, and$K(e) = F_1(e) \cap F_2(e) \cap \cdots \cap F_n(e) \neq \phi$, for all $e \in C$. Let $e \in C = A_1 \cap A_2 \cap \cdots \cap A_n$. Since $e \in A_1$ and $(F_1, A_1) \in S\ell(L), F_1(e)$ is a sublattice of L, since $e \in A_2$ and $(F_2, A_2) \in S\ell(L), F_2(e)$ is a sublattice of L by induction, since $e \in A_n$ and $(F_n, A_n) \in S\ell(L), F_n(e)$ is also a sublattice of L by assumption, we have, $K(e) = F_1(e) \cap F_2(e) \cap \cdots \cap F_n(e)$ is a sublatices of L. Since $e \in C$ is arbitrary.

Proposition3.4[3]

Let$(F, A), (G, B) \in S\ell(L)$. If $A \cap B = \phi$, then their union$(F, A) \cup (G, B) \in S\ell(L)$.

The general case of the above proposition is another proposition, given as below.

Proposition 3.5

Let $(F_1, A_1), (F_2, A_2), \ldots (F_n, A_n) \in S\ell(L)$ be such that $A_1 \cap A_2 \cap \cdots \cap A_n = \phi$. Then their union $(F_1, A_1) \cup (F_2, A_2) \cup \cdots \cup (F_n,\ A_n) \in S\ell(L)$.

Proof

The union of n-number of soft set $(F_1, A_1), (F_2, A_2), \ldots (F_n, A_n)$ is given by $(F_1, A_1) \cup (F_2, A_2) \cup \cdots \cup (F_n,\ A_n) = (H, C)$ where $C = A_1 \cup A_2 \cup \cdots \cup A_n$ and

$$H(e) = \begin{cases} F_1(e), & for\ e \in A_1 - A_2 - \cdots - A_n \\ . \\ . \\ . \\ F_n(e), & for\ e \in A_n - A_1 - \cdots - A_{n-1} \\ F_1(e) \cup \cdots \cup F_n(e), & for\ e \in A_1 \cap A_2 \cap \cdots \cap A_n \end{cases}$$

Since, $A_1 \cap A_2 \cap \cdots \cap A_n = \phi$
$A_1 - A_2 - \cdots - A_n = A_1$
$A_2 - A_1 - A_3 \cdots - A_n = A_2$
.
.
.
$A_n - A_1 - A_2 - \cdots - A_{n-1} = A_n$,
and either $e \in A_1\ or\ e \in A_1 \cup A_2 \cup \cdots \cup A_n$

$$H(e) = \begin{cases} F_1(e), & for\ e \in A_1 \\ & . \\ & . \\ & . \\ F_n(e), & for\ \ e \in A_n \end{cases}$$

Since, $(F_1, A_1) \in Sl(L)$, $F_1(e)$ is a sublattice of L, for $\forall e \in A_1$. Since, $(F_2, A_2) \in Sl(L)$, $F_2(e)$ is a sublattice of L, for $\forall e \in A_2$. By induction since, also, $(F_n, A_n) \in Sl(L)$, $F_n(e)$ is a sublattice of L, for $\forall e \in A_n$. Thus $K(e)$ is a sublattice of L, for $\forall e \in C$. Therefore, $(H, C) \in Sl(L)$.
That is, $(F_1, A_1) \cup (F_2, A_2) \cup \cdots \cup (F_n,\ A_n) \in S\ell(L)$.

Proposition3.6

Let $(F, A)$and $(G, B) \in Sl(L)$ be such that $F(x) \cap G(x) \neq \phi, \forall x \in A \cap B$. Then $(F, A) \wedge\ (H, B) \in S\ell(L)$.

Proof

The AND operation of two soft sets (F, A) and (G, B) is given by $(F, A) \wedge\ (H, B) = (H, C)$ where $C = A \cap B$ and $H(x) = F(x) \cap G(y) \neq \emptyset, \forall x \in A \cap B$. Since (F, A) is a soft lattice over L, then $F(x)$is a sublattice of L for all $x \in A$. Since (G, B) is a soft lattice over L, then $G(x)$ is a sublattice of L for all $x \in B$. That is, $H(x)$ is a sublattice of L for all $x \in A \cap B$. Therefore (H, C) is soft lattice over L. that is $(F, A) \wedge\ (G, B)$ is soft lattice over L.

