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Effects of Pressure on Attenuation of Seismic Waves through Consolidated Sedimentary Rocks from Ewekoro Formation, Nigeria

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Abstract

Effects of pressure on the Attenuation of Seismic waves propagating three different rock samples from the upper crust in Ewekoro formation are investigated. The frequencywas varied from 1Hz to 100MHz while the pressure was varied from atmospheric 9.7kPa to 27.9kPa. The Continuous Wave Transmission and Spectral Amplitude wave-ratio techniques were employed to determine the Attenuation Coefficient k, for each rock sample. Attenuation Coefficients were plotted against frequency on scatter diagrams. Results show that sandstone attenuated the most at 2.73 and closely followed by limestone 2.25; while shale is the least attenuated at 1.99. In all three cases, attenuation decreases (Q-factor increases) with increasing confining shear pressure.

Keywords: Attenuation coefficient, Confining Shear Pressure, Q-Factor, Spectral Amplitude

1.0 Introduction

Attenuation as a function of pressure generally has been neglected by most investigators, yet the behavior of 1/Q(anelastic attenuation factor) with pressure can yield as much information about mechanisms as does the frequency dependence. When a rock is subjected to hydrostatic pressure, such as overburden pressure, its elastic and anelastic properties will change. The behavior of elastic properties under pressure is well known and a theoretical treatment of it may be found in [1]. According to Hooke's law, strain is linearly related to the stress applied until a certain point of stress, known as the field strength of the material. According to [2], anelasticity, is the process which describes the dissipation of energy in materials under stress.

1.1 Seismic Waves

Yield maps of the distribution of seismic velocities, interface between rock units and the reflection coefficient of the interface. Measuring the velocity of seismic waves through the earth layers provides foremost elucidation to the composition and constitution of the layers [3]. The velocities of crustal rocks vary due to rock porosity, pressure, temperature, fluid saturation and other physical parameters. Velocity structure of the Earth's interior, attenuation structure, and materials with their state in the Earth can be estimated through investigation of the propagation of seismic waves.

Basically, seismic waves comprise of body waves and surface waves. Body waves include compressional and shear waves while the surface waves are the love waves and the Rayleigh. For the purpose of this work we are focusing on compressional and shear waves**.** Compressional wave directs the motion of the particle along the direction of wave propagation. The particle motion associated with compressional wave consists of alternating compression and rarefactions during which adjacent particles of the solid are closer together and farther apart in one successive half cycle[4]. The velocity of compressional wave can be expressed in terms of other elastic constants λ, μ and ρ.

$$\psi = K + \frac{4}{3}\mu \text{ ------ ----------} \quad (1)$$

$$V_p = \left[\frac{\psi}{\rho}\right]^{1/2} \text{ -----------------} \quad (2)$$

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$$V_p = \left[\frac{K + \frac{4}{3}\mu}{\rho} \right]^{1/2} \text{- - - - - - - - - - - - -} \qquad (3)$$

Where $\lambda = \left[\frac{E}{\rho} \left(1 + \frac{2\sigma^2}{1-\sigma-2\sigma^2} \right) \right]^{1/2}$ - - - - - - (4)

Compressional wave velocity in the crust of continents beneath the sedimentary layers varies from 5km/s at shallow depths to about 7km/s at a depth of 30 - 50km. the lower velocities indicate the presence of pores and cracks more than the intrinsic velocities of the rock.

Shear waves propagate by a pure shear strain in a direction perpendicular to the direction of wave motion. The velocity of propagation of a body wave in any homogeneous, isotropic material is given by the velocity V_s of a shear body wave, which involves a pure shear strain

$$V_s = \left[\frac{\mu}{\rho} \right]^{1/2} \text{- - - - - - - - - - - - - - - - - - -} \qquad (5)$$

Individual particle motions involve oscillations about a fixed point, in a plane at right angles to the direction of wave propagation. If all the particle oscillations are confined to a plane, the shear wave is said to be plane-polarized.

It can be observed from these equations that compressional waves always travel faster than shear waves in the same medium. The ratio v_p/v_s in any material are determined solely by the value of Poisson's ratio (s) for that material

$$\frac{V_p}{V_s} = \left[\frac{2(1-\sigma)}{1-2\sigma} \right]^{\frac{1}{2}} \text{- - - - - - - - - - - - - - - -} \qquad (6)$$

The v_p/v_sratio, however, is independent of density and can be used to derive a Poisson's ratio, which is a better lithological diagnostic indicator. If this information is required, then both v_p and v_s must be determined in the seismic survey [5].

1.2 Anelastic Attenuation Factor

In reflective seismology, the anelastic attenuation factor is often expressed as a seismic Quality factor (Q-factor for short) this is inversely proportional to attenuation. Q-factor quantifies the effect of anelastic attenuation on a seismic wavelet caused by fluid movement and grain boundary friction. As seismic wave propagates through a medium, the elastic energy associated with the wave is gradually absorbed by the medium, eventually ending up as heat energy which is also referred to as absorption i.e. anelastic attenuation and will eventually cause the total disappearance of the seismic waves [7]

