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Improved Product Estimators Using Known Value of Some Population Parameters

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Abstract

In this study, three product estimators were proposed which were compared with the conventional product estimator depending on the value of α being considered. The conditions attached to these proposed product estimators are that: $0 < \alpha < 1$, $(\frac{\bar{X}}{\bar{X} + \rho_{xy}}) > 1$, $(\frac{\bar{X}}{\bar{X} + c_x}) < 1$ and $(\frac{\bar{X}}{\bar{X} + \beta_{xy}}) > 1$. Two data sets called populations I and II are used to justify this work using their derived mse's and the proposed product estimators, $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$ are said to be better and more efficient than the conventional product estimator, $\bar{y}_{pr}$ whenever $0.1 \le \alpha \le 0.9$ but $\bar{y}_{paa2}$ was best preferred.

Key words: Product, estimator, efficient, mean square error, bias, conditions

1.0 Introduction

Let N and n be the population and sample sizes respectively, $\bar{X}$ and $\bar{Y}$ be the population means for the auxiliary variable (X) and the variable of interest (Y), $\bar{x}$ and $\bar{y}$ be the sample means based on the sample drawn. If the correlation between the study variable y and the auxiliary variable x is negative (low), the product method of estimation is quite effective. This paper is interested only in product method of estimation when the correlation between the study variable y and the auxiliary variable x is negative (low).
Conventionally [1 – 2],

$$\bar{y}_{pr} = \frac{\overline{yx}}{\bar{X}} \qquad \ldots (1)$$

$$mse(\bar{y}_{pr}) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho_{yx}c_x c_y)] \qquad \ldots (2)$$

respectively.
In sample surveys, supplementary information is often used for increasing the precision of estimators [3 – 8].
Many authors [9 – 11] have used auxiliary information for improved estimation of population mean of study variable y.

2.0 On the newly Proposed Product Estimators

The proposed product estimators are given as:

$$\bar{y}_{paa1} = \alpha(\frac{\overline{yx}}{\bar{X} + \rho_{xy}}) \quad \text{… (3)}$$

$$\bar{y}_{paa2} = \alpha(\frac{\overline{yx}}{\bar{X} + c_x}) \quad \text{… (4)}$$

$$\bar{y}_{paa3} = \alpha(\frac{\overline{yx}}{\bar{X} + \beta_{xy}}) \quad \text{… (5)}$$

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where $\beta_{xy} = \frac{\rho_{xy} s_x s_y}{s^2_x} = \frac{\rho_{xy} s_y}{s_x}$, $\bar{x} = \bar{X}(1+\Delta_{\bar{x}}),\ \bar{y} = \bar{Y}(1+\Delta_{\bar{y}}),\quad \Delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}},\ \Delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}$

such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$

The biases and mean square errors of these proposed product estimators, $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$ are given as:-

$$bias(\bar{y}_{paa1}) = \alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})\bar{Y}(\frac{N-n}{Nn})(\rho_{xy} c_x c_y) \quad \text{… (6)}$$

$$mse(\bar{y}_{paa1}) = \alpha^2(\frac{\bar{X}}{\bar{X} + \rho_{xy}})^2 \bar{Y}^2(\frac{N-n}{Nn})[c^2{}_x + c^2{}_y + 2\rho_{xy} c_x c_y] \quad \text{… (7)}$$

$$bias(\bar{y}_{paa2}) = \alpha(\frac{\bar{X}}{\bar{X} + c_x})\bar{Y}(\frac{N-n}{Nn})(\rho_{xy} c_x c_y) \quad \text{… (8)}$$

$$mse(\bar{y}_{raa2}) = \alpha^2(\frac{\bar{X}}{\bar{X} + c_x})^2 \bar{Y}^2(\frac{N-n}{Nn})[c^2{}_x + c^2{}_y + 2\rho_{xy} c_x c_y] \quad \text{.. . (9)}$$

$$bias(\bar{y}_{paa3}) = \alpha(\frac{\bar{X}}{\bar{X} + \beta_{xy}})\bar{Y}(\frac{N-n}{Nn})(\rho_{xy} c_x c_y) \quad \text{… (10)}$$

$$mse(\bar{y}_{paa3}) = \alpha^2(\frac{\bar{X}}{\bar{X} + \beta_{xy}})^2 \bar{Y}^2(\frac{N-n}{Nn})[c^2{}_x + c^2{}_y + 2\rho_{xy} c_x c_y] \quad \text{… (11)}$$

provided $0 < \alpha < 1$, $(\frac{\bar{X}}{\bar{X} + \rho_{xy}}) > 1$, $(\frac{\bar{X}}{\bar{X} + c_x}) < 1$ and $(\frac{\bar{X}}{\bar{X} + \beta_{xy}}) > 1$.