Proposition3.7

Let $(F, A)$and $(G, B) \in Sl(L)$ be such that $F(x) \cap G(x) = \phi, \forall x \in A \cup B$. Then $(F, A) \vee\ (H, B) \in S\ell(L)$.

Proof

The *OR* operation of two soft sets (F, A) and (G, B) is given by $(F, A) \vee\ (G, B) = (H, C)$ where $C = A \cup B$ and $H(x) = F(x) \cup G(y), \forall x \in A \cup B$. Since (F, A) is a soft lattice over L, then $F(x)$ is a sublattice of L for all $x \in A$. Since (G, B) is a soft lattice over L, then $G(x)$ is a sublattice of L for all $x \in B$. That is, $H(x)$ is a sublattice of L for all $x \in A \cup B$. Therefore (H, C) is soft lattice over L. that is $(F, A) \vee\ \ (H, B)$ is soft lattice over L.

Definition3.2[3]

Let(F, A) and $(G, B) \in Sl(L)$.Then $(F, A)$is a soft sublattice of (G, B) if and only if$F(x) \subseteq G(x), \forall x \in A$.
The above definition can be illustrated by the following example.

Example3.4[3]

Consider the lattice L as shown in figure 3. Let $A = \{0, a, b, c, d, 1\}$. Let $B = \{0, b, c, d\}$. Define the set-valued function $F: A \to P(L)$ by $F(x) = \{y \in L: xRy \Leftrightarrow x \vee y = x \in A\}$. Then $F(0) = \{0\}$, $F(a) = \{0, a\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$,$F(d) = \{0, b, c, d\}$,$F(1) = \{0, a, b, c, d, 1\}$. Define the set-valued function $G: B \to P(L)$ by $G(x) = \{y \in L: xRy \Leftrightarrow x \vee y = x \in B\}$. Then $G(0) = \{0\}$, $G(b) = \{0, b\}$, $G(c) = \{0, c\}$, $G(d) = \{0, b, c, d\}$. Therefore, (F, A), $(G, B) \in Sl(L)$. Here $B \subseteq A$ and $G(x)$ is a sublattice of $F(x)$, for all $x \in B$. Therefore, (G, B) is a soft sublattice of (F, A).

Remark 3.2

In the above example the lattice in question have the same elements with the parameter set A. it is not must that the set parameter A to be equal in elements with the lattice L. If we let $A = \{0, b, c, d, 1\}$ and let $B = \{0, b, c, d\}$. We can still show that (G, B) is a soft sublattice of (F, A).

Example3.5

Consider the lattice L as shown in figure 3. Let $A = \{0, b, c, d, 1\}$. Let $B = \{0, b, c, d\}$. Define the set-valued function $F: A \to P(L)$ by $F(x) = \{y \in L: xRy \Leftrightarrow x \vee y = x \in A\}$. Then $F(0) = \{0\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$, $F(d) = \{0, b, c, d\}$,$F(1) = \{0, a, b, c, d, 1\}$. Define the set-valued function $G: B \to P(L)$ by $G(x) = \{y \in L: xRy \Leftrightarrow x \vee y = x \in B\}$.

Then $G(0) = \{0\}$, $G(b) = \{0, b\}$, $G(c) = \{0, c\}$, $G(d) = \{0, b, c, d\}$. Therefore, (F, A), $(G, B) \in Sl(L)$. Here $B \subseteq A$ and $G(x)$ is a sublattice of $F(x)$, for all $x \in B$. Therefore, (G, B) is a soft sublattice of (F, A).

Proposition 3.7[3]

Let(F, A), $(G, A) \in Sl(L)$. Then (F, A) is a soft sublattice of (G, A) if and only if $F(x) \subseteq G(x)$, for all $x \in A$.

Remark 3.3

It is not necessary for the two soft lattices to have the same parameter as it was stated in proposition 3.7 in [3]. Since all the elements in the set-valued function are coming from parameter set of the soft set (F, A).

Proposition 3.8

Let (F, A), $(G, B) \in Sl(L)$. Then (F, A) is a soft sublattice of (G, B) if and only if $F(x) \subseteq G(x)$, for all $x \in A$.