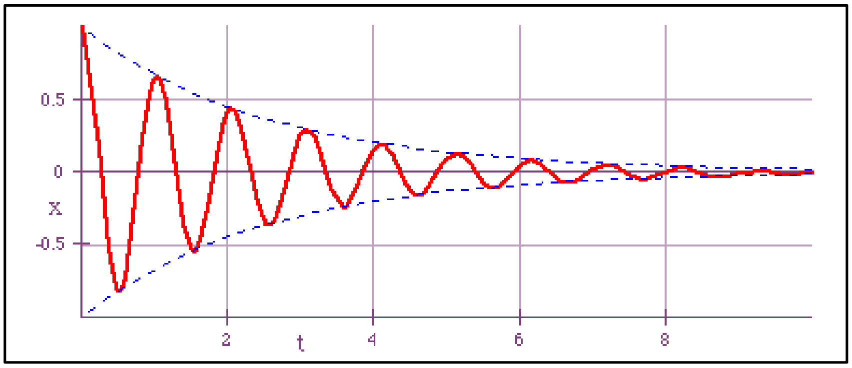

Fig. 1: Attenuation is quantified by 1/Q, in analogy to the damped harmonic oscillator (after Wiens)

$$\ddot{x} = \frac{\omega_o}{Q}\dot{x} + \omega_o^2 x = 0 \text{- - - - - - - - - - - - - - - -} \qquad (7)$$

Where $x(t) = Ae^{\frac{\omega_o t}{2Q}} \sin \omega t$ - -- - -- - - - - - - - - - (8)

$$Q - factor = 2\pi \left[\frac{E}{\delta E} \right] \text{- - - - - - - - - - - - - - - - -} \qquad (9)$$

E is the fraction of energy lost per cycle [8]

This behavior is said to be dispersive because the rate of attenuation increases with frequency. The earth preferentially attenuates higher frequencies, resulting in the loss of signal resolution as seismic wave propagates through rock materials.

1.3 Attenuation as Function of Pressure

Compressional wave velocity is strongly dependent on effective stress. For a rock buried in the earth, the confining pressure is the pressure of the overlying rock column and the pore-space pressure may be greater or less than hydrostatic pressure; or (if there is a connected porosity to the surface) equal to the confining shear pressure. The effective pressure is the difference between the confining and pore pressure.

Velocity change with pressure depends on the closing of cracks, pores and grain boundaries, which elastically stiffens the rock mineral frame[6]. In general, velocity rises with increasing confining pressure and then levels off to a “terminal velocity” when the effective pressure is high **[7].**The effect is probably due to crack closure. At low effective pressure, cracks are opened and easily closed with increase in stress. As the effective pressure increases the cracks are all closed, k goes up and the velocity increases. This also holds for changes in attenuation of the rock.

In all cases, attenuation decreases (Q increases) with increasing pressure. Experimental data verifying this are found in many write-ups[8-15].For these data and the theoretical models to be presented, the pressure given is the differential pressure,

$$P_d = P_c - P_f - - - - - - - - - \qquad (10)$$

Where P_c is the confining pressure and P_f is the pore fluid pressure. This relationship holds for all rocks, as demonstrated by laboratory tests[16-18]. The attenuation of P-waves in diabase and graywacke was measured up to a pressure of 4 kilobars (kb) by a pulse transmission method with a prevailing frequency of 0.9 MHz. Although not stated explicitly, the samples are assumed to be air-dry[9].

However, even at depth, as the pore pressure increases above hydrostatic, the effective pressure decreases as does the velocity. Over-pressured zones can be detected in a sedimentary sequence by their anomalously low velocities.

Finally, we can infer that pressure impacts on velocities are: -

(i). Increasing pore pressure softens the elastic mineral frame by opening cracks and flaws, tending to lower velocities.

(ii). Increasing pore pressure tends to make the pore fluid or gasless compressible, tending to increase velocities.

(iii). Changing pore pressure can change the saturation as gas goes in and out of solution. Velocities can be sensitive to saturation.

(iv). High pore pressures persisting over long periods of time can inhibit diagenesis and preserve porosity, tending to keep velocities low. The only way to know the pressure dependence of velocities for a particular rock is to measure it[19].

2.0 Experimental Work

The experimental set-up comprises a sine-audio signal generator which continuously generates sinusoidal waveform through the rock samples in turn with a pair of receiving and transmitting transducers on either side of the rock samples. One converts the electrical signal into a mechanical strain on one side of the rock and vice-visa on the other side of the rock sample and readings were recorded by Double-beam oscilloscope.Rock samples collected from Shagamu in Ogun state, in the western Dahomey basin,were sandstone, limestone, and shale. These rock samples were cut into a size of thickness 6.0 cm cube.

Incident waves were allowed to pass through from Y_1channel of the double beam oscilloscope while the transmitted wave passes through the Y_2 channel of the oscilloscope for comparison. Both the incident wave amplitude A_1 and the transmitted wave amplitude A_2 through the rock samples were measured simultaneously on the double-beam oscilloscope. Signals in the range 1Hz to 1MHz were investigated as they traverse the rock samples. Pressure variations were obtained using materials of 5Kg to 50Kg weight of the circular base and radius 2,8cm, placed on each rock samples as the signals traverse the rocks. The incident wave amplitude is related to the transmitted wave by the following equation

$$A_1 = A_2 e^{2kr} - - - - - \qquad (11)$$

$$In\left[\frac{A_2}{A_1}\right] = 2kr - - - - - - \qquad (12)$$

Where k is the Attenuation coefficient in inverse meter (m^{-1}) and r is the thickness of the rock samples in meters (m). Q-Absorption operates with the amplitude spectrum of a seismic pulse with frequencies f that obeys the relation 13. The travel time of the pulse at t=0is denoted by t0 and A_o(f) is the un-attenuated amplitude spectrum of the pulse

$$|A_t(f)| = |A_o(f)| e^{\left(-\pi f t /_Q\right)} - - - - - \qquad (13)$$

$$In\left[\frac{A_2}{A_!}\right] = -\frac{\pi f (t_2 - t_1)}{Q} - - - - - \qquad (14)$$

3.0 Results and Discussion

The static and dynamic biaxial test implemented at a confining pressure range of 9.4 kPa, 18.4 kPa and 28 kPa dynamic load for the three rock samples were specified with a capricious frequency set from 1Hz to 100MHz. The data obtained from the attenuation experiment on the sandstone, limestone and shale were recorded and calculations of the attenuation coefficient for these rock samples were made using equation (16). The data were plotted using 2010 Microsoft excel. The results obtained show how the rock samples behave under different pressure conditions at room temperature.

