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Estimating the Depth, Dip and Velocity from Refracted Arrivals for a Dipping Refractor

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Abstract

Calculating depth thickness, dip and velocity can be very tedious when involving a dipping refractor. Properly programmed computers can, of course, relieve much of this, but the calculations still requires a lot of input. This paper however shows the complete derivation of the depth thickness, dip and velocity, and how the knowledge of the derivation will assist the geophysicist minimize the interpretational ambiguities and cumbersome computations associated with seismic refraction data.

Keywords: Depth thickness, dip, velocity, dipping refractor, seismic refraction data.

1.0 Introduction

In practice, the refracting earth interface is often not horizontal. The assumption of flat layers often leads to errors in estimating the velocity and depth. From the refraction theory [1, 2], the inverse of the slope of the line associated with the refracted wave arrivals is equal to the bedrock velocity V_1. Also note that the inverse of the slope of the line associated with the direct wave arrivals is equal to the velocity of the weathering layer V_o. However when the refractor is dipping, it turns out that the inverse of the slope of the refracted arrival is no longer equal to the bedrock velocity V_1 (Figure 1). An extra parameter – dip of the refractor, needs to be estimated [3].
This paper presents a stepwise approach to estimating the refractor dip Φ, the bedrock velocity V_1 and finally the depth to the bedrock at shot/receiver stations.

2.0 Theory

Seismic refraction data is always represented on a T-X plot. The vertical scale of the plot is corrected refraction time, and the horizontal scale is the distance between the shot and the receiver corresponding to the trace from which the time was read. For each distance, there will be two times corresponding to refraction times from down-dip and up-dip directions [4,5].In the case of a flat refractor, the straight line in Figure 1 have slopes that are reciprocal of the refraction velocities. However in a situation where the refractors are not flat but dipping, relationship between the dip angle Φ, apparent velocities (V_{1u} and V_{1d}) would be required to estimate the actual velocities between the layers [6].

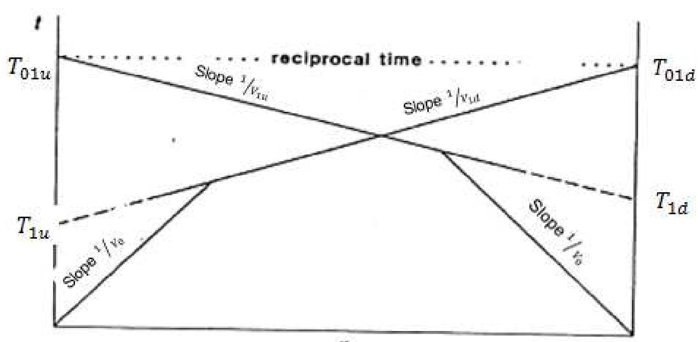

Figure 1: T-X plot for head-wave arrivals from a dipping refractor in the forward and reverse directions along a refraction profile line

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3.0 Methodology

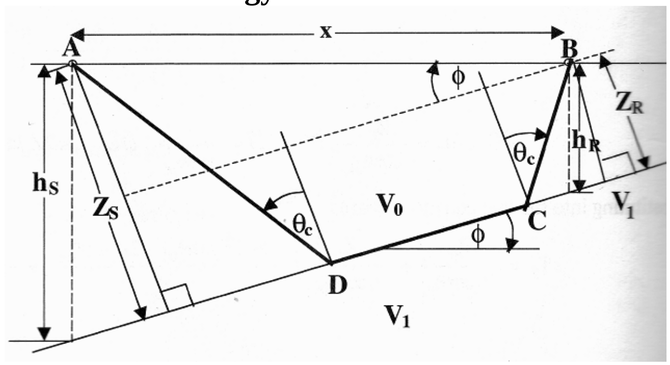

Figure 2: Earth Model for a single dipping layer. *Here V_0= weathering velocity, V_1= bedrock velocity, h_S= depth to bedrock at source station, h_R= depth to bedrock at receiver station, Z_S=layer thickness at source station, Z_R= layer thickness at receiver station, θ_c= critical angle, Φ= dip angle x= offset distance*

Figure 2 shows an earth model for a single dipping layer and the annotation used in the following derivations. As shown in the T-X plot of Figure 1, when the refractor dips the refraction velocity in the up-dip direction appears to be different from that in the down-dip direction [7]. Using this model we derived the expression for the refraction time of a single dipping layer in which $V_1>V_0$.

