

## **Development of One-dimensional Temperature Distribution Model in a Hydraulic Press Platen**

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### *Abstract*

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*Hydraulic press is an essential compressive pressure processing equipment, which has a wide range of applications in polymer manufacturing industry. The rate of heat transfer through the hydraulic press plate has an extremely significant effect on polymer processing. This paper is on the development of a model to predict temperature distribution in a hydraulic press platen.*

*To achieve this, a physical structure was constructed to enable complete finite element analysis. Then steady-one-dimensional heat transfer ordinary differential equation, relevant assumptions and suitable boundary conditions are deduced on the geometric domain. The domain governing equation is then solved using finite element method. Obtained analytical solution from the governing equation compared to finite element model results shows that the model approximates very fast to the analytical solution as the number of element is increased. With this model, therefore, temperature distribution characteristics can be accurately predicted during design and service stage.*

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**Keywords:** Hydraulic press, Finite Element Method, Heat conduction, Temperature

### **1.0 Introduction**

Hydraulic press is an essential compressive pressure processing machine, which has a wide range of applications in production or manufacturing industry. The rate of heat supplied and transferred through a hydraulic press plate plays a vital role in the transfer of heat to the polymer material processed.

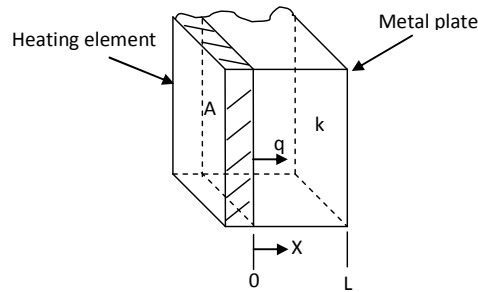
Heat transfer equation is a second-order partial differential equation that describes physical heat transfer phenomenon mathematical. In this application, the heat equation is expressed in one dimensional based on the given assumptions; which made it easier to apply the finite element method, a numerical method of solution to differential equations.

Finite element method of analysis has been used in the past to solve one dimensional problem. FE method was adopted to solve one dimensional heat equation in order to understand the basis behind FE method using quadratic B-spline and cubic B-spline. The solutions obtained compared with analytical solution are found to agree well the one dimensional analytical solution [1]. Galerkin B-spline finite element was used in [2] to obtain numerical solutions to one dimensional heat equation by reducing the initial boundary value problem to ordinary differential equations. The results they obtained were found to agree favourably with analytical solutions in literatures. Also, solutions to one-dimensional heat equation were achieved by the derivation of analytical heat equation and numerical explicit centred difference scheme. A computer program was then used on the heat equation which enabled error estimate to be used as basis of comparison [3]. More so, a one dimensional transient heat equation was solved analytically and numerically and the solutions obtained were found to compare favourably [4].

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## 2.0 Model Geometry



**Figure 1:** heat transfer section

## 3.0 Problem Statement

In a hydraulic press heat transfer takes place from the upper plate to the polymer substance. The rate of heat transfer in such system is very important in during design and operation, therefore the knowledge of how temperature varies in the direction of heat transfer is essential. Consequently, it becomes imperative to develop a model to predict the temperature distribution in one dimension in such a plate.

## 4.0 Mathematical Formulation

The study domain is subjected to the following assumptions:

1. Heat conduction is steady with no heat generation.
2. Heat conduction is one dimensional, along x-axis only ( $A \gg L$ ).
3. Thermal heat conduction across any section is uniform.
4. Thermal conductivity is constant (isotropic material).
5. Element length is uniform over domain region.
6. Heat transfer by radiation negligible with other five sides of plate well insulated so the entire heat energy generated by heating element is transferred at uniform rate along x-axis alone.

With these assumptions, the governing three-dimensional heat transfer equation is reduced to a second-order differential equation in one-dimensionas:

$$-k \frac{d^2 T}{dx^2} = q \quad \Omega(0 \leq x \leq L) \quad (1)$$

Subject to the boundary conditions

$$\begin{aligned} T(0) &= T_o \\ T(L) &= T_L \end{aligned} \quad (2)$$

Where, k is isotropic thermal conductivity of the material (W/m.K), T is the unknown temperature, L (m) is plate domain,  $q_o$ (W/m<sup>3</sup>)is the inner plate surface heat generated, and  $T_L$ (°K) is the outer platen surface temperature.

