Journal of the Nigerian Association of Mathematical Physics
Volume35,(May, 2016), pp 209 – 214
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Analytic Method for Determination of LumbarSpine's Curvature(Lordosis) and the Vertebral Rotational Range of Motion (ROM)

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Abstract

In this paper, consideration is given to the geometric and mechanical properties of the lumbar spine to derive the mathematical equations that can be used to estimate the lordosis and the rotational range of motion of the lumbar spine. the equations are simple and strait forward. only two parameters- the length of the lumbar spine and of the chord formed by the lumbar spine are required and these can easily be determined from a radiographic image of the lumbar spine. Our results made it possible for us to define two types of range of motion which can allow for more comprehensive analysis of abnormality.

Keywords: Lumbar spine, Curvature, Lordosis, Range of motion,Osculating circle, MATLAB.

1.0 Introduction

The increasing cases of low back pain globally has lead to a wave of findings into the causes and aetiologyof this problem [1]. Mechanical factors have been considered as predisposing factors[2-4]. To investigate this claim, biomechanical modelling has been extensively utilised. To date, each model developed falls in either of the two existing theories- equilibrium and stability theory. While equilibrium theory seeks to distribute loads among various anatomical members of the lumbar spine, the stability seeks to establish structural tolerance [5].The techniques often used suffer from one limitation or the other[2,3]. Furthermore, a number of phenomena of the lumbar spine still remained difficult to investigate using these theories. For instance, determination of center of mass which is key to understanding mechanics of many system has been a big question, understanding coupled motion among other things have been controversial[2,6]. In a bit to develop a complete and more simplified theory, we approach thetheory development from the perspective of structural mechanics. We first developed the equation of lumbar spine in three degrees-of-freedom and used it to determine and investigate the center of mass in the sagittal plane and the reaction in the inter-vertebral disc [7-9]. In order to have a complete analysis and better understand of the dynamics of the center of mass, we moved on to develop the equation in six-degrees-of-freedom[10]. In furtherance of our objective of developing a more complete, simple and accurate theory, the present work developed equations that can address a question that remains unresolved-the range of motion (ROM).Researchers have found it difficult to achieve consensus on the issue of methodology for measuring motion, or the cut-off value or values beyond which the motion segment should be diagnosed as having a lumbar segmental motion disorder (LSMD) [11-27]. This, may be due to complications in measurement techniques which result from the various models used [28].The need to study lumbar spine abnormal kinematic behaviour is based on the postulation that abnormal kinematics behaviour is associated with low back pain (LBP) [11, 29-31]. The abnormality is either segmental rigidity or instability [30-34]. To ascertain this postulation, accurate and easy method for estimating LSMD is required. To proceed further, it is worthwhile to understand what the lumbar spine is made of and its functionality.

The lumbar spine consist the five vertebrae between the rib cage and the pelvis. They are the largest segments of the vertebral column and are characterized by the absence of the foramen transversarium within the transverse process, and by the absence of facets on the sides of the body. They are designated L1 to L5, starting at the top. The lumbar vertebrae help support the weight of the body, and permit movement [35]. This area is commonly called the "lower back". The lumbar vertebrae are the largest of the vertebrae because of their weight-bearing function supporting the torso and head [36]. The joints between the vertebrae are separated by disc. The presents of the joint allows a vertebra six-degree-of –freedom hence, each vertebra can

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translate or rotate relative to another. The extent to which this relative motion occurs is known as range of motion [28,37]. One obvious geometric fact about lumbar spine is its curve (Figure 1A). During any rotational motion, the lumbar spine is subjected to both compression stress at one end and extension stress at another leading to a neutral length midway of the depth of the lumbar spine(Figure 1B). The length of neutral axis usually remains unchanged[38].