From Figure 2, the refraction time from A to B is:

$$T_1 = \frac{AD}{V_0} + \frac{CD}{V_1} + \frac{BC}{V_0} ------------ (1)$$

From trigonometrical . ratios

$$AD = \frac{Z_S}{cos\theta_c}, BC = \frac{Z_R}{cos\theta_c}, and\ CD = xcos\Phi - (Z_S + Z_R)tan\theta_c$$

Substituting this into the starting equation (1), gives

$$T_1 = \frac{Z_S}{V_0 cos\theta_c} + \frac{Z_R}{V_0 cos\theta_c} + \frac{xcos\Phi - (Z_S + Z_R)tan\theta_c}{V_1}$$

$$= \frac{xcos\Phi}{V_1} + (Z_S + Z_R)\left(\frac{1}{V_0 cos\theta_c} - \frac{tan\theta_c}{V_1}\right)$$

$$= \frac{xcos\Phi}{V_1} + \frac{Z_S + Z_R}{V_0 cos\theta_c}\left(1 - \frac{V_0 sin\theta_c}{V_1}\right)$$

$$provided\ V_1 > V_0, sin\theta_c = \frac{V_0}{V_1}\ (Snell`s\ Law)\ and\ from\ trig.identities\ sin^2\theta + cos^2\theta = 1$$

$$T_1 = \frac{xcos\Phi}{V_1} + \frac{(Z_S + Z_R)cos\theta_c}{V_0} ------------- (2)$$

Shooting up-dip (from A to B), substitute $Z_R = Z_S - xsin\Phi$ into equation (2) to get

$$T_{1u} = \frac{xcos\Phi}{V_1} + \frac{(2Z_s - xsin\Phi)cos\theta_c}{V_0} = x\left(\frac{cos\Phi}{V_1} - \frac{sin\Phi\ cos\theta_c}{V_0}\right) + \frac{2Z_s\ cos\theta_c}{V_0}$$

Recall that $sin\theta_c = \frac{V_0}{V_1}$, rewrite V_1 in terms of V_0

$$T_{1u} = x\left(\frac{sin\theta_c cos\Phi}{V_0} - \frac{sin\Phi cos\theta_c}{V_0}\right) + \frac{2Z_s cos\theta_c}{V_0} = x\left(\frac{sin\theta_c cos\Phi - sin\Phi cos\theta_c}{V_0}\right) + \frac{2Z_s cos\theta_c}{V_0}$$

$$from\ trigonometrical\ identities\ \sin(A - B) = sinAcosB - sinBcosA$$

$$T_{1u} = \frac{xsin(\theta_c - \Phi)}{V_0} + \frac{2Z_s cos\theta_c}{V_0} ------------- (3)$$

Let $Z_s = h_s cos\Phi$. Substituting this into equation (3) gives

$$T_{1u} = \frac{xsin(\theta_c - \Phi)}{V_0} + \frac{2h_s cos\theta_c cos\Phi}{V_0} ------- (4)$$

Equation (4) is analogous to the equation of a straight line $y = mx + c$. Hence

$$T_{1u} = \frac{x}{V_{1u}} + T_{01u} ----------- (5)$$

With the inverse slope (V_{1u}) defined as

$$V_{1u} = \frac{V_0}{sin(\theta_c - \Phi)} ---------- (6)$$

And the intercept time (T_{01u}) given by

$$T_{01u} = \frac{2h_s cos\theta_c cos\Phi}{V_0} ---------(7)$$

Shooting down-dip (from B to A), substitute $Z_S = Z_R + xsin\Phi$ into equation (2) to get

$$T_{1d} = \frac{xcos\Phi}{V_1} + \frac{(2Z_R + xsin\Phi)cos\theta_c}{V_0} = x\left(\frac{cos\Phi}{V_1} + \frac{sin\Phi\, cos\theta_c}{V_0}\right) + \frac{2Z_R\, cos\theta_c}{V_0}$$