The experimental data show that sandstone attenuated in a logarithmic decrement, though it is in the negative region under atmospheric conditions, but it increases as the pressure increased. The varying confining pressure at different frequencies shows that Q-factor increasing from the range (-59.1 to -44.3) at a lower frequency to a higher range of (-39.4 to -16.29).

This indicates that attenuation decreases (Q increases) with increasing shear pressure on the sandstone as shown in Figure 2. At Atmospheric pressure (ATP) the absorption coefficient was at 1.49% and at higher confining shear pressure (CSP), the absorption coefficient increases to 2.73%, which is a result of shear pressure on the sandstone with an average mass density of 2.33gcm^{-3}.

Limestone attenuated in a logarithmic decrement in the negative region under atmospheric conditions. The varying confining pressure at different frequencies shows that Q-factor increases from the range (-46.5 to -31.3) at a lower frequency to a higher range of (-31.4 to -13.9). This indicates that attenuation decreases (Q increases) with increasing shear pressure on the limestone as shown below inFigure 3. At ATP the absorption coefficient was at 1.48% and at higher confining pressure, the absorption coefficient increases to 2.25%, which is as a result of shear pressure on the limestone with an average mass density of 3.79gcm^{-3}.

Shale attenuated slowly in a logarithmic decrement at a negative region under atmospheric conditions. The varying confining pressure at different frequencies also shows that Q-factor increases from the range (-26.9 to -17.4) at lower frequency to a higher range of (-18.0 to -8.7). This indicate that attenuation decreases (Q increases) with increasing shear pressure on the shale as shown below in thefigure 4. At ATP the absorption coefficient was at 1.49% and at higher confining pressure, the absorption coefficient increases to 1.99%, which is a result of shear pressure on the shale with an average mass density of 2.68gcm^{-3}.

The correlation of this rock sample was also determined to understand their attenuation absorption rate under the same confining shear pressure. It was shown that sandstone attenuated the most at 2.73 and closely followed by limestone in 2.25 while shale is the least attenuated at 1.99, as shown in Figures 5,6,7 and 8. Although, all in the negative direction. This result at room temperature and atmospheric pressure condition is in good agreement with the findings of other seismologists elsewhere [6, 18, 19, 20].

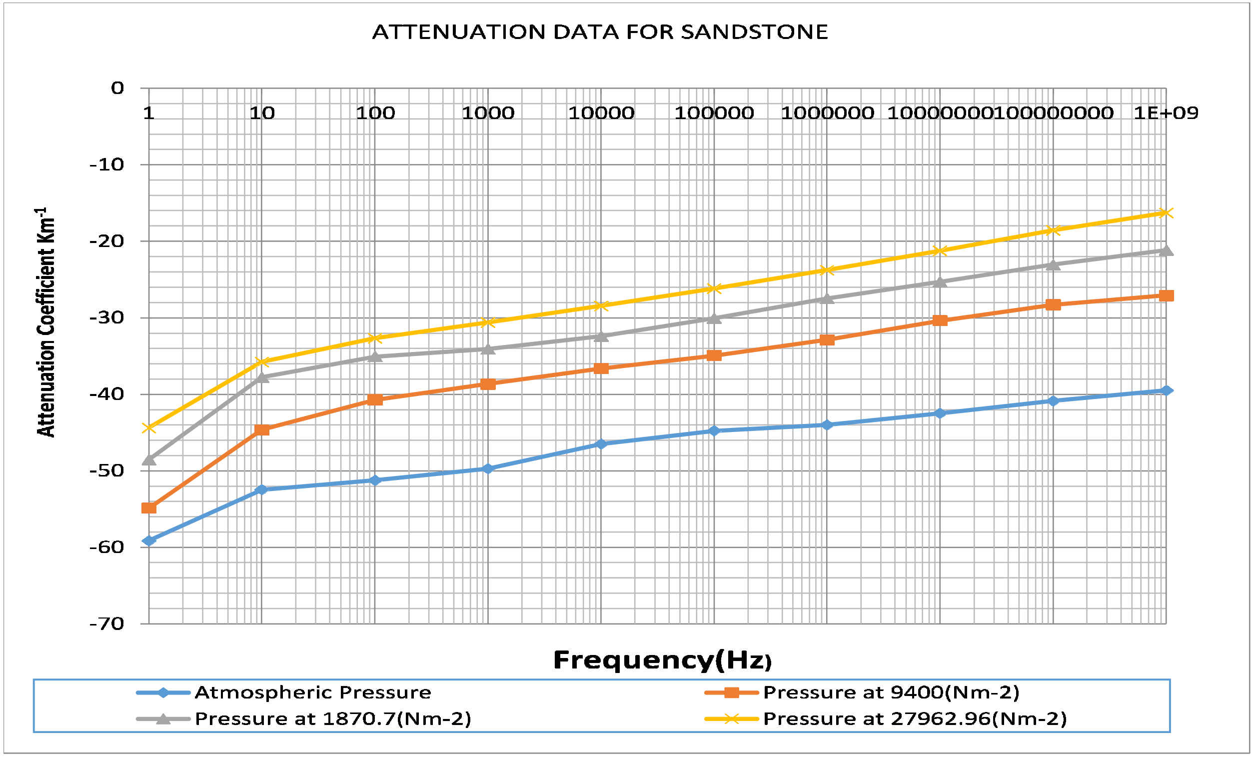

Figure 2: Q-factor of sandstone increases with varying Confining Shear Pressure at different frequency

