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Modelfor Layered Reservoir Subject toBottomWaterDriveUsing Horizontal Wells

E.D Akpobi, E.S Adewoleand O.A Olafuyi

Department Of Petroleum Engineering,University Of Benin,Benin City,Nigeria.

Abstract

In this paper,mathematical models describing the pressure behaviour of two layered reservoir subject to a bottom waterdrive is developedandpresented.TheSource Function Method wasutilised in this study.The nature of the interface between the layers (permeable or impermeable)was also investigated. Results from the study showed thatas layer permeability ratio (k_1:k_2) decreases with increasing well length, layers withhigher permeability responded with increased pressure for impermeable interface (nocrossflow systems) and decreased pressure for permeable interface(crossflowsystem). A number of flow periods prevailed, but for crossflow system,both layers exhibited a compulsory steady state flow period at latetimes.Results from this study canbe applied in well test analysis procedures.

Keywords: Model, layered reservoir, waterdrive,horizontal well.

1.0 Introduction

Layered reservoirs have been the focus of researchers for decades because of their unique nature.Apart from having more than one layer, differences may exist in the formation/fluid characteristicsespecially in the layer permeability.Also the nature of their interfaces need to be ascertained, whether it is permeable or impermeable.Studies in layered reservoir using vertical well[1-3],are more popular than their horizontal counterpart[4,5].Whether the reservoiris modelled asa commingled, crossflowor composite system, the main objectives includedetermination of analytical expression that will aid in the estimation of flow rate,pressure, flow periods,skin factor,and permeability e. t.c.

Bottom water drive reservoirs exhibit movement of water, in a direction that is perpendicular to the well. Past work done on waterdrive reservoirs were mostly in single well systems and in the area of production, conning, estimating water breakthrough time and critical rates[6].And recently, Liu Jia et. al [7] studied the rate, water production and watercut in a layered reservoir with vertical well.Using horizontal wells, a study of the behaviour of layered system with both gas cap and an aquifer was carried out in[10]. There is no available literature for pressure models for layered system in reservoir subject to bottom water drive using horizontal wells. Hence this study will fill the existing gap by developingappropriatemathematical models describing the dimensionless pressure response in individual layer and for the extended reservoir case.The Source and Greens functions method is used in developing the model.The source functions are selected based on flow periods, the well and reservoir geometry according to suggestion made in[8],in conjunction withthe NuemannProduct Rule.The flow periodestimation was done foreach data set using the technique in [13].All model equations were solved numerically [9, 14].

2.0 Reservoir Description and Model Formulation

Figure 2 shows the schematic of the model that consist of 2 horizontal wells each centrally located in the top and bottom layers of a rectangular reservoir.The wells are assumed to be parallel to a no flow boundary at the top, and a constant pressure boundary at the bottom,the lateral boundaries are assumed to be far away from the well ($x_e \gg l$ **and** $y_e \gg l$).The vertically stackedtwo layered bottom waterdrive reservoir has an interface which may or maynot be permeable.Each layer contains fluid of small and constant compressibility.Pressure is above bubble point in the reservoir and production rate is assumed to be constant.Energy from the fluids is derived from the external recharging constant pressure boundary at the bottom of the reservoir. Skin and wellbore effects are not considered.

Corresponding author:E.D Akpobi, E-mail:ituabhor.odesanya@fulokoja.edu.ng, Tel.: +2347067121363

Dimensionless Pressure Computation

In this work, dimensionless parameters are used to make the solution more general(see Appendix).Infinite conductivity wellboreinner boundary condition is assumed (X_D =0.732), and the well is modelled as a line source, at the wellbore.

$$y_{WD} = y_D = r_{WD} \tag{2.1}$$

$$h_D = z_{WD} + z_D \tag{2.2}$$

Sources in a horizontal well are three dimensional in nature, andcan be visualised as the product of the one dimensional sources in the three principal axis using Neumann Product rule.

$$S(X_D, Y_D, Z_D, \tau) = S(X_D, \tau).S(Y_D, \tau).S(Z_D, \tau) \tag{2.3}$$

Note $S(X_D, Y_D, Z_D, \tau)$ and $P_D(X_D, Y_D, Z_D, \tau)$ are used to represent source functions and dimensionless pressure respectively.

Dimensionless pressure canbe represented as.

$$P_D(X_D, Y_D, Z_D, \tau) = 2\prod h_D \int_0^{t_D} S(X_D, \tau).S(Y_D, \tau).S(Z_D, \tau)\partial\tau \tag{2.4}$$

For a layered system Equation 2.4 is reproduced with the inclusion of a factor E_j(modification factor) that accounts for the dual nature of the interface as a result of cross flow between the layers, as shown below

$$P_D(X_D, Y_D, Z_D, \tau) = 2\prod h_D E_j \int_0^{t_D} S(X_D, \tau).S(Y_D, \tau).S(Z_D, \tau)\partial\tau \tag{2.5}$$

Where j =1or 2

Using the source numbers (in Roman numerals) and the axis direction (in letters) [8, 11], Instantaneous Source and Greens function for horizontal wells in this system are assembled depending on the reservoir geometry (external boundaries), flow periods that may prevail and type of well completion.