## 5.0 Method of Solution

The one dimensional differential equation is solved numerically using the finite element method [5]procedure.

### 5.1 Domain Symmetry Discretization Into Four Elements

The characteristic of the temperature field, T, is modelled using temperature profile normal to the y-axis (i.e. temperature T changes in x direction only). Therefore, the domain  $\Omega(0 \leq x \leq L)$  is subdivided into N (= 3)linear elements mesh along the x-axis. A typical element e will have interval of h (=  $x_{i+1} - x_i$ ) along the x-axis. Assume that the domain is discretized into N = 3linear elements mesh then  $h = (L - 0)/3$ .

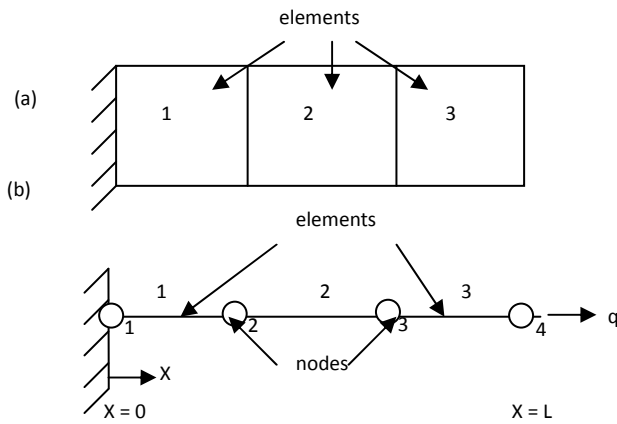


Fig 2: (a) domain discretization into four elements mesh (b) elements nodal points

### 5.2 Derivation of Element Equations for Typical Element E In The Mesh

The residual of the governing equation is

$$0 = k \frac{d^2 T}{dx^2} + q_o \tag{3}$$

Subject to boundary conditions

$$T_o = 1525,$$

$$T_L = 1000$$

The Galerkin scheme is therefore given as

$$\int_{\Omega^e} w R d\Omega = 0 \tag{4}$$

Where the Galerkin integral is

$$\int_{\Omega^e} w \left[ k \frac{d^2 T}{dx^2} + q \right] d\Omega = 0 \tag{5}$$

The weak form of this equation is

$$\begin{aligned} 0 &= \int_0^h \left( k \frac{d\psi_i^e}{dx} \sum_{j=1}^n T_j^e \frac{d\psi_j^e}{dx} dx + w_i q \right) dx - \left[ k \psi_i^e \frac{dT}{dx} \right]_0^h \\ &= T_j^e \sum_{j=1}^n \int_0^h k \frac{d\psi_i^e}{dx} \frac{d\psi_j^e}{dx} dx + q \int_0^h w_i dx - \left[ k \psi_i^e \frac{dT}{dx} \right]_0^h \end{aligned} \tag{6}$$

In matrix form

$$[K_{ij}^e] \{T^e\} = \{F_i^e\} - \{Q_i^e\} \tag{7}$$

Where, w is the weight function.

Two elements solution is

$$T_2 = \frac{qh^2}{2k} + \frac{1}{2}(T_o + T_L) = \frac{6300 \times 0.75^2}{2 \times 1200} + \frac{1}{2}(1525 + 1000) = 1.477 + 1262.5 = 1263.98^\circ K$$

Where,

$$T_o = T_1 = 1525^\circ K, T_L = T_3 = 1000^\circ K, q = 6300 \frac{W}{m^3}, k = 1200 \frac{W}{m \cdot ^\circ K}, h = \frac{L}{2} = \frac{1.5m}{2} = 0.75m$$

And the three elements solutions are

$$T_2 = \frac{qh^2}{2k} + \frac{1}{3}(2T_o + T_L) = \frac{6300 \times 0.75^2}{2 \times 1200} + \frac{1}{3}(2 \times 1525 + 1000) = 1.477 + 1350 = 1351.48^\circ K$$

$$T_3 = \frac{qh^2}{2k} + \frac{1}{3}(T_o + 2T_L) = \frac{6300 \times 0.75^2}{2 \times 1200} + \frac{1}{3}(1525 + 2 \times 1000) = 1.477 + 1175 = 1176.48^\circ K$$

