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Scaling Group Analysis of Heat and Mass Transfer of Magnetohydrodynamic Pressure-Driven Flow Past a Permeable Plate with Inclined Magnetic Field

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Abstract

This study investigates steady free convective heat and mass transfer of magnetohydrodynamic pressure-driven flow past a permeable plate with inclined uniform magnetic field. The governing partial differential equations of the model are reduced to a system of coupled non-linear ordinary differential equations by applying a scaling group of transformations. The coupled differential equations are solved numerically using the Weighted Residual method. The results obtained were presented graphically to illustrate the influence of various parameters on the dimensionless velocity, temperature, concentration and pressure drop. Finally, the effects of Skin friction, Nusselt and Sherwood numbers which are of physical and engineering interest are also presented and discussed.

Keywords: Magnetohydrodynamics; pressure-driven; Permeable plate; Scaling group transformations; Weighted Residual method

1.0 Introduction

The fundamental concept of magnetohydrodynamics is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and changes the magnetic field itself. A pressure-driven flow of heat and mass transfer in MHD poiseuille flow past a permeable plate are being studied widely due to its importance in MHD power generators, petroleum reservoirs, chemical catalytic reactor, reducing drag, Aeronautical engineering fields, nuclear waste disposal and others.

In view of these applications, Youssef et al. [1] analyzed two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability by using Lie-group method. They neglected the magnetic terms and pressure gradient in their analysis. An analytical analysis was performed to study the steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field with constant pressure gradient by Manyonge et al. [2] and reported that the velocity decreases in the presence of an inclined magnetic field, suction/injection rates, pressure gradient and Hartmann number. Also, the study of boundary layer flow of a non-Newtonian power-law fluid in a convergent channel was carried out by Pramanik [3]. Ignoring the pressure gradient and magnetic terms, and reduced the partial differential equations to a nonlinear differential equation using scaling group of transformations. Mohammad et al. [4] examined the Viscous flow through expanding or contracting gaps with permeable walls using Optimal Homotopy Asymptotic method. They neglected the magnetic terms but highlighted the effect of Reynolds number on the pressure distribution.

The problem of free convection under the influence of a magnetic field has attracted the interest of many researchers in view of its applications in geophysics and astrophysics. Makinde [5] investigated MHD boundary layer flow with heat and mass transfer over a moving vertical plate in the presence of magnetic field and a convective heat exchange at the surface with the surrounding while Uwanta and Sarki [6] studied heat and mass transfer with variable temperature and exponential mass diffusion. However,the authors neglected the effect of pressure gradient in their studied. Alireza et al. [7] dealt with the problem of steady two-dimensional MHD stagnation point flow towards a permeable stretching sheet with chemical reaction. The problem was solved using Optimal Homotopy Asymptotic method and compared their results with fourth order Runge-Kutta method. Hossain and Samand [8] studied heat and mass transfer of a MHD free convection flow along a stretching sheet with chemical reaction, radiation and heat generation in the presence of transfer magnetic field. The problem under consideration was transformed using similarity solution and analyzed by applying Nachtsheim Swigert shooting iteration technique with sixth order Runge-Kutta integration scheme. Scaling transformation for heat and mass transfer effects on steady MHD free convection dissipative flow past an inclined porous surface was investigated by Reddy [9]. The results showed that the velocity increases with an increase in the thermal and solutal Grashof numbers but decreases with an increase in Schmidt and Prandtl numbers. However, the pressure, heat source and reaction rate terms were not taken into consideration the study.

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A pressure driven flow in a magnetohydrodynamics heat and mass transfer problems are important in many processes such as purification of cruid oil, fluid droplets sprays, energy transfer in a wet cooling tower, flow in a desert cooler and possible applications in many industries and many more. Time dependent pressure gradient effects on unsteady MHD couette flow and heat transfer of a casson fluid was carried out by Sayed-Ahmed et al. [10]. The fluid is acted upon by a uniform and exponential decaying pressure gradient, an external uniform magnetic field is applied perpendicular to the plates with the fluid motion subjected to a uniform suction and injection, while Farooq et al. [11] studied steady poiseuille flow and heat transfer of couple stress fluids between two parallel inclined plates with variable viscosity. they used Reynold's model for temperature dependent viscosity. Thiagarajan and Sangeetha [12] investigated nonlinear MHD boundary layer flow and heat transfer past a stretching plate with free stream pressure gradient in presence of variable viscosity and thermal conductivity.

Most of the above studies, neglected the influence of inclined magnetic field and the contributions of fluid pressure. Consequently, the present research investigates the combined effects of inclined magnetic field and pressure drop in a steady convective heat and mass transfer in a magnetohydrodynamicpressure-driven flow. Scaling group of transformation was applied to reducing the number of independent variables.