Layers experience same early flow period since pressure response in the reservoir is infinite acting at this time and the effect of the boundaries/ interface has not been felt.

Early time Radial flow period

S(x, y, z) = ii(x). i(y) . i(z) (2.6)

No Crossflow System

There is no communication between the layers because the interface is sealed hence each layer is independent. Sources describing the flow periods possible for horizontal wells in layer one and two are:

LAYER 1 (bottom layer)

a. Intermediate flow periods

S(x, y, z) = ii(x). iii(y). i(z) (2.7)

S(x, y, z) =vii(x). i(y). i(z) (2.8)

S(x, y, z) =vii(x).i(y).vi(z) (2.9)

b. Late time flow period

S(x, y, z) =vii(x).iii(y).vi(z) (2.10)

LAYER 2 (top layer)

a. Intermediate flow periods

S(x, y, z) = ii(x). iii(y). i(z) (2.11)

S(x, y, z) =vii(x). i(y). i(z) (2.12)

S(x, y, z) =vii(x).i(y).iii(z) (2.13)

b. Late time flow period

S(x, y, z) =vii(x).iii(y).iii(z) (2.14)

Crossflow System

Interface is permeable, hence there is communication between layers in the reservoir.Important parameters that needs to be computed for the system include

1. **Modification Factor** E that accounts for the contributions of both layers at the interface as a result of the dual nature of the z-axis(see appendix)
2. **Crosflow Coefficient (**β); is a factor used to correct for differences in response time of the layers to the same transient regime for crossflow systems.

$$\beta = \frac{\phi_i \mu_i C_{ti} L_i^2 K_{i+1}}{\phi_{i+1} \mu_{i+1} C_{ti+1} L_{i+1}^2 K_i} \tag{2.15}$$

Total Permeability of the system

$$K_t = \frac{K_1 h_1 + K_2 h_2 .... K_n h_n}{h_1 h_2 ...... h_n} \quad (2.16)$$

Fluid mobility ratio

$$M = \frac{K_1 h_1 / \mu_1}{K_2 h_2 / \mu_2} \quad (2.17)$$

Total pay thickness of the system

$$h_{Dt} = h_{D1} + h_{D2} \quad (2.18)$$

LAYER 1(bottom layer)

a. Intermediate flow periods

S(x, y, z) =E_1. ii(x). iii(y). i(z) (2.19)

S(x, y, z) =E_1.vii(x). i(y). i(z) (2.20)

S(x, y, z) =E_1.vii(x).i(y). iv(z) (2.21)

b. Late time flow period

S(x, y, z) =E_1.vii(x). iii(y). iv(z) (2.22)

LAYER 2(top layer)

a. Intermediate flow periods

S(x, y, z) =E_2. ii(x). iii(y). i(z) (2.23)

S(x, y, z) =E_2.vii(x).i(y).i(z) (2.24)

S(x, y, z) =E_2.vii(x).iii(y).i(z) (2.25)

b. Late time flow period

S(x, y, z) =E_2.vii(x).iii(y) .vi(z) (2.26)

The principle of superposition in time is applied to the flow periods that may prevailand the full dimensionless pressure (P_{Dj}) for each layer is developed using Eqs. (2.3)-(2.5).

$$P_{Dj} = P_{D1} + P_{D2} + P_{D3} + ... P_{Dn} \quad (2.27)$$

Where j depicts the layer and n is the last flow period.

Equation. 2.28 (early radial flow) is common to all layers for both systems.

$$P_{D1=} -\frac{\alpha}{8L_D}\sqrt{\frac{k}{K_y}\frac{k}{K_Z}}\mathrm{Ei}(\frac{-r_{WD})^2}{4\tau_D}) \quad (2.28)$$

Hence the full dimensionless pressure P_{D1} expression for layer1(Nocrossflow)is