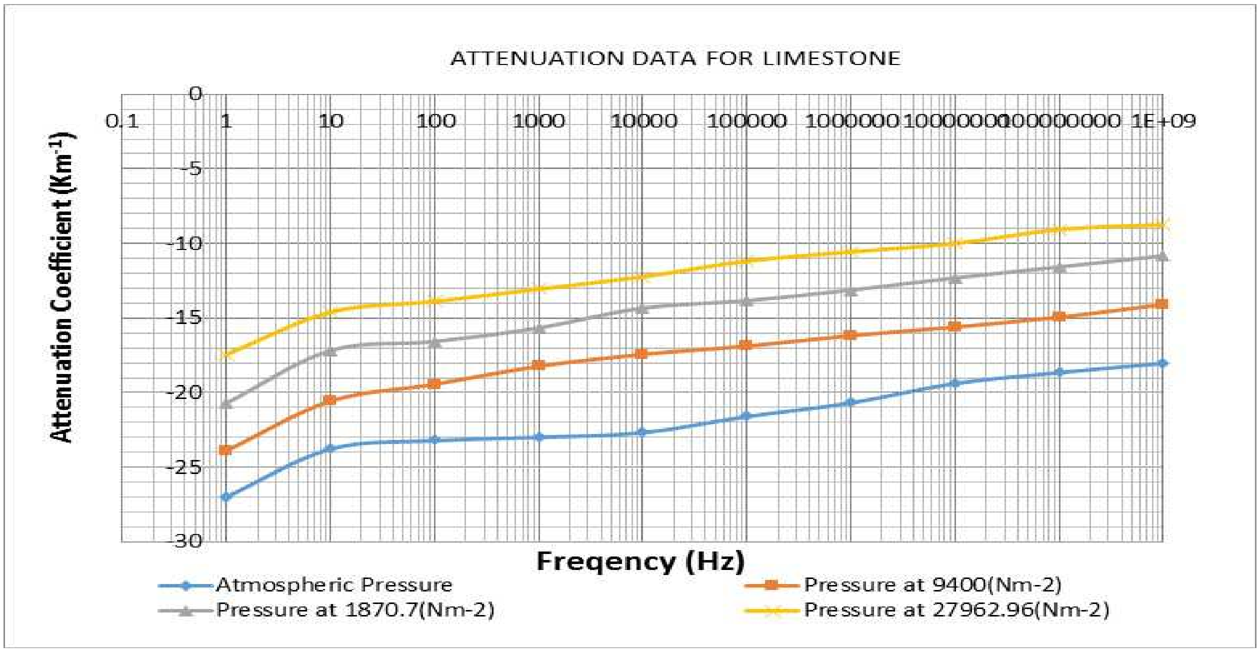

Figure 3: Q-factor of Limestone increases with varying Confining Shear Pressure at different frequency

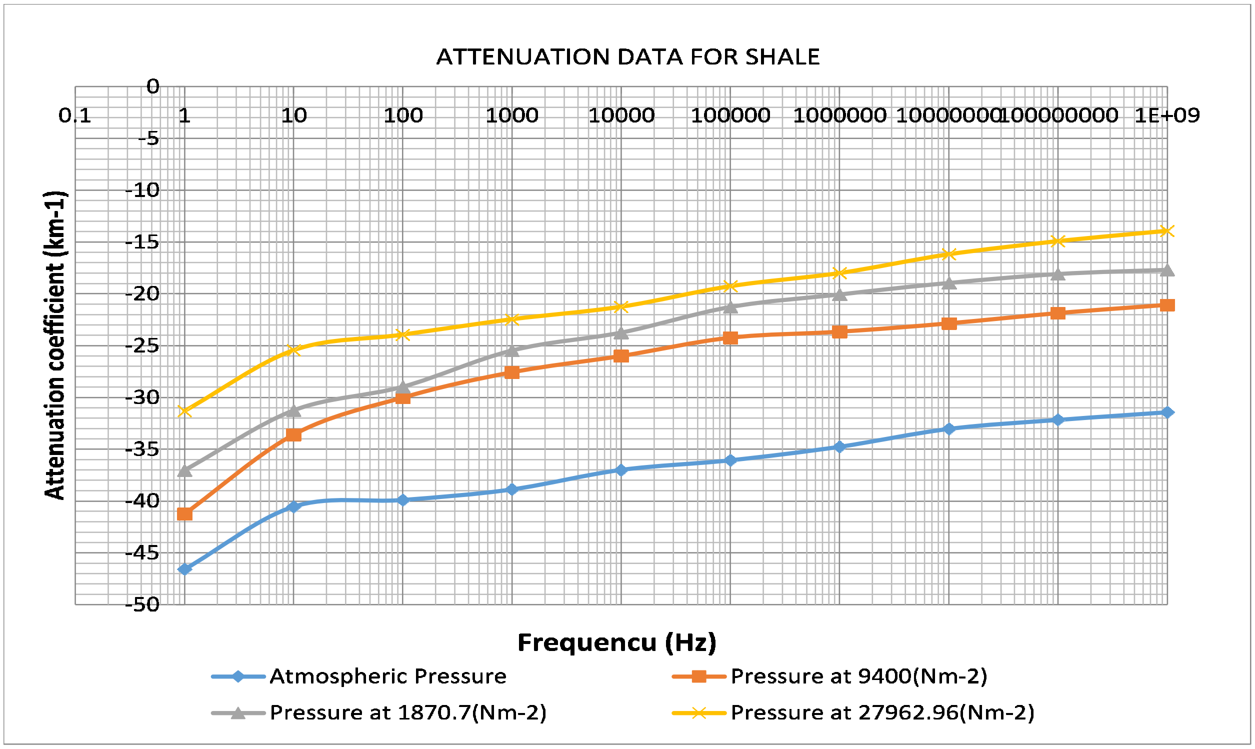

Figure 4: Q-factor of Shale increases with varying Confining Shear Pressure at different frequency.