Proof

Let us assume that (F, A) is a soft sublattice of (G, B). Then $A \subseteq B$ that implies that$F(x) \subseteq G(x)$, for all $x \in A$. Conversely, let $F(x) \subseteq G(x)$, for all $x \in A$. Since $(F, A) \in Sl(L)$, $F(x)$ is a sublattice of L, for all $x \in A$. Since $(G, A) \in Sl(L)$, $G(x)$ is a sublattice of L, for all $x \in A$. Therefore, $F(x)$ becomes a sublattice of $G(x)$, for all $x \in A$. Also $A \subseteq B$. Thus (F, A) is a soft sublattice of (G, B).

Corollary 3.1

Every soft lattice is a soft sublattice of itself. That is, (F, A) is a sublattice of (F, A).

proof

Let (F, A) be a soft sublattice of (F, A), then $A \subseteq A$ that implies that $F(x) \subseteq F(x)$, for all $x \in A$. Since (F, A) is a soft lattice of L, then $F(x)$ is a sublattice of L, for all $x \in A$. Therefore, (F, A) is a soft sublattice of itself.

Definition3.3[3]

Let (F, A) be a soft lattice over L_1. Let f be a lattice homomorphism from L_1 to L_2. Then $\big(f(F)\big)(x) = f\big(F(x)\big)$, for all $x \in A$.

Proposition3.9 [3]

Let (F, A) and (G, B) be soft lattices over L_1 such that (F, A) be a soft sublattice of (G, B). If f is a homomorphism from L_1 to L_2, then $(f(F), A)$ is a soft sublattice of $(f(G), B)$.

Definition3.4[3]

Let (F, A) and (G, B) be two soft lattices over L_1 and L_2 respectively. Let $f: L_1 \to L_2$ and $g: A \to B$. Then (f, g) is said to be a soft lattice homomorphism if

(1) f is a lattice homomorphism from L_1 onto L_2,

(2)g is a mapping from A onto B,

(3) $f\big(F(x)\big) = G(g(x))$, for all $x \in A$.

Then (F, A) is said to be a soft lattice homomorphic to (G, B) and it is denoted by $(F, A) \sim (G, B)$.

Example 3.6[3]

Consider the lattice L_1 and L_2 as shown in figure 4 and Figure 5 respectively. Let $A = \{0, a, b, 1\}$ and $B = \{0^1, 1^1\}$. Define the set-valued function F by $F(x) = \{y \in L_1: xRy \Leftrightarrow x \vee y = x \in A\}$. Then $F(0) = \{0\}$, $F(a) = \{0, a, \}$, $F(b) = \{0, b\}$, $F(1) = \{0, a, b, 1\}$.Then $(F, A) \in Sl(L)$.Define the set-valued function $G(x) = \{y \in L_2: xRy \Leftrightarrow x \vee y = x \in B\}$. Then $G(0^1) = \{0^1\}$, $G(1^1) = \{0^1, 1^1\}$. Then $(G, B) \in Sl(L_2)$. Define $f: L_1 \to L_2$ by $f(0) = 0^1$, $f(a) = 0^1 f(b) = 1^1$, $f(1) = 1^1$. Define $g: A \to B$ by $g(0) = 0^1$, $g(a) = 0^1 g(b) = 1^1$, $g(1) = 1^1$. Then f is a lattice homomorphism from L_1 ontoL_2 and g is a mapping from A onto B. Also $f\big(F(x)\big) = G\big(g(x)\big)$, for all $x \in A$. Hence (F, A) is a soft lattice homomorphism to (G, B).

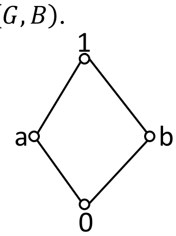

Figure 4

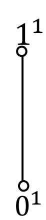

Figure 5

Definition3.5[3]

Let (F, A) be a soft set over L. then (F, A) is said to be a soft distributive lattice over L if $F(x)$ is a distributive sublattice of L, for all $x \in A$.

Example 3.6[3]

Consider the lattice L as shown in Figure6. Let $A = \{a, b, c\}$. Define the set-valued function F by $F(x) = \{y \in L: xRy \Leftrightarrow x \vee y = x \in A\}$.Then$F(a) = \{0, a\}$, $F(b) = \{0, a, b\}$,$F(c) = \{0, a, b, c\}$. Hence $F(x)$ is a distributive sublattice of L, for all $x \in A$. Therefore, (F, A) is soft distributive lattice over L.