Where,

$$T_o = T_1 = 1525^\circ K, T_L = T_3 = 1000^\circ K, q = 6300 \frac{W}{m^3}, k = 1200 \frac{W}{m \cdot ^\circ K}, h = \frac{L}{2} = \frac{1.5m}{2} = 0.5m$$

## 6.0 Determination of Analytical Solution

The governing equation is

$$-k \frac{d^2 T}{dx^2} = q \quad \Omega(0 \leq x \leq L) \quad (8)$$

Subject to the boundary conditions

$$T(0) = T_o \quad (9)$$

$$T(L) = T_L$$

The exact solution function is

$$T(x) = \frac{x}{L} \left[ \frac{qL^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + (T_L - T_o) \right] + T_o \quad (10)$$

The nodal solutions by analytical approach are

$$T(0) = 1525, T(0.5) = 1352, T(1.0) = 1177, T(1.5) = 1000$$

## 7.0 Results and Discussion

Figure 3 is a graph of temperature against domain space which compares exact (triangle) and FE solutions for 2(circle) and 3(diamond) elements. Each graph shows that temperature decreases linearly in the direction of increasing space (i.e. in the direction of heat flow) along the heated platen. This graph shows that the FE solutions (circle and diamond) converged very fast to exact solution (triangle) as the number of elements is increased.

In figure 4 is shown how temperature values are distributed along the domain at different locations. It revealed that temperature values 1525, 1439, 1353, 1178, 1090 and 1000°K at the nodes for 3 elements mesh are the same for the nodal values of the exact solution; which is an indication that FE solution converged very fast with the exact solution.

Figures 5 and 6 are three dimensional views of temperature distributions for exact and FE solutions. With these views is clearly seen how temperature is distributed for the exact (foreX-strip), 3 elements mesh (middle Y-strip) and 2 elements mesh (last Z-strip). In Figure 6 is shown that exact and 3 elements mesh solutions converged and that temperatures are linearly distributed compared to the 2 elements mesh.

