Journal of the Nigerian Association of Mathematical Physics
Volume35,(May, 2016), pp 413 – 416
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On the Modified Adomian Decomposition Method Applied to a Transformed Fredholm Integro-Differential Equation of the Second Kind

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Abstract

In this paper, FredholmIntegro-differential equations are reduced to an equivalent Fredholm integral equation of the second kind and the exact solutions are obtained using the Modified Adomian Decomposition Method (MADM) in order to reduce the large computational workexperienced in some other methods.

1.0 Introduction

In 1980 a new method to solve linear and nonlinear functional equations was introduced and has since been termed as the Adomian Decomposition Method (ADM) and has been subject of many investigations [1]. The method generates a series form solutions whose terms are determined by a recursive relation. Some fundamental works on various aspect of modification of Adomian decomposition method are given in the literature [1-3]. Homotopy analysis and variational iteration methodswere also used to solve nonlinear integro-differentialequations [4]

Taylor's polynomials were used to study the solution of the integro-differential equations [5].The used of modified decomposition method, Lagrange interpolation and the traditional decomposition methods to solve nonlinear integral equations were also extensibly discussed in [5-7]. However, it is well known that large computational work is the main disadvantage of some of the methods mentioned above. It is also known that these traditional non perturbation methods cannot ensure convergence of solution series; they are in fact only valid for weakly non linear problems [6].

The basic motivation of this paper is to; employ a transformational method combining it with modified Adomian decomposition methodin order to overcome the cumbersomeness of the huge computational work experienced in some other methods. The former will be used to transform integro-differential equation to an integral equation of the second kind, while the later will be aimed at solving the transformed equation to obtain its exact solutions. The rest of the paper is organized as follows: In section 2, a brief discussion of the MADM and the transformation procedure is presented. In section3implementation of the methods is considered by solving examples. Section 4 ends the paper with a brief conclusion.

2.0 Methodology

The linear Fredholmintegro-differential equation is given by

$$u^n(x) = f(x) + \lambda \int_a^b k(x,t)\, u(t)dt\,, \qquad (2.1)$$

where$u^n(x)$ is the nth derivative of the unknown function $u(x)$ that will be determined, $k(x,t)$ is the kernel of the integration, $f(x)$ is an analytic function and $\boldsymbol{\lambda}$ is a parameter, the function $u(t)$ appears linearly under the integral sign [4]

To illustrate the MADMwe consider the standard Adomian decomposition method (ADM) as given by [6]

$$u_0(x) = f(x)$$

$$u_1(x) = \lambda \int_a^b k(x,t)u_0(t)dt$$

$$u_2(x) = \lambda \int_a^b k(x,t)u_1(t)dt$$

$$u_3(x) = \lambda \int_a^b k(x,t)u_2(t)dt$$

$$u_{n+1}(x) = \lambda \int_a^b k(x,t)\, u_n(t)dt, n \geq 0 \qquad (2.2)$$

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In view of (2.2) the component $u_n(x),\ n \geq 0$can easily be evaluated.
However, the MADAM introduces a slight variation to (2.2) that will lead to the determination of the component$u(x)$ in an easier and faster manner.
We set the Inhomogeneous term $f(x)$ as a sum of two partial functions, namely$f_1(x) and f_2(x)$, i.e.

$$f(x) = f_1(x) + f_2(x) \qquad (2.3)$$

In view of(2.3), we introduce a qualitative change in theformation of (2.3) as established by [7].
We identify the zeroth component $u_0(x)$ by one part of$f(x)$, the other part of $f(x)$ can be added to the component $u_1(x)$ among other terms

$$u_0(x) = f_1(x),$$
$$u_1(x) = f_2(x) + \lambda \int_a^b k(x,t)u_0(t)dt,$$
$$u_2(x) = \lambda \int_a^b k(x,t)u_1(t)dt$$
$$u_3(x) = \lambda \int_a^b k(x,t)u_2(t)dt$$
$$u(x) = u_0(x) + \ u_n(x), n \geq 1 \qquad (2.4)$$

3.0 Transformation Analysis

The transformation analysis as presented [8-10] contained the n-fold integral formula as:

$$\int_0^x \int_0^x u(t)dtdt = \int_0^x (x-t)u(t)dt,$$
$$\int_0^x \int_0^x \int_0^x u(t)dtdtdt = \frac{1}{2!}\int_0^x (x-t)^2 u(t)dt,$$
$$\int_0^x \int_0^x \dots \int_0^x u(t)dt = \frac{1}{(n-1)!}\int_0^x (x-t)^{n-1}u(t)dt \qquad (3.1)$$