$$P_{D1}=\left[\begin{aligned}&\left[\frac{\alpha}{8L_D}\sqrt{\frac{K}{K_y}}Ei\left(-\frac{r_{WD}{}^2}{4\tau_D}\right)\right]+\\&\left[\sqrt{\Pi}\int_{t_{De1}}^{t_{De2}}\left\{\begin{aligned}&\left(erf\left(\frac{\sqrt{\frac{K}{K_X}}+X_{D1}}{2\sqrt{\tau_D}}\right)+erf\left(\frac{\sqrt{\frac{K}{K_X}}-X_{D1}}{2\sqrt{\tau_D}}\right)\right)\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\Pi y_{wD}}{Y_{eD}}\cos\frac{n\Pi Y_W}{Y_{eD}}\right).\\&\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D1}-z_{WD1})^2}{4\tau_D}}\end{aligned}\right\}\partial\tau\right]+\\&\left[\sqrt{\Pi}\int_{t_{De2}}^{t_{De3}}\left\{\begin{aligned}&\left(1+\frac{4X_{eD}}{\Pi}\sum_{m=1}^{\infty}\exp^{\left[-\frac{m^2\Pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\frac{m\Pi x_f}{2X_{eD}}\cos\frac{m\Pi x_{wD1}}{X_{eD}}\cos\frac{m\Pi x_{D1}}{X_{eD}}\right).\\&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\Pi y_{wD}}{Y_{eD}}\cos\frac{n\Pi Y_W}{Y_{eD}}\right)..\\&\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D1}-z_{WD1})^2}{4\tau_D}}\end{aligned}\right\}\partial\tau\right]+\\&\left[\frac{2\Pi}{X_{eD}Y_{eD}}\int_{t_{De3}}^{t_D}\left\{\begin{aligned}&\left(1+\frac{4X_{eD}}{\Pi}\sum_{m=1}^{\infty}\exp^{\left[-\frac{m^2\Pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\frac{m\Pi x_f}{2X_{eD}}\cos\frac{m\Pi x_{wD1}}{X_{eD}}\cos\frac{m\Pi x_{D1}}{X_{eD}}\right).\\&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\Pi y_{wD}}{Y_{eD}}\cos\frac{n\Pi Y_W}{Y_{eD}}\right).\\&\left(\sum_{i=i}^{\infty}\exp^{\left[-\frac{(2i-1)^2\Pi^2\tau_D}{4h_D{}^2}\right]}\sin\frac{(2i-1)\Pi z_{wD1}}{2h_D}\sin\frac{(2i-1)\Pi z_{D1}}{2h_D}\right)\end{aligned}\right\}\partial\tau\right]\end{aligned}\right] \quad (2.29)$$

Dimensionless pressure P_{D2} expression for layer2 (Nocrossflow) is

$$P_{D2}=\left[\begin{aligned}&\left[\frac{\alpha}{8L_D}\sqrt{\frac{K}{K_y}}Ei\left(-\frac{r_{WD}{}^2}{4\tau_D}\right)\right]+\\&\left[\frac{\sqrt{\Pi}}{2}\int_{t_{De1}}^{t_{De2}}\left\{\begin{aligned}&\left(erf\left(\frac{\sqrt{\frac{K}{K_X}}+X_{D2}}{2\sqrt{\tau_D}}\right)+erf\left(\frac{\sqrt{\frac{K}{K_X}}-X_{D2}}{2\sqrt{\tau_D}}\right)\right).\\&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{h_D{}^2}}\cos\frac{n\Pi z_{wD2}}{h_{D2}}\cos\frac{n\Pi z_{D2}}{h_{D2}}\right).\\&\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D2}-z_{WD2})^2}{4\tau_D}}\end{aligned}\right\}\partial\tau\right]+\\&\left[\frac{\sqrt{\Pi}}{2}\int_{t_{De2}}^{t_{De3}}\left\{\begin{aligned}&\left(1+\frac{4X_{eD}}{\Pi}\sum_{m=1}^{\infty}\exp^{\left[-\frac{m^2\Pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\frac{m\Pi x_f}{2X_{eD}}\cos\frac{m\Pi x_{wD2}}{X_{eD}}\cos\frac{m\Pi x_{D2}}{X_{eD}}\right).\\&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{h_D{}^2}}\cos\frac{n\Pi z_{wD2}}{h_{D2}}\cos\frac{n\Pi z_{D2}}{h_{D2}}\right).\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D2}-z_{WD2})^2}{4\tau_D}}\end{aligned}\right\}.\partial\tau\right]+\\&\left[\frac{2\Pi}{X_{eD}Y_{eD}}\int_{t_{De3}}^{t_D}\left\{\begin{aligned}&\left(1+\frac{4X_{eD}}{\Pi}\sum_{m=1}^{\infty}\exp^{\left[-\frac{m^2\Pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\frac{m\Pi x_f}{2X_{eD}}\cos\frac{m\Pi x_{wD2}}{X_{eD}}\cos\frac{m\Pi x_{D2}}{X_{eD}}\right).\\&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\Pi y_{wD2}}{Y_{eD}}\cos\frac{n\Pi Y_{D2}}{Y_{eD}}\right).\\&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{h_D{}^2}}\cos\frac{n\Pi z_{wD2}}{h_D}\cos\frac{n\Pi z_{W2}}{h_D}\right)\end{aligned}\right\}\partial\tau\right]\end{aligned}\right] \quad (2.30)$$