2.0 Formulation of the Problem

Investigation was carried out to examine free convective heat and mass transfer of two dimensional MHD poiseuille flow past a permeable plate under the influence of uniform inclined magnetic field and pressure gradient. The motion of the fluid is maintained by both pressure gradient and gravity, and the flow is assumed to be in the X-direction with Y-axis normal to it. The magnetic field of uniform strength B_0 is introduced at angle α lying in the range $0 \prec \alpha \prec \frac{\pi}{2}$ in the direction of the flow. The plate is maintained at the temperature and species concentration T_w, C_w and free stream temperature and species concentration T_∞, C_∞ respectively. The geometry and equations governing the steady heat and mass transfer of two-dimensional magnetohydrodynamics poiseuille fluid flow past a permeable plate with inclined magnetic field are as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

(1)

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{1}{\rho}\sigma B_0^2 U sin^2\alpha-\frac{1}{\rho}\frac{\partial P}{\partial X}+\nu\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+g\beta_T\left(T-T_\infty\right)+g\beta_C\left(C-C_\infty\right)$$

(2)

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\nu\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$

(3)

$$\rho C_p\left(U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}\right)=k\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right)+Q_0\left(T-T_\infty\right)$$

(4)

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=D\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right)-\gamma\left(C-C_\infty\right)$$

(5)

The corresponding initial and boundary conditions are as follows:

$$U=0,V=v_w,P=0,T=T_\infty+(T_w-T_\infty)AX,C=C_\infty+(C_w-C_\infty)BX \quad at \quad Y=0$$
$$U=0,T=T_\infty,C=C_\infty \quad as \quad Y\to\infty$$

(6)

Where U, V, P, C, and T are velocity component in the X direction, velocity component in the Y direction, pressure, concentration of species in the fluid, temperature of the fluid respectively. A and B are constants defined as $A=B=\frac{1}{l}$, B_0 is the magnetic field strength, α is the angle of inclination of the magnet, v_w is the permeability of the porous surface such that $v_w \succ 0$ indicates wall injection and $v_w \prec 0$ indicates wall suction respectively.

The physical quantities ν, ρ, σ, D, C_p, k, Q_0 and γ are the fluid kinematics viscosity, density, electric conductivity of the fluid, mass diffusion coefficient, specific heat at constant pressure, thermal conductivity, rate of specific internal heat generation or absorption and reaction rate coefficient respectively. g is the gravitational acceleration, β_T and β_C are the thermal and concentration expansion coefficients respectively.

Introducing the following non-dimensional quantities

$$x=\frac{X}{l},y=\frac{Y}{l},u=\frac{Ul}{\nu},v=\frac{Vl}{\nu},p=\frac{Pl^2}{\rho\nu^2},H_a=lB_0\sqrt{\frac{\sigma}{\mu}},\theta=\frac{T-T_\infty}{T_w-T_\infty},P_r=\frac{\mu C_p}{k},$$
$$S_c=\frac{\nu}{D},G_r=\frac{l^3gB_T\left(T_w-T_\infty\right)}{\nu^2},G_c=\frac{l^3gB_C\left(C_w-C_\infty\right)}{\nu^2},\phi=\frac{C-C_\infty}{C_w-C\infty},\lambda=\frac{l^2\gamma}{\nu},$$
$$Q=\frac{l^2Q_0}{\mu C_p},f_w=-\frac{v_wl}{\nu}$$

(7)

Substituting the non-dimensional quantities (7) into equations (1)-(6), to obtain

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(8)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-H_a^2 sin^2\alpha u-\frac{\partial p}{\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+G_r\theta+G_c\phi \tag{9}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{10}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+Q\theta \tag{11}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=\frac{1}{S_c}\left(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)-\lambda\phi \tag{12}$$

The corresponding initial and boundary conditions are as follows:

$$\begin{aligned}&u=0, v=\frac{v_w l}{v}, p=0, \theta=x, \phi=x \quad at \quad y=0\\&u=0, \theta=0, \phi=0 \quad as \quad y\to\infty\end{aligned} \tag{13}$$

where x and y are dimensionless coordinate, u and v are the dimensionless velocity, θ and ϕ are the dimensionless temperature and concentration, p is the pressure, H_a the Hartmann number, G_r is the thermal Grashof number, G_c is the solutal Grashof number, M is the incline magnetic terms parameter, P_r is Prandtl, S_c is Schmidt number, Q is the heat source and λ is the concentration parameter respectively. f_w is the non-dimensional wall mass transfer coefficient such that $f_w \succ 0$ indicates wall suction and $f_w \prec 0$ indicates wall injection or blowing respectively.

Introducing the stream function $u=\frac{\partial \psi}{\partial y}$, $v=-\frac{\partial \psi}{\partial x}$, continuity equation is automatically satisfied and equations (9)-(13) become

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=-H_a^2 sin^2\alpha\left(\frac{\partial \psi}{\partial y}\right)-\frac{\partial p}{\partial x}+\left(\frac{\partial^3 \psi}{\partial x^2\partial y}+\frac{\partial^3 \psi}{\partial y^3}\right)+G_r\theta+G_c\phi \tag{14}$$

$$-\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial x\partial y}=-\frac{\partial p}{\partial y}-\left(\frac{\partial^3 \psi}{\partial x^3}+\frac{\partial^3 \psi}{\partial x\partial y^2}\right) \tag{15}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+Q\theta \tag{16}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y}=\frac{1}{S_c}\left(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)-\lambda\phi \tag{17}$$

subject to the initial and boundary conditions

$$\frac{\partial\psi}{\partial y}=0,\frac{\partial\psi}{\partial x}=f_w, p=0,\theta=x,\phi=x \quad at \quad y=0$$
$$\frac{\partial\psi}{\partial y}=0,\theta=0,\phi=0 \quad as \quad y\to\infty \tag{18}$$