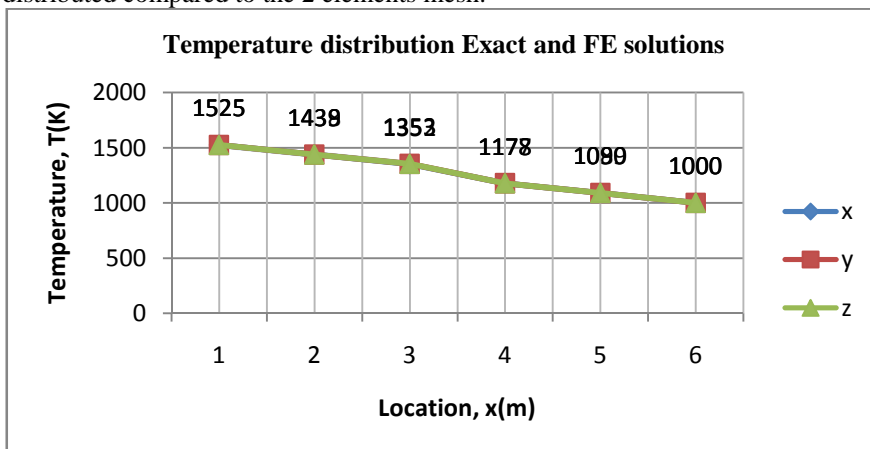


Fig 3: Comparison of Exact and FE Solutions

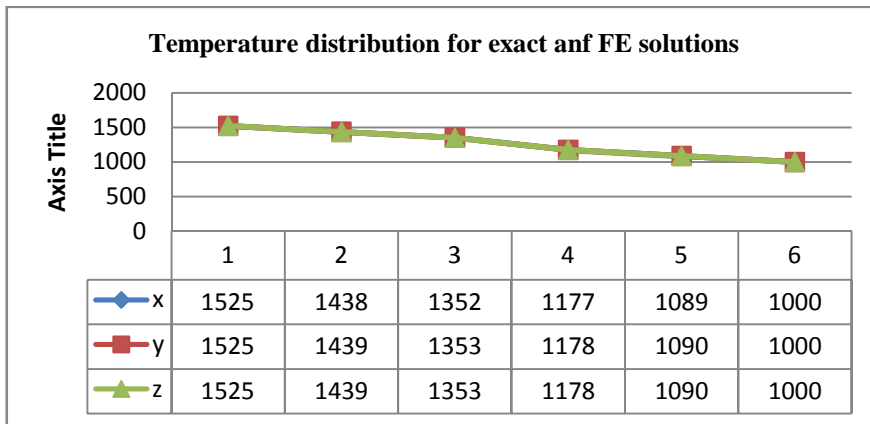


Fig. 4: Three Elements Distribution

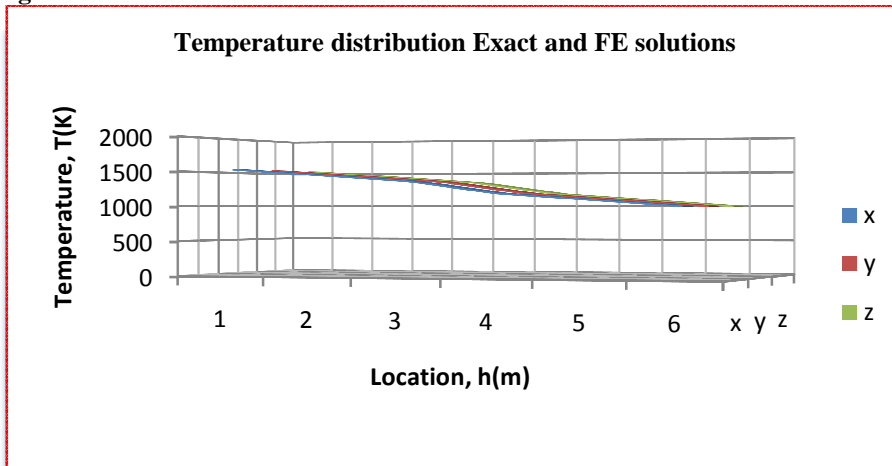


Fig. 5: 3D view of exact and FE solutions

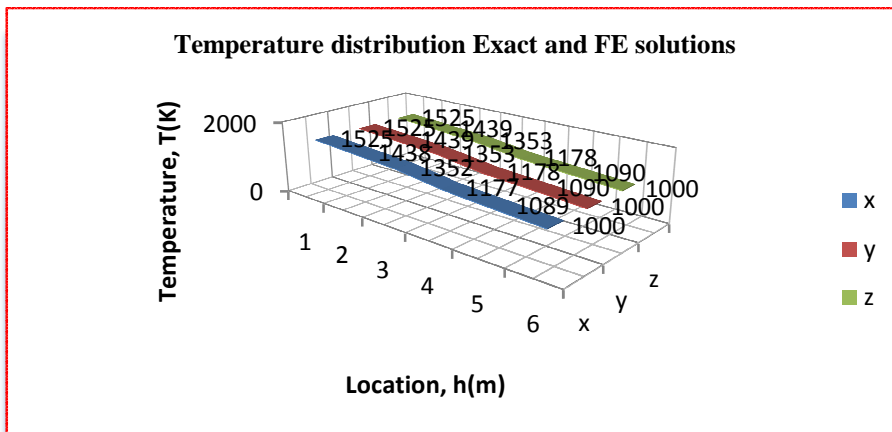


Fig. 6: 3-D view of exact and FE solutions

In Table 1 is shown nodal values of temperature (in bold print) for exact and FE solutions. The table shows that the 3 elements nodal values coincide with those of the exact at points 0.00, 0.50, 1.00 and 1.5 respectively.

**Table 1:** Comparison of finite element and exact solutions

Nodal values (x) (m)	FE solution ( 2 elements) ( <sup>o</sup> K)	FE solution ( 3 elements) ( <sup>o</sup> K)	Exact solution ( <sup>o</sup> K)
0.00	<b>1525.00</b>	<b>1525.00</b>	<b>1525.00</b>
0.25	1438.00	1439.00	1439.00
0.50	1351.00	<b>1352.00</b>	<b>1352.00</b>
0.75	<b>1264.00</b>	1265.00	1265.00
1.00	1176.00	<b>1177.00</b>	<b>1177.00</b>
1.25	1088.00	1089.00	1089.00
1.50	<b>1000.00</b>	<b>1000.00</b>	<b>1000.00</b>

Note: Nodal values of temperature are in bold print.