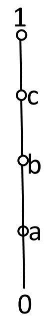

Figure 6

Proposition3.10[3]
Let L be a distributive lattice. Then every $(F, A) \in Sl(L)$ is a soft distributive lattice over L.
Proof
Let $(F, A) \in Sl(L)$. Then $F(x)$ is a sublattice of L, for all $x \in A$. Since L is distributive and every soft lattice of a distributive lattice is distributive,$F(x)$ is a distributive sublattice of L, for all $x \in A$. Therefore, (F, A) is a soft distributive lattice over L.
Remark3.4[3]
The converse of the above proposition is not true. That is, if (F, A) is a soft distributive lattice over L then L need not to be a distributive lattice.
Example3.7[3]
Consider the lattice L as shown in Figure 2. Let $A = \{a, b, c, d\}$. Define the set-valued function F by $F(x) = \{y \in L: xRy \Leftrightarrow x \wedge y = y\}$. Then $F(a) = \{0, a\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$, $F(d) = \{0, d\}$. Here $F(x)$ is a distributive sublattice of L, for all $x \in A$. Therefore, $(F, A)$is a soft distributive lattice over L. But L is not a distributive lattice, since L has a sublattice isomorphic to M_5.
Remark 3.5
The converse of the above proposition *3.10* in [3] can be made true always if L has no sublattice isomorphic to M_5 or N_5. That is suppose thatL has no sublattice isomorphic to M_5 or N_5 if (F, A) is a soft distributive lattice over L then L is always distributive lattice.
M_5 and N_5 are two five –element lattices structure depicted by the following diagram below

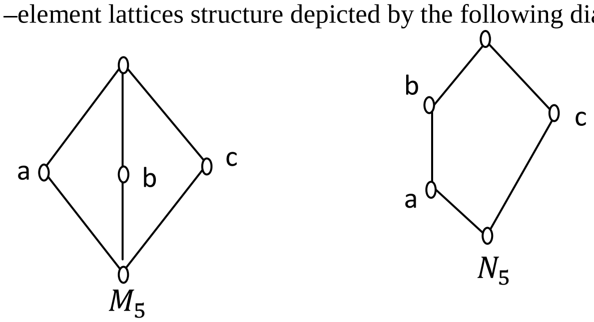

Figure 7
In neither case is $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, so neither M_5 nor $N_5$is a distributive lattice. For N_5 we also see that $a \leq b$ but $a \vee (b \wedge c) \neq b \wedge (a \vee c)$, so $N_5$is not modular.
Theorem3.1 [3]
A soft lattice (F, A) over L is distributive if and only if $F(x)$ has no sublattice isomorphic to N_5 or M_5. For all $x \in A$.
Theorem3.2[3]
(1) let (F, A) be a soft distributive lattice over L and (G, B) be a soft sublattice of (F, A). Then (G, B) is a soft distributive lattice over L.
(2) let (F, A) be a soft distributive lattice over L_1 and (G, B) be a soft lattice homomorphic image of (F, A) over L_2. Then (G, B) is a soft distributive lattice over L_2.
Proposition 3.11
Let (F, A) and (G, B) be two soft distributive lattices over L, then $(F, A) \cap (G, B)$ is also a soft distributive lattice over L.
Proof
Let (F, A) be a soft distributive lattice over L, then $F(x)$ is a distributive sublattice of L, and let $(G\ B)$ be a soft lattice over L, then $G(y)$ is a distributive sublattice of L, for all $x \in A$ and for all $y \in B$. Since $(F, A)$and (G, B) are soft distributive lattices over L, that implies that $F(x) \cap G(y)$ is a sublattice of L. therefore, $(F, A) \cap (G, B)$ is a soft distributive lattice over L.
Proposition 3.12
Let (F, A) and (G, B) be two soft distributive lattices over L, then $(F, A) \cup (G, B)$ is also a soft distributive lattice over L.