Introducing simplified form of Lie-group transformations namely, the scaling group of transformations (Mukhopadhyay et al. [13] and Bhattacharyya et al. [14]),

$$\nabla: x^*=xe^{\varepsilon\alpha_1}, y^*=ye^{\varepsilon\alpha_2}, \psi^*=\psi e^{\varepsilon\alpha_3}, u^*=ue^{\varepsilon\alpha_4}, v^*=ve^{\varepsilon\alpha_5},$$
$$p^*=pe^{\varepsilon\alpha_6}, \theta^*=\theta e^{\varepsilon\alpha_7}, \phi^*=\phi e^{\varepsilon\alpha_8} \tag{19}$$

where α_1, α_2, α_3, α_4, α_5, α_6, α_7, and α_8, are transformation parameters and ε is a small parameters. Equation (19) may be considered as a point-transformation which transforms coordinate $(x,y,\psi,u,v,\theta,\phi)$ to the coordinate $(x^*,y^*,\psi^*,u^*,v^*,\theta^*,\phi^*)$.

Substituting the transformations equation (19) into equations (14) to (18), this gives

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{*2}}\right)=-H_a^2 sin^2\alpha e^{\varepsilon(\alpha_2-\alpha_3)}\frac{\partial\psi^*}{\partial y^*}-e^{\varepsilon(\alpha_1-\alpha_6)}\frac{\partial p^*}{\partial x^*}$$
$$+e^{\varepsilon(2\alpha_1+\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial x^{*2}\partial y^*}+e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial y^{*3}}+e^{-\varepsilon\alpha_7}G_r\theta^*+e^{-\varepsilon\alpha_8}G_c\phi^* \tag{20}$$

$$e^{\varepsilon(2\alpha_1+\alpha_2-2\alpha_3)}\left(-\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^{*2}}+\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}\right)=-e^{\varepsilon(\alpha_2-\alpha_6)}\frac{\partial p^*}{\partial y^*}-e^{\varepsilon(3\alpha_1-\alpha_3)}\frac{\partial^3\psi^*}{\partial x^{*3}}-$$
$$e^{\varepsilon(\alpha_1+2\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial x^*\partial y^{*2}} \tag{21}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right)=\frac{1}{P_r}e^{\varepsilon(2\alpha_1-\alpha_7)}\frac{\partial^2\theta^*}{\partial x^{*2}}+\frac{1}{P_r}e^{\varepsilon(2\alpha_2-\alpha_7)}\frac{\partial^2\theta^*}{\partial y^{*2}}+$$
$$e^{-\varepsilon\alpha_7}Q\theta^* \tag{22}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_8)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\phi^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\phi^*}{\partial y^*}\right)=\frac{1}{S_c}e^{\varepsilon(2\alpha_1-\alpha_8)}\frac{\partial^2\phi^*}{\partial x^{*2}}+\frac{1}{S_c}e^{\varepsilon(2\alpha_2-\alpha_8)}\frac{\partial^2\phi^*}{\partial y^{*2}}-$$
$$e^{-\varepsilon\alpha_8}\lambda\phi^* \tag{23}$$

The corresponding initial and boundary conditions gives

(24)

The system will remain invariant under the group of transformation ∇ and the following relations are obtained.

$$\alpha_3=\alpha_4=\alpha_7=\alpha_8=\alpha_1,\ \alpha_2=\alpha_5=\alpha_6=0 \tag{25}$$

Thus the set of transformations ∇ reduces to one parameter group of transformations as

$$x^* = xe^{\varepsilon\alpha_1}, y^* = y, \psi^* = \psi e^{\varepsilon\alpha_1}, u^* = ue^{\varepsilon\alpha_1}, v^* = v, p^* = p, \theta^* = \theta e^{\varepsilon\alpha_1}, \phi^* = \phi e^{\varepsilon\alpha_1} \qquad (26)$$

Then, the absolute invariant gives:

$$f(\eta) = x^{*-1}\psi^*, p_d(\eta) = p^*, \theta(\eta) = x^{*-1}\theta^*, \phi(\eta) = x^{*-1}\phi^* \qquad (27)$$

Therefore, the similarity transformations is obtained as:

$$\eta = y^*, \psi^* = x^* f(\eta), p^* = p_d(\eta), \theta^* = x^*\theta(\eta), \phi^* = x^*\phi(\eta) \qquad (28)$$

Substituting the similarity variables (28) into equations (20)-(24), the following system of non-linear differential equations are obtained.

$$f''' + ff'' - f'^2 - H_a^2 sin^2\alpha f' + G_r\theta + G_c\phi = 0 \qquad (29)$$

$$-p_d' = f'' + ff' \qquad (30)$$

$$\theta'' + P_r f\theta' - P_r f'\theta + P_r Q\theta = 0 \qquad (31)$$

$$\phi'' + S_c f\phi' - S_c f'\phi - S_c\lambda\phi = 0 \qquad (32)$$

The corresponding initial and boundary conditions take the form:

$$\begin{aligned} &f = f_w, f' = 0, p_d = 0, \theta = 1, \phi = 1 \ \ at \ \ \eta = 0 \\ &f' = 0, \theta = 0, \phi = 0 \ \ as \ \ \eta \to \infty \end{aligned} \qquad (33)$$

Integrating equation (30) with the initial and boundary conditions with $f_w = 1$ and pressure drop $-p_d = G$, hence $G = f' + \frac{1}{2}f^2 - \frac{1}{2}$ (34)