The full dimensionless pressure P_{D1} equation for Layer one (crossflow system)is

$$P_{D1}=\left[\begin{array}{l}\left[\dfrac{\alpha}{8L_D}\sqrt{\dfrac{K}{K_y}}Ei\left(-\dfrac{r_{WD}{}^2}{4\tau_D}\right)\right]+\\[2ex]\left[\sqrt{\Pi}E_1\displaystyle\int_{t_{De1}}^{t_{De2}}\left\{\begin{array}{l}erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}+X_{D1}}{2\sqrt{\tau_D}}\right)+erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}-X_{D1}}{2\sqrt{\tau_D}}\right).\\1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\Pi y_{wD}}{Y_{eD}}\cos\dfrac{n\Pi Y_W}{Y_{eD}}.\\\dfrac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(Z_{D1}-Z_{WD1})^2}{4\tau_D}}\end{array}\right\}\partial\tau\right]+\\[2ex]\left[\sqrt{\Pi}E_1\displaystyle\int_{t_{De2}}^{t_{De3}}\left\{\begin{array}{l}\left(1+\dfrac{4X_{eD}}{\Pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\Pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\Pi x_f}{2X_{eD}}\cos\dfrac{m\Pi x_{wD1}}{X_{eD}}\cos\dfrac{m\Pi x_{D1}}{X_{eD}}\right).\\\left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\Pi y_{wD1}}{Y_{eD}}\cos\dfrac{n\Pi Y_{W1}}{Y_{eD}}\right)\\\dfrac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(Z_{D1}-Z_{WD1})^2}{4\tau_D}}\end{array}\right\}\partial\tau\right]+\\[2ex]\left[\dfrac{4\Pi E_1}{X_{eD}Y_{eD}}\displaystyle\int_{t_{De3}}^{t_D}\left\{\begin{array}{l}\left(1+\dfrac{4X_{eD}}{\Pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\Pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\Pi x_f}{2X_{eD}}\cos\dfrac{m\Pi x_{wD1}}{X_{eD}}\cos\dfrac{m\Pi x_{D1}}{X_{eD}}\right).\\\left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\Pi y_{wD}}{Y_{eD}}\cos\dfrac{n\Pi Y_W}{Y_{eD}}\right).\\\left(\displaystyle\sum_{n=1}^{\infty}\exp^{\left[--\frac{n^2\Pi^2\tau_D}{h_{D1}{}^2}\right]}\sin\dfrac{n\Pi Z_{wD1}}{h_{D1}}\sin\dfrac{n\Pi z_{D1}}{h_{D1}}\right)\end{array}\right\}\partial\tau\right]\end{array}\right] \quad (2.31)$$

The full dimensionless pressure P_{D2}equation for Layer 2 (Crossflow) is

$$P_{D2}=\left[\begin{array}{l}
\left[\dfrac{\alpha}{8L_D}\sqrt{\dfrac{K}{K_y}}Ei\left(-\dfrac{r_{WD}{}^2}{4\tau_D}\right)\right]+\\
\left[\sqrt{\Pi}\,\beta E_2\displaystyle\int_{t_{De1}}^{t_{De2}}\left\{\begin{array}{l}
erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}+X_{D2}}{2\sqrt{\beta\tau_D}}\right)+erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}-X_{D2}}{2\sqrt{\beta\tau_D}}\right).\\
\left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\beta\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\Pi y_{wD2}}{Y_{eD}}\cos\dfrac{n\Pi Y_{D2}}{Y_{eD}}\right).\\
\dfrac{1}{2\sqrt{\beta\tau_D}}\exp^{-\frac{(Z_{D2}-Z_{WD2})^2}{4\beta\tau_D}}
\end{array}\right\}\partial\tau\right]+\\
\left[\sqrt{\Pi}\,\beta E_2\displaystyle\int_{t_{De2}}^{t_{De3}}\left\{\begin{array}{l}
\left(1+\dfrac{4X_{eD}}{\Pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\Pi^2\beta\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\Pi x_f}{2X_{eD}}\cos\dfrac{m\Pi x_{wD2}}{X_{eD}}\cos\dfrac{m\Pi x_{D2}}{X_{eD}}\right).\\
\left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\beta\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\Pi y_{wD2}}{Y_{eD}}\cos\dfrac{n\Pi Y_{D2}}{Y_{eD}}\right).\\
\dfrac{1}{2\sqrt{\beta\tau_D}}\exp^{-\frac{(Z_{D2}-Z_{WD2})^2}{4\beta\tau_D}}
\end{array}\right\}\partial\tau\right]+\\
\left[\dfrac{4\Pi\,\beta E_2}{X_{eD}Y_{eD}}\displaystyle\int_{t_{De3}}^{t_D}\left\{\begin{array}{l}
\left(1+\dfrac{4X_{eD}}{\Pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\Pi^2\beta\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\Pi x_f}{2X_{eD}}\cos\dfrac{m\Pi x_{wD2}}{X_{eD}}\cos\dfrac{m\Pi x_{D2}}{X_{eD}}\right).\\
\left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\Pi^2\beta\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\Pi y_{wD2}}{Y_{eD}}\cos\dfrac{n\Pi Y_{D2}}{Y_{eD}}\right).\\
\left(\displaystyle\sum_{n=1}^{\infty}\exp^{\left[-\frac{(2n-1)^2\Pi^2\beta\tau_D}{4h_{D2}{}^2}\right]}\sin\dfrac{(2n-1)\Pi z_{wD2}}{2h_{D2}}\sin\dfrac{(2n-1)\Pi z_{D2}}{2h_{D2}}\right)
\end{array}\right\}\partial\tau\right]
\end{array}\right] \qquad (2.32)$$