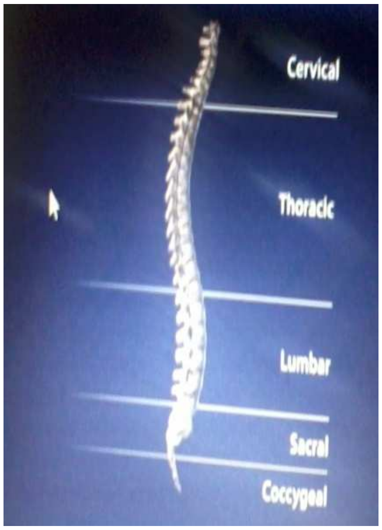

Figure 1A: The Structure of the Spine Showing the Various Sections

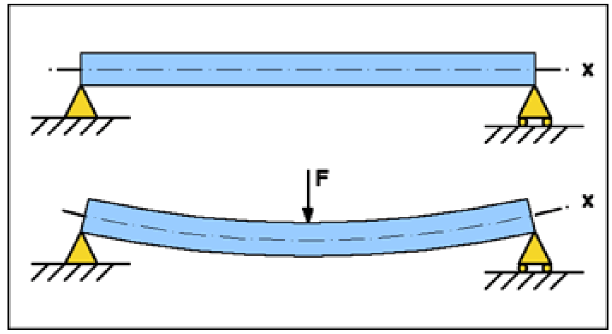

Figure 1B: A Beam Under Bending Moment Showing Neutral Axis X

2.0 Method

The length l of the circumference of a circle with radius r is given by

$$l = 2\pi r \qquad (1)$$

This means that every flexible material of length l can be bent into a circle of radius r. The size of the circle formed is dependent on the length l. If we have a material of length l that can form a circle of radius r,and we choose to form a circle of radius $R > r$ instead, we will end up with an arc whose length is

$$l = \frac{\theta}{360} 2\pi R \qquad (2)$$

Where θ is the angle subtended at the center of an osculating circle [39,40] to the arc whose arc length is equal to l.

Let us represent the lumbar spine in Figure 1A with arc AB of the circle in Figure 2. If l is the length of the neutral axis [38], then we have

$$l = \frac{\alpha}{360} 2\pi R \qquad (3)$$

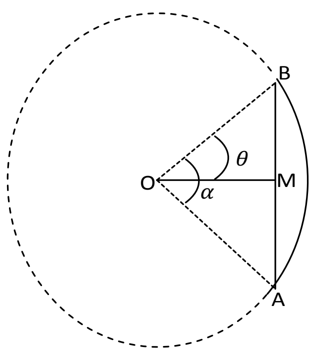

Figure 2: Geometric representation of lumbar spine.

$\overline{AB}$ is a chord of the osculating circleABA which divides the circle into major arc AOB and minor arc AB.

$\overline{OA} = \overline{OB} = R$, the radius of the circle and

$$\theta = \frac{1}{2}\alpha \qquad (3)$$

Let s be the length of chord $\overline{AB}$, the length $\overline{MB}$ is $\frac{1}{2}s$. Using triangle OMB we have

$$sin\theta = \frac{s}{2R} \quad (4)$$

Substituting equation (3) in equation (4) we have

$$sin\frac{1}{2}\alpha = \frac{s}{2R} \quad (5)$$

Making R the subject formula in equation (5) and substitute in equation (2) and have

$$l = \frac{s\alpha}{2sin\frac{1}{2}\alpha} \quad (6)$$

Since equation (6) is nonlinear in α, solving for α could be a bit challenging. To simplify equation (6) we approximate $sin\frac{1}{2}\alpha$ with a polynomial of first degree inα. We used MATLAB programme in MATLAB R2008b to evaluate $sin\frac{1}{2}\alpha$ within the interval$\alpha \in [0°, 70°]$ and fit a polynomial using excel (Figure 3). The choice for the interval is informed by the fact that the bend on the lumbar curve is not sharp hence the curvature will be small [41] which imply small values of α by observing that

$$\alpha = \frac{360l}{2\pi R} = \frac{360l\kappa}{2\pi} = l\kappa \quad (7)$$

where κ is curvature defined as [38]

$$\kappa = \frac{1}{R} \quad (8)$$

Also, the maximum ROM reported in literatures fall within the chosen interval [25,30,31].

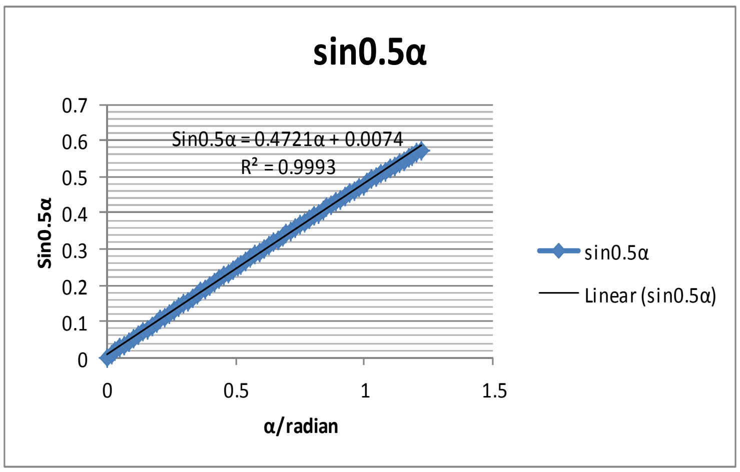

Figure 3: Linear regression of $sin\frac{1}{2}\alpha$ on α

The regression equation is

$$sin\frac{1}{2}\alpha = 0.4721\alpha + 0.0074 \quad (9)$$

Substituting equation (9) in (6) and simplifying, we have

$$0.9442l\alpha + 0.0148l = s\alpha \quad (10)$$

Equation (10) has the solution of the form

$$\alpha = \frac{0.0148l}{s - 0.9442l} \quad (11)$$

To find the curvature, we substitute equation (11) in equation (7) and simplify as