3.0 Weighted Residual Method

The idea of weighted residual method [15,16] is to seek an approximate solution, in form of a polynomial to the differential equation of the form

$$L[u(x)] = f \ \ in \ \ the \ \ domain \ \ T, \quad B_\mu[u] = \gamma_\mu \ \ on \ \ \partial T \qquad (35)$$

where $L[u]$ denotes a differential operator linear or non-linear involving spatial derivatives of dependent variables u, f is known function of position, $B_\mu[u]$ represents the approximate number of boundary conditions and T is the domain with boundary ∂T. The problem of finding an approximate solution of the boundary above is often done by assuming an approximation to the solution $u(x)$, an expression of the form

$$u(x) \approx w(x, a_1, a_2, a_3 \ldots a_n) \qquad (36)$$

such that for arbitrary values $a_1, a_2, a_3 \ldots a_n$, the boundary conditions are satisfied.
Applying WRM to equations (29), (31), (32) and (33). We assume a polynomial with unknown coefficients or parameters to be determined later, this polynomial is called the trial function.

$$f(\eta) = \sum_{i=0}^{n} a_i\eta^i, \quad \theta(\eta) = \sum_{i=0}^{n} b_i\eta^i, \quad \phi(\eta) = \sum_{i=0}^{n} c_i\eta^i$$

(37)

Use the boundary conditions (33) on the trial functions also substituting the trial functions into equations (29), (31) and (32) to obtain the residual equations

$$
\begin{aligned}
f_r = {} & 6a_3 + 24a_4\eta + 60a_5\eta^2 + 120a_6\eta^3 + 210a_7\eta^4 + 336a_8\eta^5 + 504a_9\eta^6 + 720a_{10}\eta^7 \\
& + 990a_{11}\eta^8 + 1320a_{12}\eta^9 + \left(a_{12}\eta^{12} + a_{11}\eta^{11} + a_{10}\eta^{10} + a_9\eta^9 + a_8\eta^8 + a_7\eta^7\right. \\
& \left. + a_6\eta^6 + a_5\eta^5 + a_4\eta^4 + a_3\eta^3 + a_2\eta^2 + a_1\eta + a_0\right)\left(132a_{12}\eta^{10} + 110a_{11}\eta^9\right. \\
& \left. + 90a_{10}\eta^8 + 72a_9\eta^7 + 56a_8\eta^6 + 42a_7\eta^5 + 30a_6\eta^4 + 20a_5\eta^3 + 12a_4\eta^2 + 6a_3\eta + 2a_2\right) \\
& - \left(12a_{12}\eta^{11} + 11a_{11}\eta^{10} + 10a_{10}\eta^9 + 9a_9\eta^8 + 8a_8\eta^7 + 7a_7\eta^6 + 6a_6\eta^5 + 5a_5\eta^4\right. \\
& \left. + 4a_4\eta^3 + 3a_3\eta^2 + 2a_2\eta + a_1\right)^2 - H_a^2 sin^2\alpha\left(12a_{12}\eta^{11} + 11a_{11}\eta^{10} + 10a_{10}\eta^9 + 9a_9\eta^8\right. \\
& \left. + 8a_8\eta^7 + 7a_7\eta^6 + 6a_6\eta^5 + 5a_5\eta^4 + 4a_4\eta^3 + 3a_3\eta^2 + 2a_2\eta + a_1\right) + G_r\left(b_{12}\eta^{12}\right. \\
& + b_{11}\eta^{11} + b_{10}\eta^{10} + b_9\eta^9 + b_8\eta^8 + b_7\eta^7 + b_6\eta^6 + b_5\eta^5 + b_4\eta^4 + b_3\eta^3 + b_2\eta^2 \\
& \left. + b_1\eta + b_0\right) + G_c\left(c_{12}\eta^{12} + c_{11}\eta^{11} + c_{10}\eta^{10} + c_9\eta^9 + c_8\eta^8 + c_7\eta^7 + c_6\eta^6 + c_5\eta^5\right. \\
& \left. + c_4\eta^4 + c_3\eta^3 + c_2\eta^2 + c_1\eta + c_0\right)
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
\theta_r = {} & 2b_2 + 6b_3\eta + 12b_4\eta^2 + 20b_5\eta^3 + 30b_6\eta^4 + 42b_7\eta^5 + 56b_8\eta^6 + 72b_9\eta^7 + 90b_{10} \\
& \eta^8 + 110b_{11}\eta^9 + 132b_{12}\eta^{10} + P_r\left(a_{12}\eta^{12} + a_{11}\eta^{11} + a_{10}\eta^{10} + a_9\eta^9 + a_8\eta^8 + a_7\eta^7\right. \\
& \left. + a_6\eta^6 + a_5\eta^5 + a_4\eta^4 + a_3\eta^3 + a_2\eta^2 + a_1\eta + a_0\right)\left(12b_{12}\eta^{11} + 11b_{11}\eta^{10} + 10b_{10}\eta^9\right. \\
& \left. + 9b_9\eta^8 + 8b_8\eta^7 + 7b_7\eta^6 + 6b_6\eta^5 + 5b_5\eta^4 + 4b_4\eta^3 + 3b_3\eta^2 + 2b_2\eta + b_1\right) \\
& - P_r\left(12a_{12}\eta^{11} + 11a_{11}\eta^{10} + 10a_{10}\eta^9 + 9a_9\eta^8 + 8a_8\eta^7 + 7a_7\eta^6 + 6a_6\eta^5 + 5a_5\eta^4\right. \\
& \left. + 4a_4\eta^3 + 3a_3\eta^2 + 2a_2\eta + a_1\right)\left(b_{12}\eta^{12} + b_{11}\eta^{11} + b_{10}\eta^{10} + b_9\eta^9 + b_8\eta^8\right. \\
& \left. + b_7\eta^7 + b_6\eta^6 + b_5\eta^5 + b_4\eta^4 + b_3\eta^3 + b_2\eta^2 + b_1\eta + b_0\right) + P_rQ\left(b_{12}\eta^{12} + b_{11}\eta^{11}\right. \\
& + b_{10}\eta^{10} + b_9\eta^9 + b_8\eta^8 + b_7\eta^7 + b_6\eta^6 + b_5\eta^5 + b_4\eta^4 + b_3\eta^3 + b_2\eta^2 \\
& \left. + b_1\eta + b_0\right)
\end{aligned}
\tag{39}
$$