The derivatives of the equations were derived by differentiating Eqs.(2.11)-(2.14) using

$$P_D{}'=\partial p_D/\partial\ln t_D \qquad (2.33)$$

3.0 Validation

If the reservoir layers were to be isotropic, and well 2 was used as the reference well. Itwas observed that there isgood agreement in the values of dimensionless pressures for the early time period as shown in Table1, slight variations were observed as time progressed. The variations may be due to different methods of resolving the equations.

Table1: Validation of model at R_{wD}=Y_{WD}=2E-03, L_D=10,Z_{wD}=0.5, X_D=0.732,Y_D=0, Z_D=0.

T_D	Reference [16]	Reference [15]	This work
10^{-5}	.1124	.11245	.11245
10^{-4}	.17007	.17001	.17002
10^{-3}	.2288	.22758	.2276
10^{-2}	.3495	.29547	.4951
10^{-1}	.66767	.66853	.7728
1	1.3763	1.34005	1.055

4.0 Results and Discussion

Table2: Well /Reservoir properties for examples

Parameters	EXAMPLE 1	2	3
L ,ft	1 000	1 200	2 000
h , ft	200	120	100
Z_w ,ft	100	60	50
X_e ,ft	60 000	80 000	16000
Y_e , ft	50 000	50 000	10000
Elev, ft	20	20	20
T_{De1}	0.01 to 0.132	0.002 to 0.13	.0004 to 0.13
T_{De2}	0.027 to 0.169	0.008 to 0.17	0.002 to .17
T_{De3}	1.56 to 1000	1.56 to 1000	1.56 to 1000

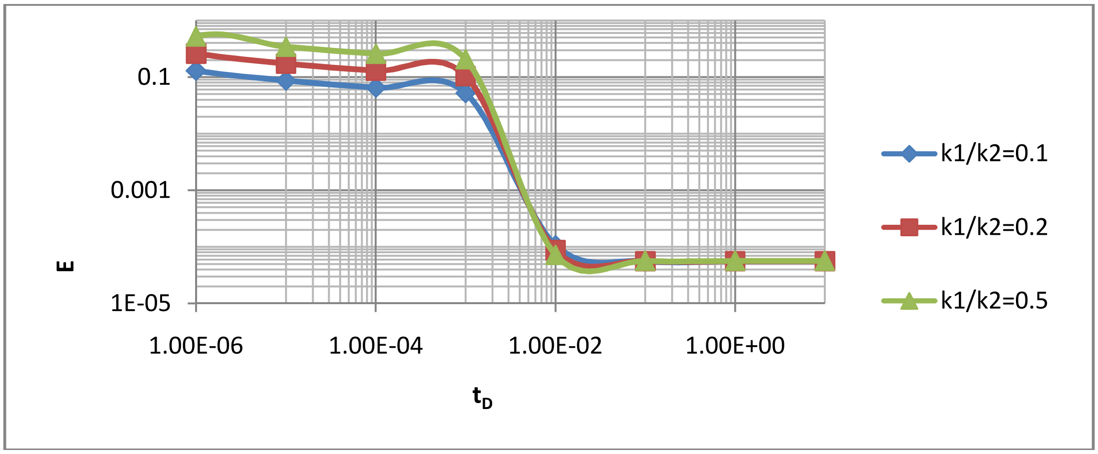

Figure1: Modification Factor E computed at the interface For Examples (Crossflow Systems)

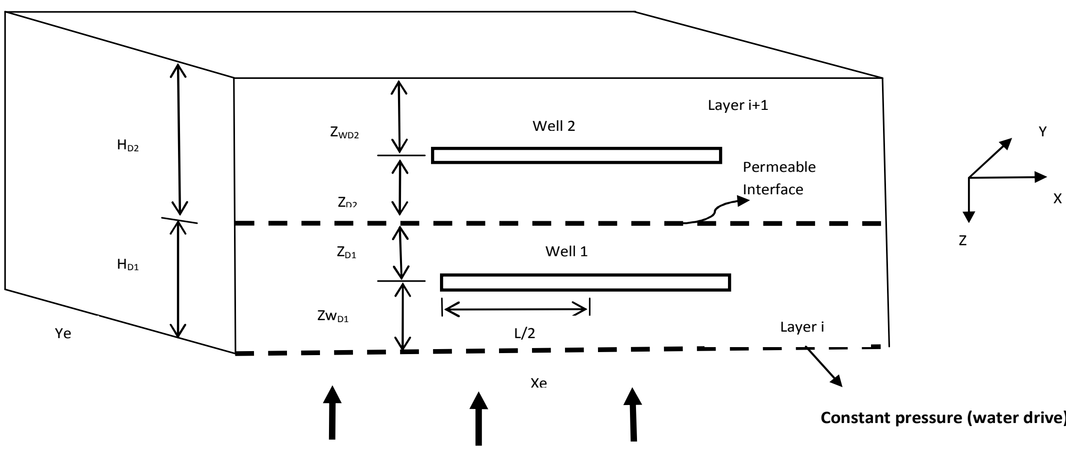

Figure2: Schematic of two horizontal wells in a layered reservoir with bottom water drive (aquifer)