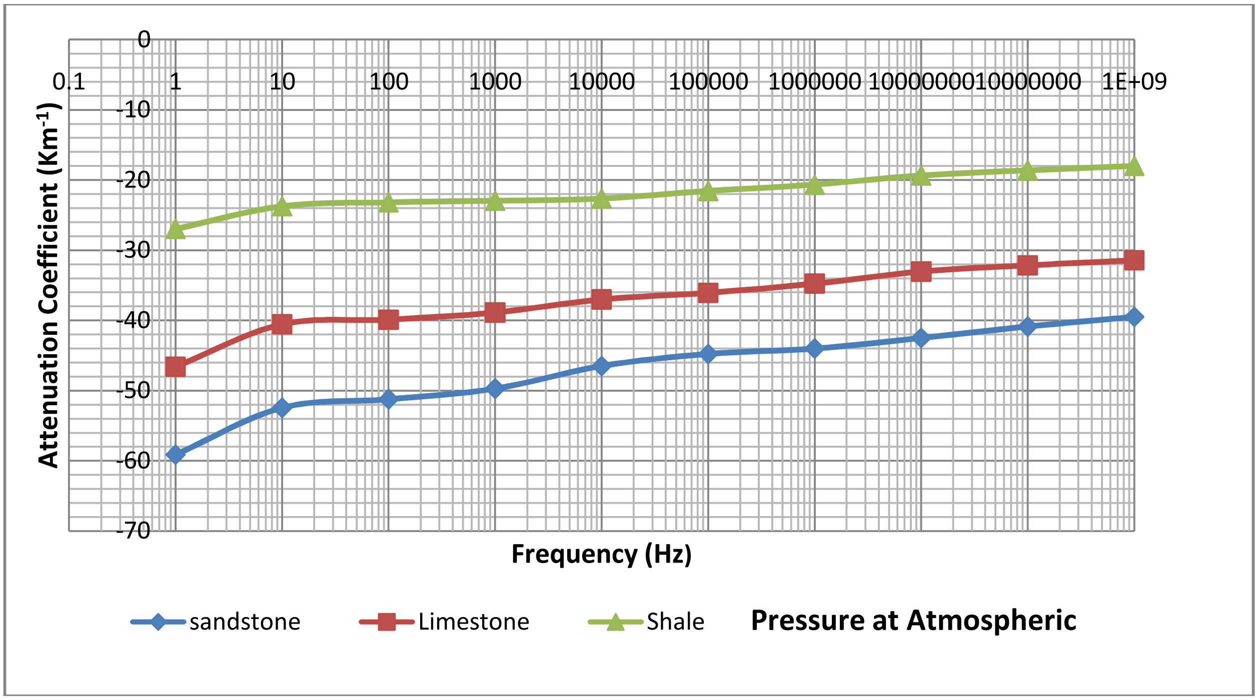

Figure 5: Q-factor of the three rocks samples increases with respect to frequency for CSP at Atmospheric Pressure. The Q-factor of sandstone, limestone and Shale increases by 1.493%, 1.489% and 1908% respectively.