$$\begin{aligned}\phi_r = \; & 2c_2+6c_3\eta+12c_4\eta^2+20c_5\eta^3+30c_6\eta^4+42c_7\eta^5+56c_8\eta^6+72c_9\eta^7+90c_{10}\eta^8\\ & +110c_{11}\eta^9+132c_{12}\eta^{10}+S_c\left(a_{12}\eta^{12}+a_{11}\eta^{11}+a_{10}\eta^{10}+a_9\eta^9+a_8\eta^8+a_7\eta^7\right.\\ & \left.+a_6\eta^6+a_5\eta^5+a_4\eta^4+a_3\eta^3+a_2\eta^2+a_1\eta+a_0\right)\left(12c_{12}\eta^{11}+11c_{11}\eta^{10}+10c_{10}\eta^9\right.\\ & \left.+9c_9\eta^8+8c_8\eta^7+7c_7\eta^6+6c_6\eta^5+5c_5\eta^4+4c_4\eta^3+3c_3\eta^2+2c_2\eta+c_1\right)\\ & -S_c\left(12a_{12}\eta^{11}+11a_{11}\eta^{10}+10a_{10}\eta^9+9a_9\eta^8+8a_8\eta^7+7a_7\eta^6+6a_6\eta^5+5a_5\eta^4\right.\\ & \left.+4a_4\eta^3+3a_3\eta^2+2a_2\eta+a_1\right)\left(c_{12}\eta^{12}+c_{11}\eta^{11}+c_{10}\eta^{10}+c_9\eta^9+c_8\eta^8+c_7\eta^7\right.\\ & \left.+c_6\eta^6+c_5\eta^5+c_4\eta^4+c_3\eta^3+c_2\eta^2+c_1\eta+c_0\right)-S_c\lambda\left(c_{12}\eta^{12}c_{11}\eta^{11}+c_{10}\eta^{10}\right.\\ & \left.+c_9\eta^9+c_8\eta^8+c_7\eta^7+c_6\eta^6+c_5\eta^5+c_4\eta^4+c_3\eta^3+c_2\eta^2+c_1\eta+c_0\right)\end{aligned}$$

(40)

Minimizing the residual error by forcing equations (38)-(40) to zero at some set of collocation points within the domain in order to obtain the unknown coefficients.

Substituting the values into the trial functions to obtain the tangential velocity, temperature and concentration equations respectively.

(41)

(42)

(43)

Differential equation (41) to obtain

$$\begin{aligned}f'(\eta)= \; & 4.198568492\eta-9.142898253\eta^2+10.12054463\eta^3-7.351298285\eta^4\\ & +3.775950145\eta^5-1.394160825\eta^6+0.3638434712\eta^7-0.0637670133\eta^8\\ & +0.00670644390\eta^9-0.00031256324\eta^{10}-0.00000123348\eta^{11}\end{aligned}$$

(44)

Also substituting for f and f' in (34) with the corresponding constant values to obtain the pressure drop as,

$$
\begin{aligned}
G(\eta) = {} & -0.5000000000 + 4.198568492\eta - 9.142898253\eta^2 + 10.12054463\eta^3 \\
& -7.351298285\eta^4 + 3.775950145\eta^5 - 1.394160825\eta^6 + 0.3638434716\eta^7 \\
& -0.06376701332\eta^8 + 0.006706443900\eta^9 - 0.0003125631885\eta^{10} \\
& -0.000001233432062\eta^{11} + 1/2\big(1.000000000 + 2.099284246\eta^2 \\
& -3.047632751\eta^3 + 2.530136157\eta^4 - 1.470259657\eta^5 \\
& +0.6293250241\eta^6 - 0.1991658321\eta^7 + 0.04548043395\eta^8 \\
& -0.007085223702\eta^9 + 0.0006706443900\eta^{10} - 0.00002841483532\eta^{11} \\
& -0.0000001027860052\eta^{12}\big)^2
\end{aligned}
\tag{45}
$$

SKIN FRICTION

(46)

NUSSELT NUMBER

SHERWOOD NUMBER

(48)

The process of weighted residual method are repeated for different values of G_r, G_c, H_a, α, Q, P_r, S_c and λ.

4.0 Results

The computational results obtained compared well with Runge-kutta method as indicated in table1.