Proposition 3.13
Let (F, A) and (G, B) be two soft distributive lattices over L, then $(F, A) \vee (G, B)$ is also a soft distributive lattice over L.
Proposition 3.14
Let (F, A) and (G, B) be two soft distributive lattices over L, then $(F, A) \wedge (G, B)$ is also a soft distributive lattice over L.
Definition3.6[3]
Let (F, A) be a soft set over L. Then (F, A) is said to be a soft modular lattice over L if $F(x)$ is a modular sublattice of L, for all $x \in A$.
Example3.8[3]
Consider the lattice L as shown in Figure 1. Let $A = \{0, a, b, c, 1\}$. Define the set-valued function $F(x) = \{y \in L: xRy \Leftrightarrow x \wedge y = 0\}$. Then$F(0) = L$, $F(a) = \{0, b, c, f\}$, $F(b) = \{0, a, c, e\}$, $F(c) = \{0, a, b, d\}$, $F(1) = \{0\}$. Here$F(x)$ is a modular sublattice of L, for all $x \in A$. Therefore, (F, A) is a soft modular lattice over L.
Proposition3.15[3]
Let L be a modular lattice. Then every $(F, A) \in Sl(L)$ is a soft modular lattice over L.
Remark 3.6[3]
The converse of the above proposition is not true. That is, if (F, A) is a soft modular lattice over L then L need not be a modular lattice.
Remark 3.7
The converse of the above *Proposition 2.15[3] 3*can be made true always if L has no sublattice isomorphic to N_5. That is suppose that L has no sublattice isomorphic to N_5 if (F, A) is a soft modular lattice over L then L is alwaysmodular lattice.
Theorem3.3[3]
A soft lattice (F, A) over L is modular if and only if $F(x)$ has no sublattice isomorphic to N_5. For all $x \in A$.
Theorem 3.4[3]
(1) Let (F, A) be a soft modular lattice over L and (G, B) be a soft sublattice of (F, A). Then (G, B) is a soft modular lattice over L.
(2) Let (F, A) be a soft modular lattice over L_1 and (G, B) be a soft lattice holomorphicimage of(F, A) over L_2. Then (G, B) is a soft modular lattice overL_2.
Proposition 3.16
Let (F, A) and (G, B) be two soft modular lattices over L, then $(F, A) \cap (G, B)$ is also a soft modular lattice over L.
Proof
Let (F, A) be a soft modular lattice over L, then $F(x)$ is a modular sublattice of L, and let $(G\ B)$ be a soft lattice over L, then $G(y)$ is a modular sublattice of L, for all $x \in A$ and for all $y \in B$. Since (F, A) and (G, B) are soft modular lattices over L, that implies that $F(x) \cap G(y)$ is a modular sublattice of L. therefore, $(F, A) \cap (G, B)$ is a soft modular lattice over L.
Proposition 3.17
Let (F, A) and (G, B) be two soft modular lattices over L, then $(F, A) \cup (G, B)$ is also a soft modular lattice over L.
Proposition 3.18
Let (F, A) and (G, B) be two soft modular lattices over L, then $(F, A) \vee (G, B)$ is also a soft modular lattice over L.
Proposition 3.19
Let (F, A) and (G, B) be two soft modular lattices over L, then $(F, A) \wedge (G, B)$ is also a soft modular lattice over L.

4.0 Conclusion

In this chapter, we have introduced the concept of soft lattices and studied some of their algebraic properties. We focused on the generalization of union and intersection operations between the soft lattices.Soft distributive lattices and soft modular lattices were also introduced and characterization theorems for them have been obtained. Soft lattice ideals and soft lattice homomorphism were also introduced and their properties have been studied with some remarks.

5.0 References

[1] A. M. Ibrahim and A. O. Yusuf. Development of Soft Set Theory, American Int. Journal of Contemporary Research, Vol. 2 No. 9, 205-210 (2012).

[2] D. A. Molodtsov. Soft set theory- First results, *Computers Mathematics with Application*s, 37 (4/5),19-31, (1999).

[3] E. K. R. Nagarajan and G. Meenambigai. An Application of Soft Set to Lattices, *Kragujevac Journal of Mathematics*, Vol. 35, No. 1, 75-87 (2011).

[4] H. Aktas and N. Cagman. Soft sets and Soft groups, *Information Sciences* 177, 2726- 2735 (2007)

[5] I. F. Feng, Y. B. Jun, and X. Zhao. Soft semirings, *Computers Mathematics with Application*s, Vol. 56, 2621- 2628, (2008).

[6] M. Irfan Ali, F. Feng, X. Liu, W. K. Min and M. Shabir. On some New operations in Soft Set Theory, *Computers Mathematics with Application*s, Vol. 57, 1547-1553, (2009).

[7] P. K. Maji, A.R. Roy and R. Biswas. An application of soft sets in a decision making problem, *Computers and Mathematics with applications*, 44 (8/9), 1077-1083, (2002).