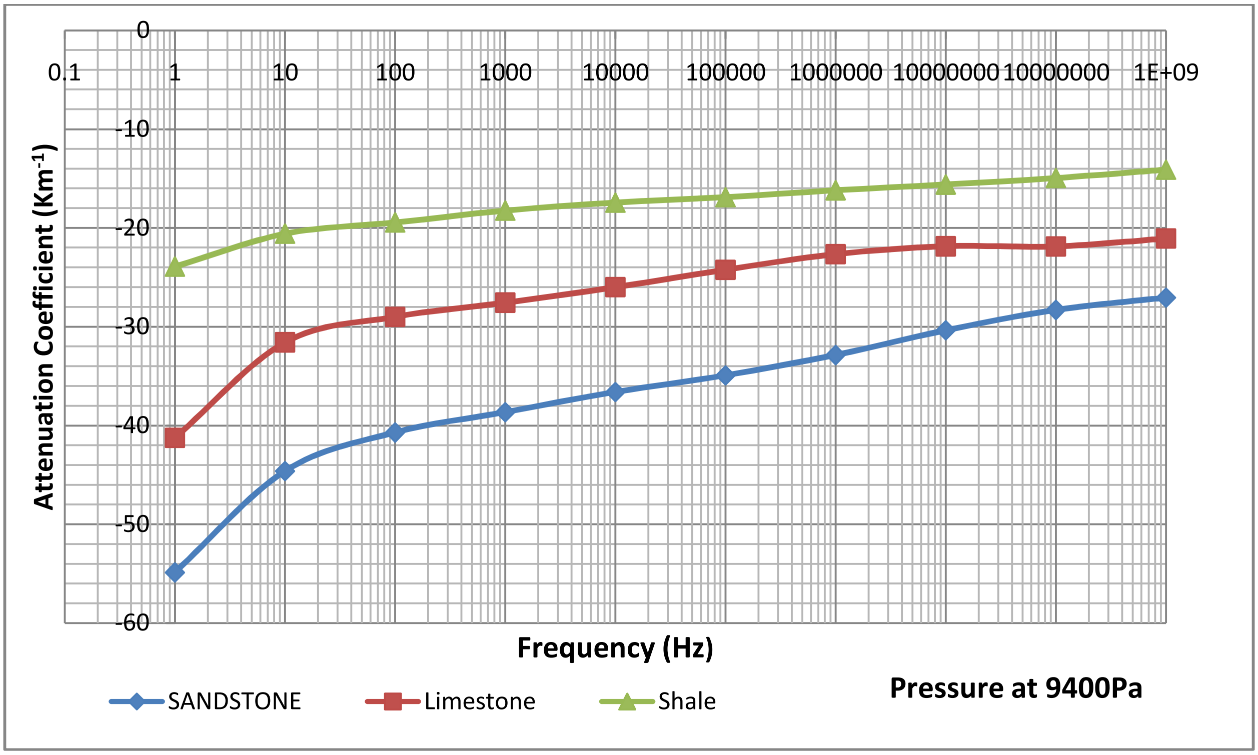

Figure 6: Q-factor of the three rocks samples increases with respect to frequency for CSP at 9.4KPa. The Q-factor of sandstone, limestone and Shale increases by 2.026%, 1.957% and 1.694% respectively with CSP

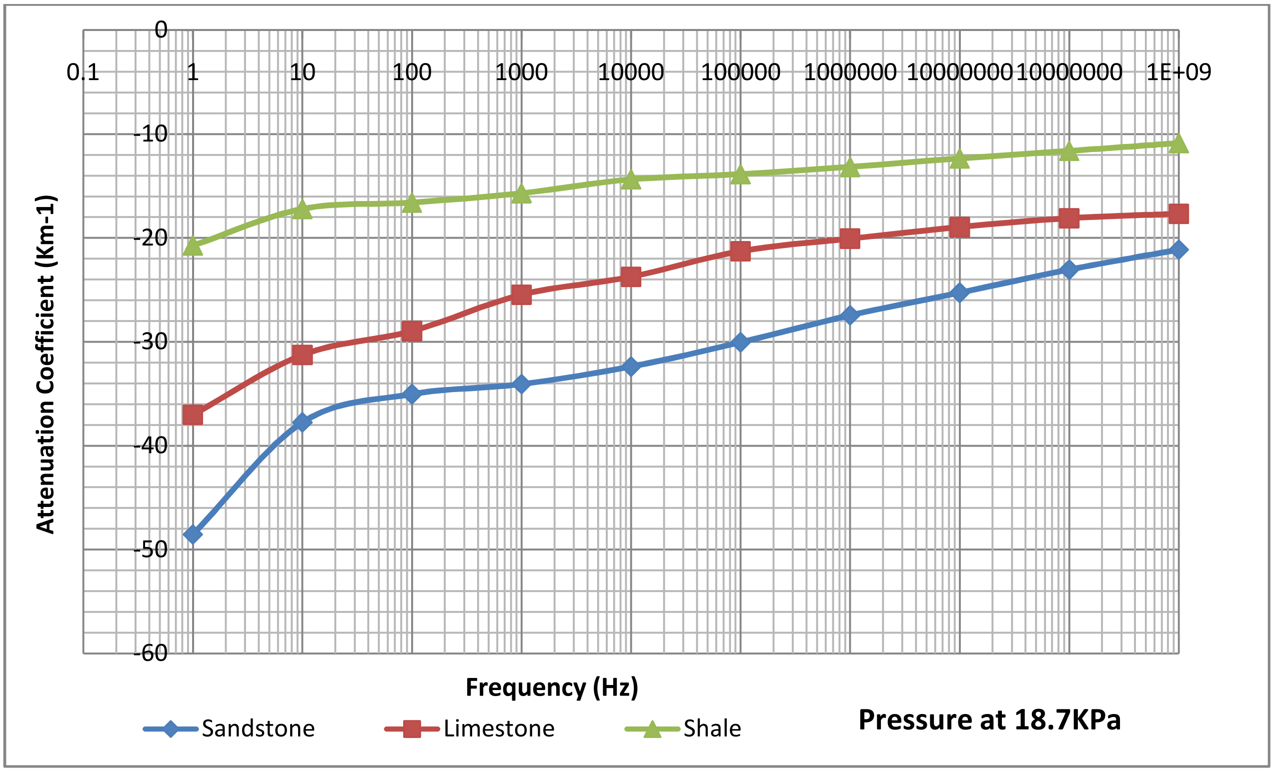

Figure 7: Q-factor of the three rocks samples increases with respect to frequency for CSP at 18.7KPa. The Q-factor of sandstone, limestone and Shale increases by 2.295%, 2.094% and 1.908% respectively with CSP