## 8.0 Conclusion

This study on finite element method revealed that it is an effective numerical tool for developing models for predicting temperature distribution in the heated platen of a hydraulic press.

The study revealed that the FE analysis of 2 and 3 elements showed that as the number of elements are increased FE results converged fast to the exact result. Also, the solutions at the nodes are observed to be the same with exact values at the nodes, which is a good indication that the model is suitable for determination of temperature distribution in a hydraulic press platen. Finally, in case of further analysis on this type of hydraulic press, the heat flux to the mould should be varied with different platen material considered.

## 9.0 Appendix A

### 1. Finite element solution determination

The residual, R, of the governing equation is

$$q = -k \frac{d^2 T}{dx^2} \quad (A1)$$

The Galerkin scheme is therefore given as

$$\int_{\Omega^e} w R d\Omega = 0 \quad (A2)$$

Where the Galerkin integral is

$$\int_{\Omega^e} w \left[ -k \frac{d^2 T}{dx^2} + q \right] d\Omega = 0 \quad (A3)$$

Let us assume that the exact solution T(x) approximates the finite element solution T<sup>e</sup>(x) as

$$T(x) \approx T^e(x) = \sum_{j=1}^n T_j^e \psi_j^e; i = j = 1, 2, \dots, n \quad (A4)$$

Differentiating (4) with respect to x yields

$$\frac{dT}{dx} = \sum_{j=1}^n T_j^e \frac{d\psi_j^e}{dx} \quad (A5)$$

Then let  $w = \psi_i^e$  and differentiating with respect to x will give

$$\frac{dw}{dx} = \frac{d\psi_i^e}{dx} \quad (A6)$$

Where,

w = the weight function.

T(x) = the exact solution

T<sup>e</sup>(x) = the FE solution (approximate solution)

T<sub>j</sub><sup>e</sup>(x) = the unknown nodal temperature values

ψ<sub>j</sub><sup>e</sup> = approximation functions selected

n = number of nodes in each element

For linear-two-nodes formulation the approximation functions are given as [5].

$$\psi_i^e = \frac{x_B - x}{x_B - x_A} = 1 - \frac{x}{h}; \frac{d\psi_i^e}{dx} = -\frac{1}{h} \quad (\text{A7})$$

$$\psi_j^e = \frac{x - x_A}{x_B - x_A} = \frac{x}{h}; \frac{d\psi_j^e}{dx} = \frac{1}{h}$$

Where  $x$  is the local element coordinate and  $x_B - x_A$  is element length  $h$ .

The weak form of equation (3) is

$$0 = \int_0^h \left( k \frac{d\psi_i^e}{dx} \sum_{j=1}^n T_j^e \frac{d\psi_j^e}{dx} dx + w_i q \right) dx - \left[ k \psi_i^e \frac{dT}{dx} \right]_0^h \quad (\text{A8})$$

$$= T_j^e \sum_{j=1}^n \int_0^h k \frac{d\psi_i^e}{dx} \frac{d\psi_j^e}{dx} dx + q \int_0^h w_i dx - \left[ k \psi_i^e \frac{dT}{dx} \right]_0^h$$

$$k_{ij}^e = \int_0^h k \frac{d\psi_i^e}{dx} \frac{d\psi_j^e}{dx} dx$$

$$f_i^e = q \int_0^h w_i dx \quad (\text{A9})$$

$$Q_i^e = \left[ k \psi_i^e \frac{dT}{dx} \right]_0^h = Q_o^e - Q_h^e$$

In matrix form

$$[K_{ij}^e] \{T^e\} = \{F_i^e\} - \{Q_i^e\} \quad (\text{A10})$$

Local element matrix

$$k_{ij}^e = \int_0^h k \frac{d\psi_i^e}{dx} \frac{d\psi_j^e}{dx} dx \quad (\text{A11})$$

$$k_{11}^e = kA \int_0^h \left( -\frac{1}{h} \right) \left( -\frac{1}{h} \right) dx = \frac{kA}{h}$$

$$k_{12}^e = kA \int_0^h \left( -\frac{1}{h} \right) \left( \frac{1}{h} \right) dx = -\frac{kA}{h} = k_{21}^e \quad (\text{A12})$$

$$k_{22}^e = kA \int_0^h \left( \frac{1}{h} \right) \left( \frac{1}{h} \right) dx = \frac{kA}{h}$$