Recall that $sin\theta_c = \frac{V_0}{V_1}$, rewrite V_1 in terms of V_0

$$T_{1d} = x\left(\frac{sin\theta_c cos\Phi}{V_0} + \frac{sin\Phi cos\theta_c}{V_0}\right) + \frac{2Z_R cos\theta_c}{V_0} = x\left(\frac{sin\theta_c cos\Phi + sin\Phi cos\theta_c}{V_0}\right) + \frac{2Z_R cos\theta_c}{V_0}$$

$from\ trigonometrical\ identities\ \sin(A+B) = sinAcosB + sinBcosA$

$$T_{1d} = \frac{xsin(\theta_c + \Phi)}{V_0} + \frac{2Z_R cos\theta_c}{V_0} ------------ (8)$$

Let $Z_R = h_R cos\Phi$. Substituting this into equation (8) gives

$$T_{1d} = \frac{xsin(\theta_c + \Phi)}{V_0} + \frac{2h_R cos\theta_c cos\Phi}{V_0} -------- (9)$$

In a similar manner equation (9) is analogous to the equation of a straight line $y = mx + c$. Hence

$$T_{1d} = \frac{x}{V_{1d}} + T_{01d} -------------- (10)$$

With the inverse slope (V_{1d}) defined as

$$V_{1d} = \frac{V_0}{sin(\theta_c + \Phi)} -----------(11)$$

And the intercept time (T_{01d}) given by

$$T_{01d} = \frac{2h_R cos\theta_c cos\Phi}{V_0} ---------- (12)$$

4.0 Results

From equations (6) and (11), we note that different apparent velocities are obtained when shooting up-dip and down-dip. To compute the refractor dip and the bedrock velocity (V_1), rewrite equations (6) and (11)

$$\theta_c - \Phi = sin^{-1}\frac{V_0}{V_{1u}} -------------(13)$$

$$\theta_c + \Phi = sin^{-1}\frac{V_0}{V_{1d}} ------------- (14)$$

Subtracting equation (13) from (14), we obtain the expression for the refractor dip Φ

$$\Phi = \frac{1}{2}\left[sin^{-1}\left(\frac{V_0}{V_{1d}}\right) - sin^{-1}\left(\frac{V_0}{V_{1u}}\right)\right] ----- (15)$$

To obtain the bedrock velocity V_1, rewrite slopes from equations (6) and (11) as

$$\frac{1}{V_{1u}} = \frac{sin\theta_c cos\Phi - \sin\Phi\cos\theta_c}{V_0} -----------(16)$$

$$\frac{1}{V_{1d}} = \frac{sin\theta_c cos\Phi + \sin\Phi\cos\theta_c}{V_0} -----------(17)$$

Adding equations (16) and (17), we have

$$\frac{1}{V_{1u}} + \frac{1}{V_{1d}} = \frac{2sin\theta_c cos\Phi}{V_0} ------------ (18)$$

Recall that $sin\theta_c = \frac{V_0}{V_1}$, $V_1 = \frac{V_0}{sin\theta_c}$ --- (19)

From equation (18)

$$sin\theta_c = \frac{V_0}{2cos\Phi}\left(\frac{1}{V_{1u}} + \frac{1}{V_{1d}}\right) -----------(20)$$

Substitute equation (20) into (19) to get

$$V_1 = \frac{V_0}{\frac{V_0}{2cos\Phi}\left(\frac{1}{V_{1u}} + \frac{1}{V_{1d}}\right)} = 2cos\Phi\left(\frac{1}{\frac{1}{V_{1u}} + \frac{1}{V_{1d}}}\right) =$$

$$V_1 = 2cos\Phi\left(\frac{V_{1u}V_{1d}}{V_{1u} + V_{1d}}\right) - - - - - - - - - - - - (21)$$

Finally, we compute the depth to the bedrock at shot-receiver stations (h_S and h_R) by substituting the relation $sin\theta_c = \frac{V_0}{V_1}$into equations (7) and (12).