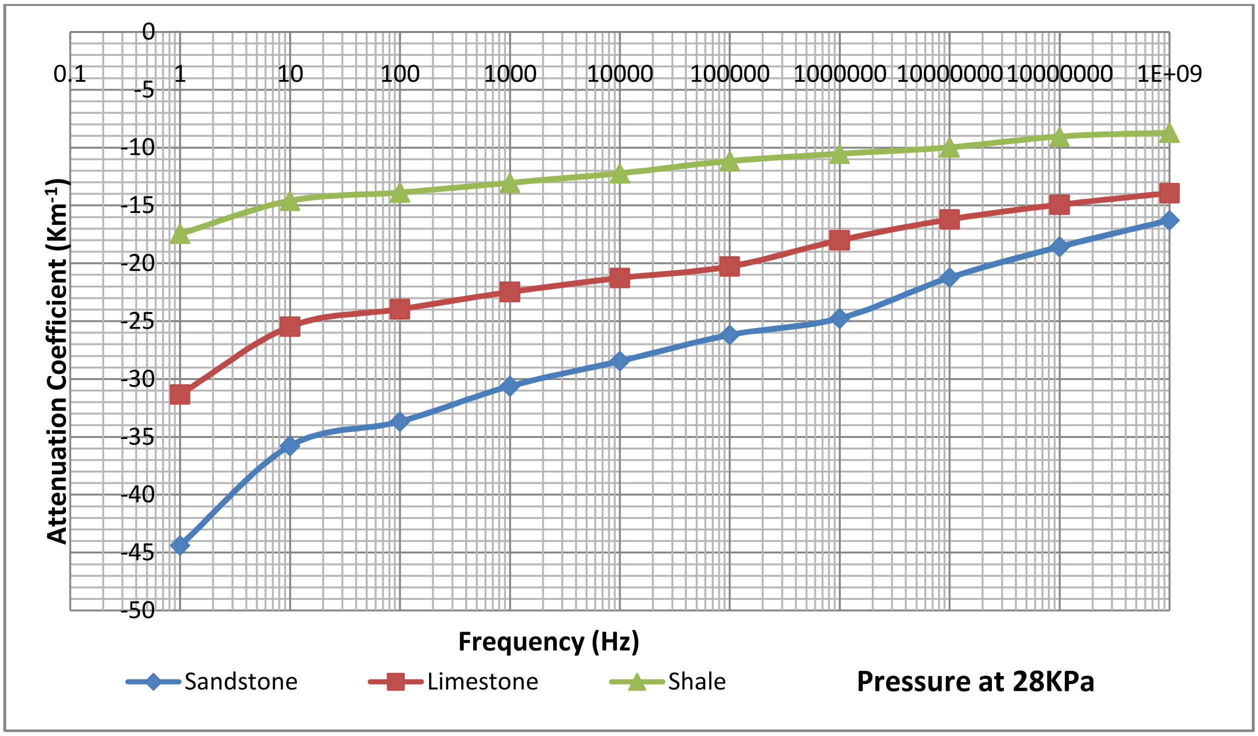

Figure 8: Q-factor of the three rock samples increases with respect to frequency for CSP at 28KPa. The Q-factor of sandstone, limestone and Shale increases by 2.724%, 2.248% and 2.000% respectively with CSP

The correlation of this rock sample was also determined to understand their attenuation absorption rate under the same confining shear pressure. It revealed that sandstone attenuated the most at 2.73 and closely followed by limestone at 2.25 while shale is the least attenuated at 1.99, as shown in Figures 2,3,4 and 5 although, all in the negative direction. This result at room temperature and atmospheric pressure condition isconsistent with the findings ofsome other scientists [3, 19,20].The results obtained gave a clearer picture of general behavior of the rocks and their attenuation trends in the presence of increasing confining shear pressure. This is caused by dilatation, the opening of micro-cracks with normal perpendicular to the biaxial confining shear stress direction. The decrease in attenuation is mainly an effect of losses occurring in the pore-space by differential motion between rock-frame and pore-fluid. It is simple to understand that attenuation is reduced when the porosity is reduced.However, local pore-pressure can alter the effect significantly. The driving force is then the differential pressure between confining pressure and the pore-pressure [21].This is caused by the progressive lowering of the apparent frequency of seismic wave with increasing distance of travel through the earth. In all cases, attenuation decreases (Q-increases) with increasing pressure.

4.0 Conclusion

Finally, it can be concluded that within limits of experimental error, that increased confining shear pressure causes the attenuation coefficient of the rock samples to decrease. It is suggested that the combined effect of fluid saturation and pressure on attenuation of this rock samples can be investigated. Rock samples from another location can be obtained and the effects of pressure should be investigated in order to study the dynamics of the earth.

Table 1: Attenuation Data for Sandstone

Attenuation Coefficient (km^{-1})

Frequency (Hz)	**Atmospheric Pressure**	**Pressure at 9400(Nm^{-2})**	**Pressure at 1870.7(Nm^{-2})**	**Pressure at 27962.96(Nm^{-2})**
1	-59.1283	-54.8733	-48.5601	-44.3717
10	-52.4466	-44.6203	-37.7733	-35.7733
100	-51.2283	-40.7117	-35.0564	-32.6751
1000	-49.7117	-38.6567	-34.0867	-30.6176
10000	-46.4867	-36.6217	-32.4083	-28.4501
100000	-44.7583	-34.9333	-30.0667	-26.1851
1000000	-43.9901	-32.8783	-27.4654	-23.7583
10000000	-42.4767	-30.3759	-25.2931	-21.2369
100000000	-40.8558	-28.3081	-23.0676	-18.5727
1000000000	-39.4757	-27.0773	-21.1517	-16.2906

Table 2: Attenuation Data for Limestone

Attenuation Coefficient (Km^{-1})

Frequency (Hz)	**Atmospheric Pressure**	**Pressure at 9400(Nm^{-2})**	**Pressure at 1870.7(Nm^{-2})**	**Pressure at 27962.96(Nm^{-2})**
1	-46.5897	-41.2544	-37.0245	-31.3167
10	-40.5502	-33.5879	-31.2587	-25.4587
100	-39.8987	-30.0014	-28.9654	-23.9564
1000	-38.8721	-27.5684	-25.4651	-22.4653
10000	-37.0001	-26.0025	-23.7367	-21.2603
100000	-36.0653	-24.2358	-21.2733	-19.2715
1000000	-34.7608	-23.6587	-20.0667	-18.0009
10000000	-33.0352	-22.8548	-18.9483	-16.2004
100000000	-32.1601	-21.8701	-18.0967	-14.9351
1000000000	-31.4301	-21.0702	-17.6783	-13.9383