Table1: Effect of G_r, G_c, Q, S_c, on τ, Nu and Sh (PP-Physical Parameters)

		WRM			4^{th} order R-K		
PP	values	τ	Nu	Sh	τ	Nu	Sh

G_r	2.5	2.86521	0.53944	1.34943	2.86359	0.53931	1.34902
	5.5	3.77012	0.69197	1.39921	3.76616	0.69173	1.39866
	7	4.19857	0.75010	1.42060	4.19315	0.74981	1.41999
Q	0.3	4.02001	1.08149	1.40258	4.01476	1.08098	1.40195
	1.0	4.19857	0.75010	1.42060	4.19315	0.74981	1.41998
	2.0	4.63757	0.01876	1.46429	4.63208	0.01877	1.46368
S_c	0.01	4.79802	0.93631	0.35822	4.79293	0.93598	0.35822
	0.1	4.65693	0.89254	0.56181	4.65179	0.89221	0.56179
	0.62	4.19857	0.75010	1.42061	4.193156	0.74981	1.41999

4.0 Discussion

The numerical computation has been carried out using the method of Weighted Residual for variations in the governing parameters, the Hartmann number H_a, angle of inclination α, thermal Grashof number G_r, solutal Grashof number G_c, Prandtl number P_r, Schmidt number S_c, heat source Q and reaction rate parameter λ. The following default parameter values are adopted for computation: $G_r = G_c$ =7, $Q = \lambda$ =1, P_r =0.72, S_c =0.62, H_a =5 and $\alpha = 30^0$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

Figures 1 and 2 illustrate the velocity and pressure profiles for various values of Hartmann number H_a. It shows that as magnetic field parameter H_a increases, the velocity and pressure profiles decreases, confirming that the magnetic field exerts an impeding force on the fluid flow.

Figures 3 and 4 present velocity and pressure profiles for different angles of inclination of the magnetic field α, while other parameters are kept at some fixed values. An increase in the angle of inclination increases the effect of the buoyancy force and consequently the driving force to the fluid flow decreases which resulted in decreasing the velocity and pressure profiles.

Figures 5, 6 and 7 show the influence of different values of the Prandtl number P_r on the velocity, pressure and temperature profiles. It is observed that an increase in the ratio of momentum diffusivity to thermal diffusivity results in the respectively decrease in velocity and pressure profiles. Figure 7 shows that an increase in the P_r results in a decrease in the thermal boundary layer thickness and reduce the average temperature within the boundary layer because smaller values of P_r are equivalent to increasing in the thermal conductivities, therefore heat is able to diffuse away from the heated plate than higher values of P_r.

Figures 8, 9 and 10 represent the effect of Schmidt number S_c on the velocity, pressure and concentration profiles. Schmidt number is the ratio of the momentum to the mass diffusivity. An increase in S_c causes reductions in the velocity, pressure and concentration profiles which are accompanied by simultaneous decrease in the velocity and concentration boundary layers. Therefore, Schmidtnumber quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic velocity and concentration boundary layers.

Table 1 represents the effect of some physical parameters on skin friction, nusselt and sherwood number. It clearly shows that the an increase in the thermal Grashof number have an accelerating effect on the skin friction, nusselt and sherwood number respectively. Increase in heat source increase skin friction and sherwood number while it decreases the Nusselt number because heat within the boundary layer reduces. Also, skin friction and nusselt number decelerate as the schmidt number increases but have an increasing effect on the sherwood number.

5.0 Conclusion

The formulated partial differential equations governing the problem are non-dimensional and reduced to a couple ordinary differential equations by using scaling and translational symmetries. The numerical solution for scaling symmetry are obtained using the Weighted Residual method. From the numerical results, it is seen that, an increase in the magnetic field parameter (Hartmann number) or the degree of inclination of the magnetic field is manifested as a decrease in the flow velocity and pressure distribution. The velocity and pressure profiles increase as the thermal Grashof number increases. In the presence of increasing Prandtl or Schmidt numbers, the velocity, pressure, temperature and concentration profiles decreases respectively.
The result of studiesof flows past a permeable surface are of great interest due to its applications in science and engineering, as well as in many transport processes in nature.