4.0 Implementation

In this section we apply the transformation and the modified Adomian decomposition methods to solve some examples to support our claims
Example 1. Consider the Volterra-Fredholm integral equation of the form

$$u(x) = \frac{1}{80}(8x^5 + 80x^2 + 5) - \frac{1}{4}\int_0^1 tu(t)dt - \frac{1}{2}\int_0^x t^2 u(t)dt \qquad (4.1)$$

equation (4.1) was solvedby [12] by the use of homotopy perturbation method
Here we use the MADM after the transformation and obtained
$u_0(x) = x^2$as the exact solution, that is

$$u_1(x) = \frac{1}{10}x^5 + \frac{1}{16} - \frac{1}{2}\int_0^1 tu_0(t)dt - \frac{1}{2}\int_0^x t^2 u_0(t)dt = 0 \qquad (4.2)$$
$$u_2(x) = 0$$
$$\vdots$$

Example 2 Consider theFredholmIntegro-differential equation of the second kind of the form:

$$u'''(x) = -cosx + x + \int_0^{\frac{\pi}{2}} xu''(t)\,dt, u(0) = 0, u'(0) = 1, u''(0) = 0 \qquad (4.3)$$

Integrating both sides of (4.3) from 0 tox three times and using the initial conditions we obtain

$$u(x) = sinx + \frac{x^4}{4!} + \frac{x^4}{4!}\int_0^{\frac{\pi}{2}} u''(t)\,dt \qquad (4.4)$$

Applying the MADM to (4.4) gives

$$u_0(x) = sinx$$
$$u_1(x) = \frac{x^4}{4!} + \frac{x^4}{4!}\int_0^{\frac{\pi}{2}} u_0''(t)dt = \frac{x^4}{4!} - \frac{x^4}{4!} = 0$$
$$u_2(x) = 0$$

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Therefore

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \ = sinx \qquad (4.5)$$

Example 3: Consider the linear FredholmIntegro-differential equation of the second kind

$$u''(x) = -sinx + x - \int_0^{\frac{\pi}{2}} xtu(t)\,dt, u(0) = 0, u'(0) = 1 \qquad (4.6)$$

Integrating both sides of (4.6) from 0 tox two times and using the initial conditions we obtain

$$u(x) = sinx + \frac{x^3}{3!} + \frac{x^3}{3!}\int_0^{\frac{\pi}{2}} tu(t)\,dt \qquad (4.7)$$

Applying the MADM to (4.7) gives

$u_0(x) = sinx$

$u_1(x) = \frac{x^3}{3!} - \frac{x^3}{3!}\int_0^{\frac{\pi}{2}} tu_0(t)dt = \frac{x^3}{3!} - \frac{x^3}{3!} = 0$

$u_2(x) = 0$

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Therefore,

$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \ = sinx \qquad (4.8)$$

Example 4: Consider the FredholmIntegro-differential equation of the second kind

$$u'(x) = xe^x + e^x - x + \int_0^1 xu(t)\,dt, u(0) = 0 \qquad (4.9)$$

Applying same procedures as started above (4.9) becomes

$$u(x) = xe^x - \frac{x^2}{2!} + \frac{x^2}{2!}\int_0^1 u(t)\,dt \qquad (4.10)$$

Applying the MADM to (4.10) gives

$u_0(x) = xe^x$

$u_1(x) = -\frac{x^2}{2!} + \frac{x^2}{2!}\int_0^1 u_0(t)dt = \frac{x^2}{2!} - \frac{x^2}{2!} = 0$

$u_2(x) = 0$

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$$u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \ = xe^x \qquad (4.11)$$

5.0 Conclusion

The main idea of this work was to combine the transformatinal and the modified Adomian decomposition methods to solve Fredholm integro-differential equations. We carefully transform the Fredholm integro-differential equation to a standard Fredholm integral equation of the second kind and we applied the MADM to obtain the exact solution. The main advantage of this approach is the fact that it gives the analytical solution in just few iterations rather than series solution as obtained in some other methods, this can be observed in example 1 equation (4.1).

6.0 References

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