The local element stiffness matrix, force vector and source vector are

$$[K^e] = \frac{kA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (\text{A13})$$

$$f_i^e = Aq \int_0^h w_i dx$$

$$f_1^1 = \frac{Aq}{2}, f_2^1 = \frac{Aq}{2}, f_1^2 = \frac{Aq}{2}, f_2^2 = \frac{Aq}{2}, f_3^2 = \frac{Aq}{2}, f_2^3 = \frac{Aq}{2}, f_3^3 = \frac{Aq}{2} \quad (\text{A14})$$

$$[F^e] = \frac{Aq}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Source vector

$$Q_i^e = \left[ k\psi_i^e \frac{dT}{dx} \right]_{\Gamma} \quad (A15)$$

$$= Q_o^e - Q_h^e$$

1. Assemblage of the two elements (of 3 nodes) equations

$$\frac{kA}{h} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = Aq \begin{Bmatrix} f_1^1 \\ f_2^1 + f_1^2 \\ f_2^2 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ Q_2^1 + Q_1^2 \\ Q_2^2 \end{Bmatrix} \quad (A16)$$

Imposition of boundary conditions

$$T(0) = T_o, T(L) = T_L \quad (A17)$$

$$\frac{kA}{h} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \frac{Aqh}{2} \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ 0 \\ Q_2^2 \end{Bmatrix} \quad (A18)$$

$$\frac{kA}{h} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \frac{qh}{2} \{2\} + \frac{kA}{h} \begin{Bmatrix} T_o \\ T_L \end{Bmatrix}$$

The solution is

$$T_2 = \frac{qh^2}{2k} + \frac{1}{2}(T_o + T_L) \quad (A19)$$

Where,

$$T_o = T_1 = 1525, T_L = T_3 = 1000 \quad (A20)$$

2. Assemblage of the three elements (of 4 nodes) equations

$$\frac{kA}{h} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} T_o \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \frac{Aqh}{2} \begin{Bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ 0 \\ 0 \\ Q_2^3 \end{Bmatrix} \quad (A21)$$

Imposition of boundary conditions

$$T(0) = T_o, T(L) = T_L$$

$$\frac{kA}{h} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \frac{Aqh}{2} \begin{Bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{Bmatrix} + \begin{Bmatrix} T_o \\ 0 \\ 0 \\ T_L \end{Bmatrix} \quad (A22)$$

$$\frac{kA}{h} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \frac{Aqh}{2} \begin{Bmatrix} 2 \\ 2 \end{Bmatrix} + \frac{kA}{h} \begin{Bmatrix} T_o \\ T_L \end{Bmatrix}$$

The solutions are

$$T_2 = \frac{qh^2}{2k} + \frac{1}{3}(2T_o + T) \quad (\text{A23})$$

$$T_3 = \frac{qh^2}{2k} + \frac{1}{3}(T_o + 2T_L)$$

Where,

$$T_o = T_1 = 1525, T_L = T_4 = 1000 \quad (\text{A24})$$

## 2. Determination of Analytical solution

Integrating the following twice

$$-k \int_0^L \int_0^L \frac{d^2 T}{dx^2} dx = q_o \quad (\text{A25})$$

Yields

$$T(x) = -\frac{qx^2}{2k} + Ax + B \quad (\text{A26})$$

So applying the first boundary condition  $T(o) = T_o$

$$T(x=0) = A(0) + B = T_o \Rightarrow B = T_o$$

Applying the second boundary condition,  $T(L) = T_L$

$$T(L) = -\frac{qL^2}{2k} + AL + B = T_L \Rightarrow A = \left( \frac{qL^2}{2k} + T_L - T_o \right) \frac{1}{L} \quad (\text{A27})$$