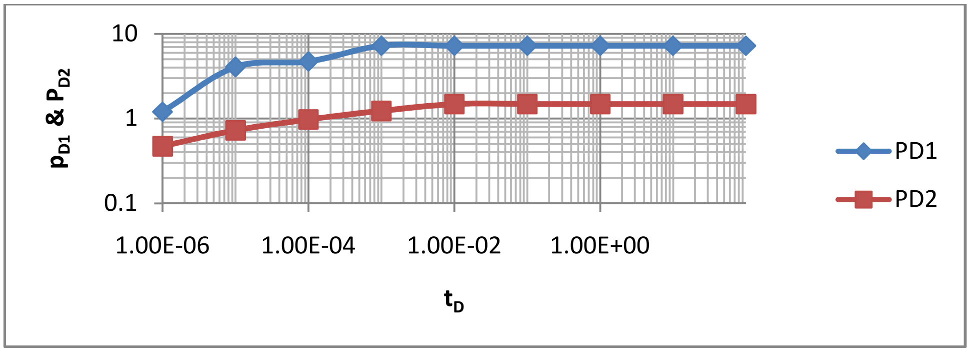

Figure 3: Dimensionless pressure for layer 1 and 2 ,Example 1 Crossflow system
β=10,K_t=5.5,h_{Dt} =0.8,E=5.76E-5

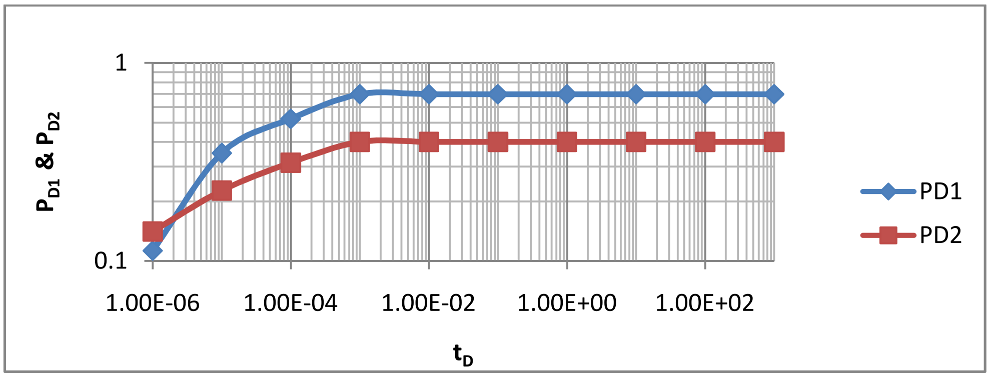

Figure 4: Dimensionless pressure for layer 1 and 2 Example2 Crossflow system
β=5,K_t=3,h_{Dt} =0.4,E=5.76E-5

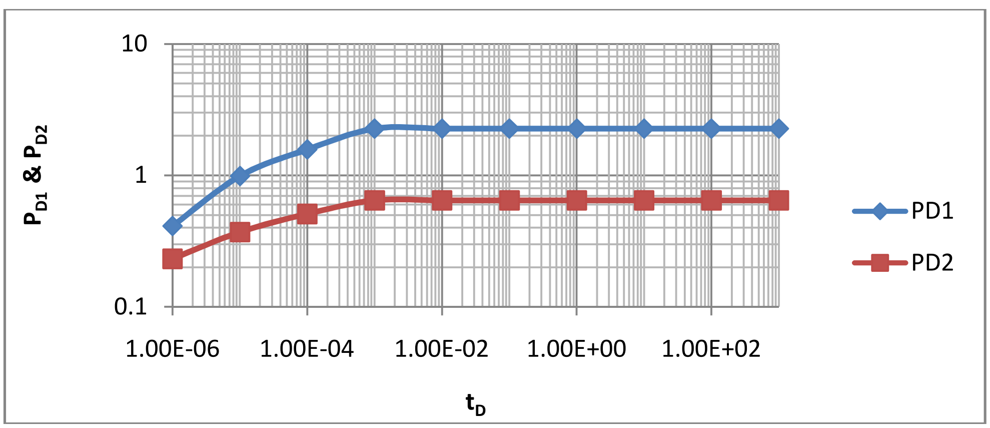

Figure 5: Dimensionless pressure for layer 1 and 2 example3 Crossflow system
β=2, K_t=1.5,h_{Dt} =0.2,E=2.85E-5