$$\kappa = \frac{0.0148}{s - 0.9442l} \quad (12)$$

Let α_n be the angle subtended by the lumbar spine at the center of the osculating circle when at a neutral position,α_f be the angle subtended by the lumbar spine at the center of the osculating circle when at a flexed position and α_e be the angle subtended by the lumbar spine at the center of the osculating circle when at an extended position, we defined two types ofrotational range of motion: Partial and Total rotational range of motion.

Partial rotational range of motion are defined quantitatively as

$$ROM_{pf} = |\alpha_n - \alpha_f| \quad (13)$$

$$ROM_{pe} = |\alpha_n - \alpha_e| \quad (14)$$

Where pf and pe mean partial at flexion and extension respectively.
Total rotational range of motion is defined quantitatively as

$$ROM = |\alpha_f - \alpha_e| = RM_{pf} + RM_{pe} \quad (15)$$

Let us now define rotational range of motion in term of curvature of the lumbar spine.
If ƙ_n is the curvature of the osculating circle to the lumbar curve when at a neutral position,ƙ_f is the curvature when at a flexed position and ƙ_e is the curvature when at an extended position then the partial rotational range of motion is defined quantitatively as

$$ROM_{pf} = |\text{ƙ}_n - \text{ƙ}_f| \quad (16)$$

$$ROM_{pe} = |\text{ƙ}_n - \text{ƙ}_e| \quad (17)$$

Where pf and pe mean partial at flexion and extension respectively.
Total rotational range of motion is defined quantitatively as

$$ROM = |\text{ƙ}_f - \text{ƙ}_e| = RM_{pf} + RM_{pe} \quad (18)$$

3.0 Results and Discussion

Equation (11) links the angle subtend at the center of an osculating circle by the lumbar curve with the easy to determine geometrical properties of the lumbar spine-the length$l$ and the chord$s$. This equation can, with the help of equation (3) allow for the determination of the radius$R$ of the osculating circle. Hence, lumbar spine can be fully geometrized. Equation (12) is the curvature of the lumbar spine. It can be claimed from the equation that the curvature is small. This is in agreement with reality as lumbar spines are naturally not sharply bent (Figure 1).
For equation (11) and (12) to make physical meaning, the denominator must be greater than zero $((s - 0.9442l) > 0$. This is because infinite curvature and angle are not realistic. Also, negative curvature and angle are meaningless. Alternatively, one can say that equation (11) and (12) make physical meaningonly if the ratio $\frac{s}{l} > 0.9442$. Since the lumbar spine is never flat even when totally flexed [42], then the ratio $\frac{s}{l}$ is always less than one. This means it is openly bounded above and below by the interval $(0.9442,1)$. This also implies that Mafuyai ratio μ [10] is openly bounded below and above by the interval $(1,1.06)$. Interestingly, rotational range of motion could as well be defined in term of mafuyai ratio since it is bounded.
The need to accurately determine the lordosis of the lumbar spine has become very important recently. This is because surgical operations in part of the spine can result in changes in the other. And whether this can affect subject recovery has been a subject of investigation [43]. Equations (11) and (12) allow us to estimate the lordosis either in terms of angle or curvature. The ease presented by this method one of its measure advantage.
Equations (13) to (18) give the various definitions of rotational range of motion. The simplicity with which ROM can be determined using this equations and possible accuracy expected is the advantage this technique has over experimental [27,28.41] techniques. With a radiograph of a lumbar spine, the length and the chord can easily be determined. This offers a cheap, simple, available and possibly the most accurate means of determining the kinematics of the lumbar spine. The beauty of our definitions of rotational range of motion is that it allows for more comprehensive understanding of where abnormality lies. That is whether it is in flexion or extension.

4.0 Conclusion

This work has provided a analytic method for the determination of lumbar spine's lordosis and rotational range of motion. its chief advantage lies with simplicity of reproducibility of methods involve in the measurements and affordability since X-ray imaging is not expensive. This work is therefore open for experimental validation.

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