Now substituting for A and B into  $T(x) = Ax + B$

$$\begin{aligned} T(x) &= -\frac{qx^2}{2k} + \left( \frac{qL^2}{2k} + T_L - T_o \right) \frac{x}{L} + T_o \\ &= -\frac{qx^2}{2k} + \frac{qL^2 x}{2kL} + (T_L - T_o) \frac{x}{L} + T_o \\ &= \frac{qL^2}{2k} \left( \frac{x}{L} - \frac{x^2}{L^2} \right) + (T_L - T_o) \frac{x}{L} + T_o \\ &= \frac{x}{L} \left[ \frac{qL^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + (T_L - T_o) \right] + T_o \end{aligned} \quad (\text{A28})$$

## 3. Problem data:

Plate thermal conductivity,  $K = 1.2\text{W/m.K}$

Surface area of plate,  $A = 15\text{m}^2$

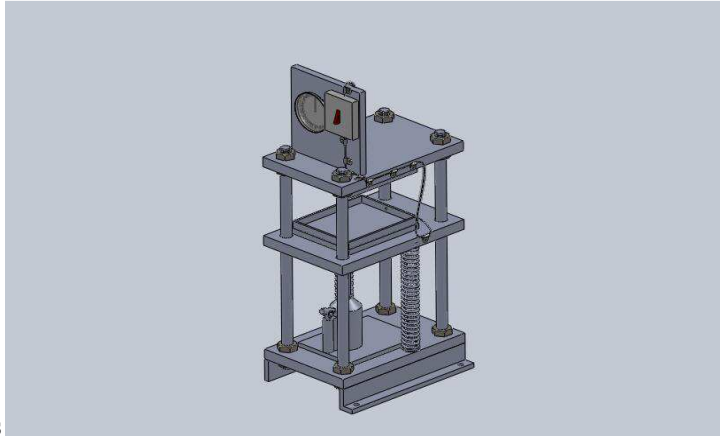
Plate thickness,  $L = 1.5\text{m}$

Outer platen surface temperature  $T_L = 1000^\circ\text{K}$

Heat generated  $q_o = 6300\text{W/m}^3$

Heat flux  $q = 420\text{W/m}^2$

## Appendix B



Hydraulic Press

## 10.0 References

- [1.] Dabral, V., Kapoor, S and Dhawan S (2011). *Numerical simulation of one dimensional heat equation; B-spline finite element method*. Indian Journal of Computer and Engineering Vol. 2, No. 2, 222-235.
- [2.] Dhawan, S. and Kumar, S. (2009). *A numerical solution of one dimensional heat equation using cubic B-spline basis functions*. International Journal of Research and Reviews in Applied Sciences Vol. 1, Issue 1, 71-77.
- [3.] Kabir M. H., Afroz A., Hossain M. M. and Gani M. O. (2010). *A finite difference method for one dimensional heat equation*. ULAB Journal of Science and Engineering Vol. 1, No. 1, 54-60.
- [4.] Mojtabi, A. and Deville, M.O. (2015). *One dimensional linear advection-diffusion equation: analytical and finite element solutions*. Journal of Computers and Fluids, Vol. 107, 189-195.
- [5.] Reddy J.N. (1993). *A Introduction to the Finite Element Method 2<sup>nd</sup> ed*. New York: McGraw-Hill, Inc.
- [6.] Cengel, Y.N. (2007). *Heat and Mass Transfer (A practical approach) 3<sup>rd</sup> ed*. New Delhi: Tata Mcgraw-Hill Publishing Co. Ltd.
- [7.] Incropera F.P. and Dewitt D.P. (2005). *Fundamentals of Heat and Mass Transfer 5<sup>th</sup> ed*. New York: John Wiley and Sons
- [8.] Badakundri U. S., Kullur S. and Kulkarni A. A. (2015). *Finite Element Analysis of Hydraulic Press Machine*. International Journal on Recent Technologies in Mechanical and Electrical Engineering (IJRMEE), Vol. 2, Issue 5, 18-24.
- [9.] More D. A., Chhaphkane N. K. and Kolhe R. (2015). *Design, Development and Optimization of Hydraulic Press*. International Journal for Research in Applied Science and Engineering Technology (IJRASET). Vol. 3, Issue VI, 902-907.
- [10.] More R. S. and Kulkarni S. R. (2015). *Finite Element Analysis and Optimization of "c" Types Hydraulic 200 tons Press*. International Journal of Research and Engineering Technology (IJRET). Vol. 2, Issue 3, 1385-1391