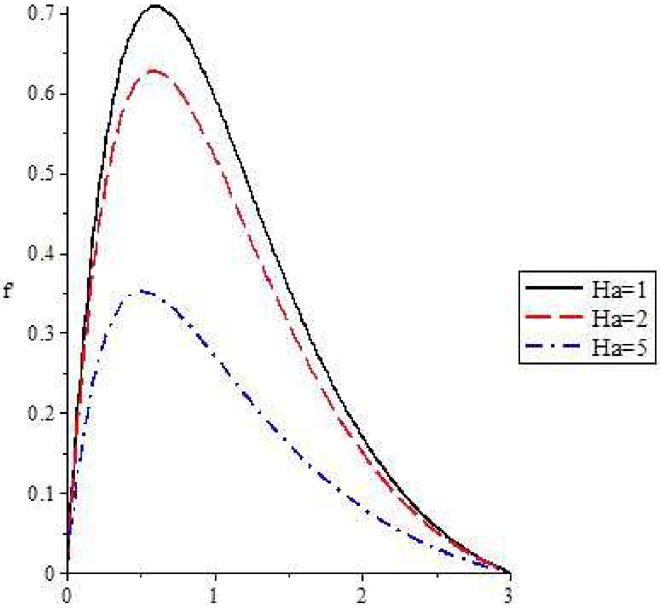

Fig. 1: Velocity profiles for different values of Ha

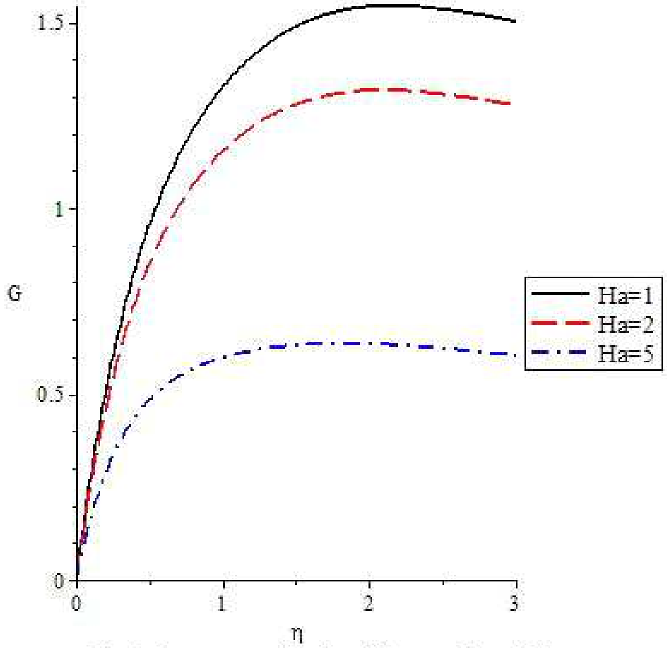

Fig. 2: Pressure profiles for different values of Ha

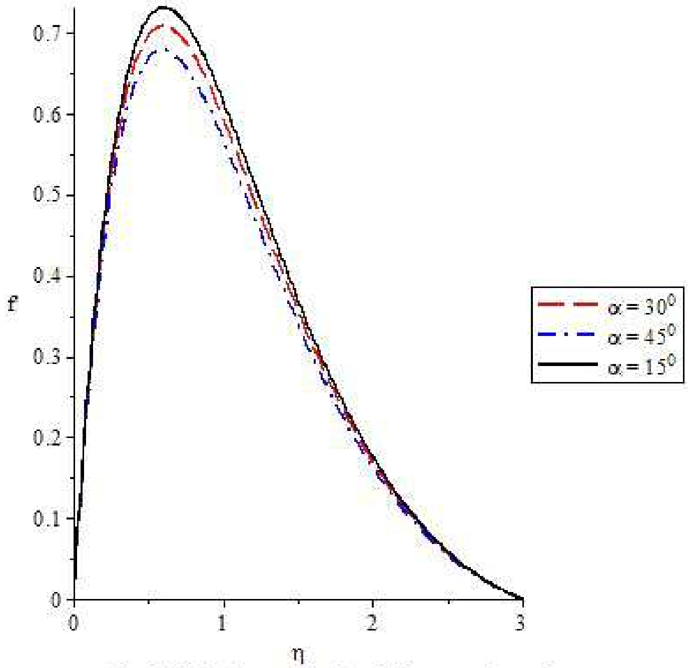

Fig. 3: Velocity profiles for different values of α

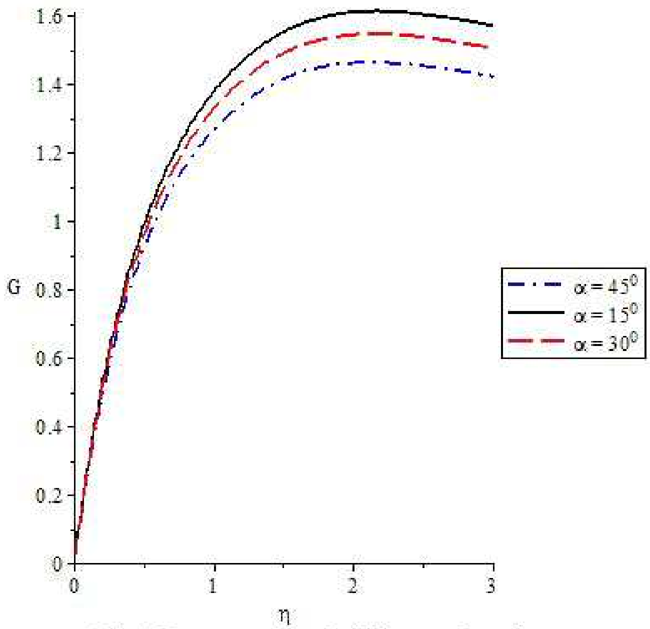

Fig. 4: Pressure profiles for different values of α

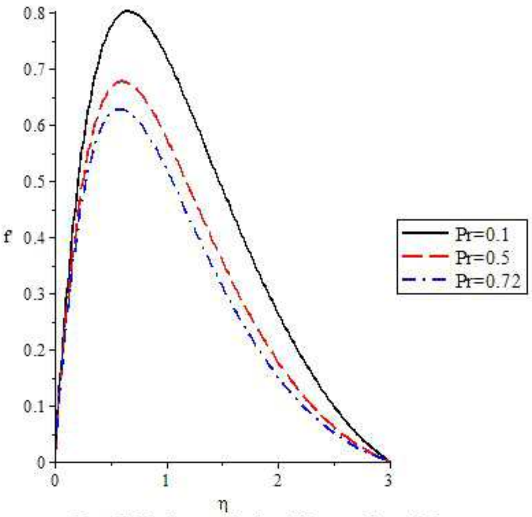

Fig. 5: Velocity profiles for different values of Pr

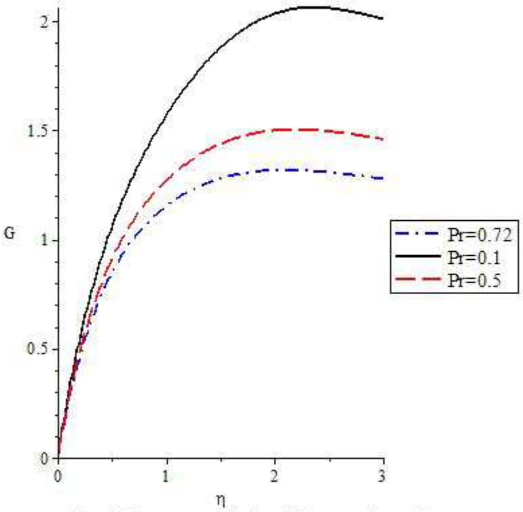