Table 3: Shows the Attenuation Data for Shale

Attenuation Coefficient (Km^{-1})

Frequency (Hz)	**Atmospheric Pressure**	**Pressure at 9400(Nm^{-2})**	**Pressure at 1870.7(Nm^{-2})**	**Pressure at 27962.96(Nm^{-2})**
1	-26.9633	-23.9047	-20.7385	-17.4433
10	-23.7452	-20.5785	-17.2058	-14.5975
100	-23.1865	-19.4457	-16.5994	-13.8745
1000	-22.9565	-18.2402	-15.6952	-13.0456
10000	-22.6508	-17.4416	-14.3633	-12.2251
100000	-21.5665	-16.8917	-13.8652	-11.1708
1000000	-20.6562	-16.1883	-13.1643	-10.5486
10000000	-19.3559	-15.6133	-12.3442	-9.9854
100000000	-18.6254	-14.9547	-11.6151	-9.0535
1000000000	-18.0003	-14.1104	-10.8695	-8.7216

5.0 References

[1] Toksoz, M. N., Cheng, C. H., and Timur. A. (1976). Velocities of seismic waves in porous rocks: Geophysics, Vol. 41, pp. 621-645.

[2] Olorode D.O and Oni, E.A. (2006). The Temperature Distribution in the Lithosphere: Laboratory Study of the Temperature Effects on the Anelastic Properties of Sandstone, Limestone and Shale. ICESA Bulletin for Geodesy and Geophysics Research in Africa. Vol 2 (2) pp 48 – 67.

[3] Olorode, D.O. (2001). Analysis of the Temperature Effects on Attenuation of seismic waves through consolidated rocks from Southwestern Nigeria.PhD Thesis, University of Ibadan.

[4] Hamada, G.M. (2004). Reservoir Fluids Identification Using Vp/Vs Ratio. Oil & Gas Science and Technology – Rev. IFP, Vol. 59 (6), pp. 649-654.

[5] Kearey P. Brooks M., and Hill I. (2002) An Introduction to Geophysical Exploration. Third Edition. Blackwell Publishing, Oxford. ISBN 632 04929 4.

[6] Birch, F. (1960) The velocity of compressional waves in rocks to 10 kilobars, 1nt. J. Geophys. Res., Vol. 65, pp. 1083 – 1102.

[7] Kern, H., and Tubia, J. (1993). Pressure and temperature dependence of P- and S-wave velocities, seismic anisotropy and density of sheared rocks from the Sierra Alpujata massif (Ronda peridotites, southern Spain), Earth Planet. Sci. Lett., Vol. 119, pp 191 – 205.

[8] Gardner, G.H.F., Wyllie, M.R.J., and Droschak, D. M. (1964). Effect of pressure and fluid saturation on the attenuation of elastic waves in sand: J Petroleum tech., Vol. 16. pp 189-198.

[9] Klima, K., Vanek, J., and Pros, Z., (1964). The attenuation of longitudinal waves in diabase and greywacke under pressure up to 4 kilobars: studia Geoph. et Geod., Vol.8, pp. 247-254.

[10] Gordon, R.B., Davis, L. A. (1968). Velocity and attenuation of seismic waves in imperfectly elastic rock: J. Geophys. Res., Vol.73, pp. 3917-3935.

[11] Levykin, A. L. (1965). Longitudinal and transverse wave: absorption and velocity in rock specimens at multilaterally pressure up to 4000km/cm^2: USSR Geophys. Ser. (Engl. transl.), Vol.1. Physics of the Solid Earth. pp. 94-98.

[12] Al-Sinawi, S. (1968) An investigation of body wave velocity, attenuation on elastic parameters of rocks subjected to pressure at room temperature: Ph.D. thesis, St. Louis University. Missouri, United States.

[13] Walsh, J. B. (1966) Seismic wave attenuation in rocks due to friction: J. Geophys. Res., Vol.71, pp. 2591-2599.

[14] Walsh, J. B., Brace, W.F., and Wawersik, W.R. (1970). Attenuation of stress waves in cedar city quartz diorite: Air Force Weapon Lab. Tech. rep. AFWL-TR-70-80.

[15] Toksoz, W.M and Johnston, D.H., and Timur, A. (1979). Attenuation of seismic waves in dry and saturated rocks. J. Laboratory measurement: Geophysics, this issue. Vol.44(4). pp. 681-690.

[16] Wyllie, M.R., Simmons, A.R., and Gardner, G.H.F. (1958). An experimental investigation of factors affecting elastic wave velocities in porous media: Geophysics, Vol.23, pp. 459-493.

[17] Nur, A., and Simmons, G. (1969). The effect of saturation on velocity in low porosity rocks: Earth Planet. Sci. lett., Vol.7, pp.183-193.

[18] Gary Mavko, (2013). Stanford Rock Physics Laboratory. Conceptual Overview of Rock and Fluid Factors that Impact Seismic Velocity and Impedance.

[19] Birch, F. (1961). The velocity of compressional waves in rocks to 10 kilobars: 2. J. Geophys. Res., Vol.66 (7), pp.2199 – 2224.

[20] Umo, J. A. (1998). Analysis of wave amplitude of broadband seismic waveforms propagated through different geological samples from Abeokuta and Ewekoro Formations, Eastern Dahomey Basin. PhD thesis, University of Ibadan, Nigeria.

[21] Reidel, M (2008) Borehole Geophysics. EPS-550.