From equation (7),

$$h_S = \frac{V_0 T_{01u}}{2cos\theta_c cos\Phi} - - - - - - - - - - - - - - (22)$$

$$from\ trigonometrical\ identities\ sin^2\theta + cos^2\theta = 1, cos\theta_c = \sqrt{1 - sin^2\theta_c}\ and\ sin\theta_c = \frac{V_0}{V_1}$$

$$\therefore\ h_S = \frac{V_0 T_{01u}}{2\left(\sqrt{1 - \frac{V_0^2}{V_1^2}}\right) cos\Phi} = \frac{V_0 T_{01u}}{2\left(\sqrt{\frac{V_1^2 - V_0^2}{V_1^2}}\right) cos\Phi}$$

$$h_S = \frac{V_1 V_0 T_{01u}}{2cos\Phi\sqrt{V_1^2 - V_0^2}} - - - - - - - - - - - - - - (23)$$

In a similar manner, from equation (12)

$$h_R = \frac{V_0 T_{01d}}{2cos\theta_c cos\Phi} - - - - - - - - - - - - - - - (24)$$

$$from\ trigonometrical\ identities\ sin^2\theta + cos^2\theta = 1, cos\theta_c = \sqrt{1 - sin^2\theta_c}\ and\ sin\theta_c = \frac{V_0}{V_1}$$

$$\therefore\ h_R = \frac{V_0 T_{01d}}{2\left(\sqrt{1 - \frac{V_0^2}{V_1^2}}\right) cos\Phi} = \frac{V_0 T_{01d}}{2\left(\sqrt{\frac{V_1^2 - V_0^2}{V_1^2}}\right) cos\Phi}$$

$$h_R = \frac{V_1 V_0 T_{01d}}{2cos\Phi\sqrt{V_1^2 - V_0^2}} - - - - - - - - - - - - - - (25)$$

5.0 Conclusion

In a dipping refractor, the depth to bedrock estimation at shot-receiver station requires the knowledge of weathering velocity (V_0), bedrock velocity (V_1), refractor dip Φ and the intercept time (T_{01u} or T_{01d}). They can be computed by way of equations (15), (21), (23) and (25).

Conclusively, the stepwise approach to deriving these parameters will provide the geophysicist the knowledge of the required input needed to maximise the use of the properly programmed computer to interpret and minimise the interpretational ambiguities associated with seismic refraction data.

6.0 References

[1] Dobrin, M.B., 1960, Introduction to geophysical prospecting: McGraw-Hill Book Co.

[2] Grant, F.S., and West, G.F., 1965, Interpretation theory in applied geophysics: McGraw-Hill Book Co.

[3] Yilmaz, O. 2005. Seismic data analysis: processing, inversion, and interpretation of seismic data. Investigations in geophysics, 2nd edition. Tulsa, OK, Society of Exploration Geophysicists

[4] Gadallah M.R.,and Fisher R.L., 2005. Applied Seismology: a comprehensive guide to seismic theory and application. 1st ed. Penwell books Corporation.

[5] Kearney, P. and Brooks, M., 1984. An Introduction to Geophysical Exploration. Blackwell Scienticfic Publication.

[6] Levin, F.K., 1971. Apparent velocity from dipping interface reflections; Geophysics, **36**,510-516

[7] Hampson, D. and Russell, B., 1984, First-break interpretation using generalized inversion: J. Can. Soc. Explor. Geophys, **20**, 40-54