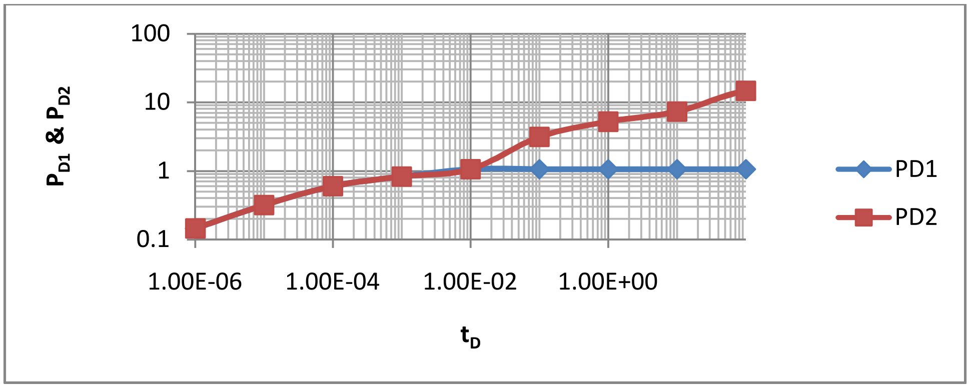

Figure 6: Dimensionless pressure for layer 1 and 2 example1Nocrossflow system
$\beta=10, K_1/K_2=0.1$

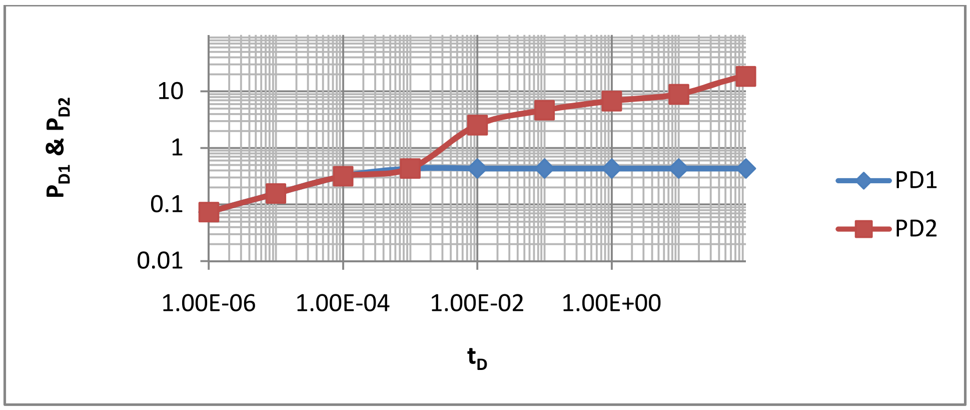

Figure 7: Dimensionless pressure for layer 1 and 2Example2Nocrossflow system
$\beta=5, K_1/K_2=0.2$

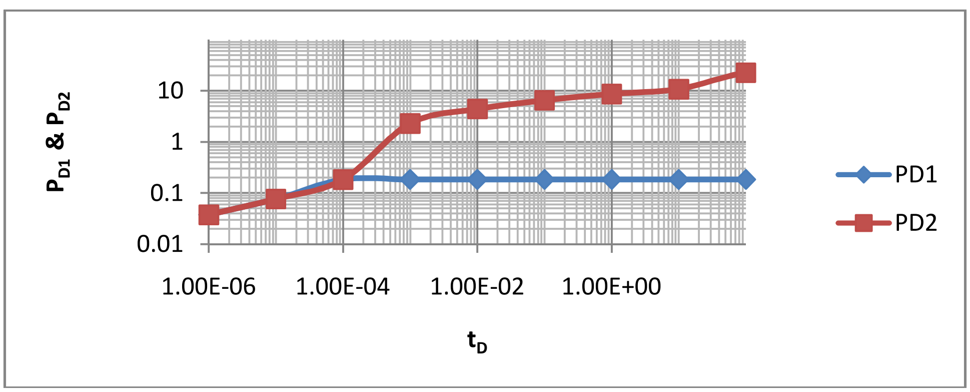

Figure 8: Dimensionless pressurefor layer 1 and 2 example3Nocrossflow system
$\beta=2, K_1/K_2=0.5$

5.0 Discussion of Results

All examples have the following well/fluid properties in common $r_w=0.375$, $C_t=1.0E\text{-}6$, $\varphi=0.2$, $\mu=0.3$, $\alpha=2$, same data is used for layer1 and 2. Other wellbore/reservoir parameters are shown in Table 2, andthe reservoir layer is isotropic
($k_x = k_y = k_z$).Figure1 indicates that the modification factor decreases with time and stabilises at a very low constant value.The time (t_{Dss}) at which it stabilises, differs with each data set.