Fig. 6: Pressure profiles for different values of Pr

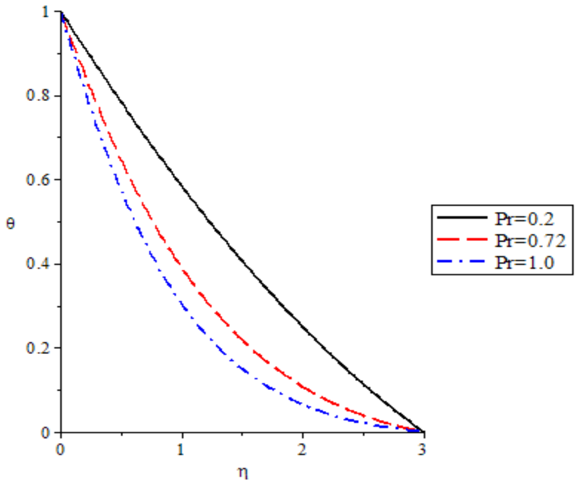

Fig. 7: Temperature profiles for different values of Pr

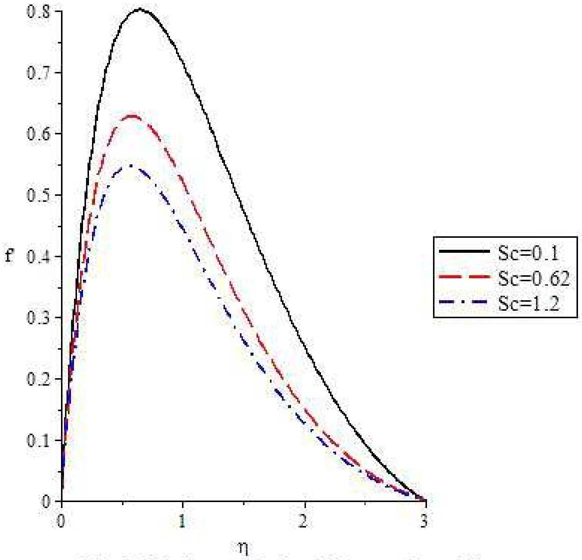

Fig. 8: Velocity profiles for different values of Sc

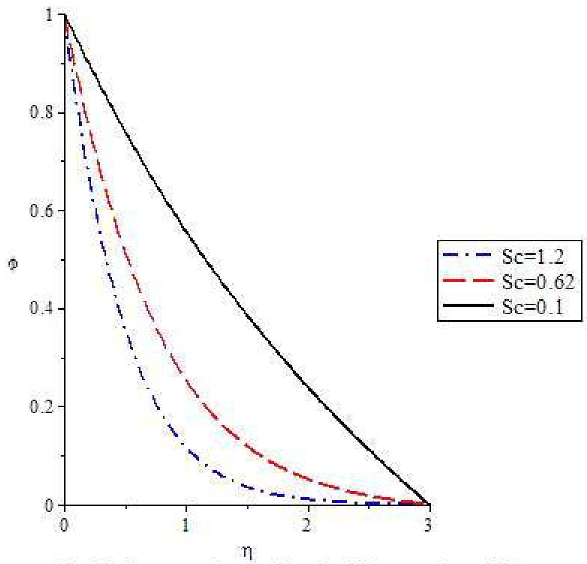

Fig. 10: Concentration profiles for different values of Sc

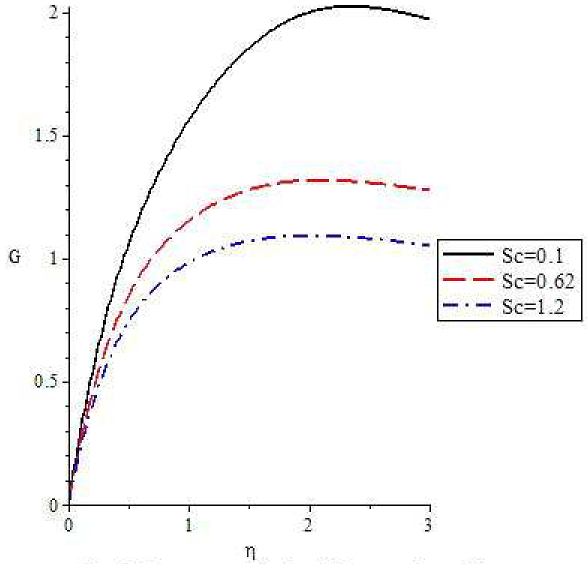

Fig. 9: Pressure profiles for different values of Sc

6.0 References

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