For crossflow systems,Figures3-5 showed that pressure were lower asdimensionless pay thickness (h_D) and layer permeability ratio(k_1:k_2) decreased .Wells in bothlayers displayed similar flow characteristics,each exhibiting a compulsory steady state flow period at late time as theeffect of the constant pressure boundary dominated well response. Fornocrossflow systems,both layers exhibit same early flow period as shown in figure 6-8 by the single straight line. This isbecausethe reservoir is infinite actingand no boundary has been felt, once this period ends a separation of the curve is observed after a short transition period. This point of separation is seen todecrease with decreasing permeability ratio. At late times, layer one pressure response becomes constant due to the effect of the bottom water drive (constant pressure boundary),whilelayer two depicts an infinite system with the appearance of thepseudo radial flow period.

6.0 Conclusion

Mathematical models for pressure distributions in a layered reservoir with bottom water drive using horizontal wellshave been developed and presented.From this study we draw the following conclusions.

As layer permeability ratio(k_1:k_2) decreases from 10 to 2 with increasing well length,the layer with higher permeability responds with increased pressure for nocrossflow systems and decreasing pressure for crossflow system.

For systems with permeable interface(crossflow) modification factor E gives an indication of the beginning of steady state period(t_{DSS}).Crossflow increases in the more permeable layer and hence a single horizontal well placed in it (layer 2 in this case)can be used to effectively drain the reservoir.

The models can be applied in well testing procedures (drawdown,build-up and interference test) for layered systems.

Nomenclature

B	formation volume factor(RB/STB)
C	compressibility (psi^{-1})
Ei	exponential integral
H	pay thickness (ft)
K	permeability (md)
L	well length (ft)
P	pressure (psi)
Psi	pound per square inch
q	flowrate (STB/D)
s	source
t	time (hr)
Δ	drop
Β	crossfow coefficient
α	flow constant
φ	porosity
μ	viscosity (cp)
τ	dummy time variable

Subscript

D	dimensionless
E	external
Eq	equivalent
Ss	steady state
T	total
W	wellbore

7.0 Reference

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8.0 Appendix

Dimensionless Parameters

Dimensionless distances in the x ,y, z direction

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \qquad \text{(Where i = x,y,z)} \qquad (1a)$$

Equivalent Wellbore Radius

$$R_{eq} = \frac{r_w}{L}\left(\sqrt{\frac{k}{k_z}} + \sqrt{\frac{k}{k_y}}\right) \qquad (2a)$$

Dimensionless time

$$t_D = \frac{Kt}{\mu\varphi C_t (L/2)^2} \qquad (3a)$$

Dimensionless pressure

$$P_D = \frac{2\pi K h \Delta p}{q\mu} \qquad (4a)$$

where

$$\Delta p(x, y, z, t) = \frac{1}{\varphi c_t}\int_0^t q_l S(X, Y. Z. t) \quad dt \qquad (5a)$$

Derivation of Modification Factor (E_j)

The instantaneoussource function (ISF) for the z-axis containing a crossflow interface is represented as

$$s(z_D, t_D) = E_j s(z_D, t_D) \qquad (6a)$$

Where the E_j are the weighting factor/modification factor,and j = 1,2.

Pressure and velocity are equal at the interface between the layers, dimensionless form pressure in a horizontal well for each layer in the reservoir[2] is given by

$$P_{DJ} = 2\pi h_{Dj} E_j \int_0^{t_D} s(x_D, \tau), s(y_D, \tau), s(z_D, \tau) d\tau \ = 1 \qquad (7a)$$

Expanding the above equations gives

$$P_{DJ} = 2\pi \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{\infty} h_{Dj} E_J \int_0^{t_D} s(x_D, \tau), s(y_D, \tau), s(z_D, \tau) d\tau \ = 1 \qquad (8a)$$

The source functions obey the orthogonality relation [2,11,12],
A series of these function in eq. (8a) can be represented by

$$f(x) = \sum_{n=1}^{\infty} a_n f_n\,(x) \quad (9a)$$

Where a_n are the weightfactors given by

$$a_n = \frac{\int_a^b f_x f_{n(x)w(x)} d_x}{\int_a^b f_n{}^2 w(x) d_x d_x} \quad (10a)$$

By same analogy also using Equations.2.13 and 2.14 for layer 1 and 2, E_j is given as:

$$E_J = \frac{W_1+W_2}{2\pi({W_1}^2+{W_2}^2)} \quad (11a)$$

$$W_1 = h_{D1}\int_{X_{wD}}^{X_{eD}}\int_0^{y_{eD}}\int_0^{h_{D1}}\left[\int_0^{t_D} s_1(x_D,\tau), s_1(y_D,\tau), s_1(z_D,\tau)d\tau\right] d_{zD}d_{yD}d_{xD} \quad (12a)$$

$$W_2 = h_{D2}\int_{X_{wD}}^{X_{eD}}\int_0^{y_{eD}}\int_{h_{D2}}^{h_D}\left[\int_0^{t_D} s_2(x_D,\tau), s_2(y_D,\tau), s_2(z_D,\tau)d\tau\right] d_{zD}d_{yD}d_{xD}. \quad (13a).$$