The proofs of (6) - (11) are as shown in section 3.0

3.0 Derivation of Biases and Mean Square Errors of $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$

Let,

$\bar{y}_{paa1} = \alpha(\frac{\overline{yx}}{\bar{X} + \rho_{xy}})$, where, $\bar{x} = \bar{X}(1 + \Delta_{\bar{x}}), \quad \bar{y} = \bar{Y}(1 + \Delta_{\bar{y}}), \quad \Delta_{\bar{y}} = \frac{\bar{y} - \bar{Y}}{\bar{Y}}, \ \Delta_{\bar{x}} = \frac{\bar{x} - \bar{X}}{\bar{X}}$

such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$ as earlier defined.

Therefore, using power series expansion,

$$\bar{y}_{paa1} = \alpha(\frac{\overline{yx}}{\bar{X} + \rho_{xy}}) = \alpha(\frac{\bar{Y}(1 + \Delta_{\bar{y}})\bar{X}(1 + \Delta_{\bar{x}})}{(\bar{X} + \rho_{xy})})$$

$$= \alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})(\bar{Y}(1 + \Delta_{\bar{y}})(1 + \Delta_{\bar{x}})) = \alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})\bar{Y}(1 + \Delta_{\bar{y}} + \Delta_{\bar{x}} + \Delta_{\bar{y}}\Delta_{\bar{x}})$$

$$bias(\bar{y}_{paa1}) = E[\alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})\bar{Y}(1 + \Delta_{\bar{y}} + \Delta_{\bar{x}} + \Delta_{\bar{y}}\Delta_{\bar{x}}) - \bar{Y}]$$

$$bias(\bar{y}_{paa1}) = E[\alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})\bar{Y}(\Delta_{\bar{y}}\Delta_{\bar{x}})]$$

Let, $E(\Delta_{\bar{y}}) = E(\Delta_{\bar{x}}) = 0, E(\Delta_{\bar{x}}^2) = \frac{S^2{}_x}{\bar{X}^2} = c^2{}_x, E(\Delta_{\bar{y}}^2) = \frac{S^2{}_y}{\bar{Y}^2} = c^2{}_y$ and $E(\Delta_x\Delta_y) = \frac{S_{xy}}{\overline{XY}} = \rho_{xy}c_x c_y$. Then,

$$bias(\bar{y}_{paa1}) = \alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})\bar{Y}(\frac{N - n}{Nn})(\rho_{xy}c_x c_y) , \quad (6)$$

$$mse(\bar{y}_{paa1}) = E[\alpha(\frac{\bar{X}}{\bar{X} + \rho_{xy}})(\bar{Y}(1 + \Delta_{\bar{x}} + \Delta^2{}_{\bar{x}} + \Delta_{\bar{y}} + \Delta_{\bar{x}}\Delta_{\bar{y}} + \Delta^2{}_{\bar{x}}\Delta_{\bar{y}}) - \bar{Y})]^2$$

$$mse(\bar{y}_{paa1}) = \alpha^2(\frac{\bar{X}}{\bar{X} + \rho_{xy}})^2\bar{Y}^2(\Delta_{\bar{y}} + \Delta_{\bar{x}})^2$$, ignoring higher orders

$$mse(\bar{y}_{paa1}) = \alpha^2(\frac{\bar{X}}{\bar{X} + \rho_{xy}})^2\bar{Y}^2(\frac{N - n}{Nn})(c^2{}_y + c^2{}_y + 2\rho_{xy}c_y c_y) , \quad (7)$$

Similarly,

$$bias(\bar{y}_{paa2}) = \alpha(\frac{\bar{X}}{\bar{X} + c_x})\bar{Y}(\frac{N - n}{Nn})(\rho_{xy}c_x c_y) , \quad (8)$$

$$mse(\bar{y}_{paa2}) = \alpha^2(\frac{\bar{X}}{\bar{X} + c_x})^2\bar{Y}^2(\frac{N - n}{Nn})[c^2{}_x + c^2{}_y + 2\rho_{xy}c_x c_y] , \quad (9)$$

$$bias(\bar{y}_{paa3}) = \alpha(\frac{\bar{X}}{\bar{X} + \beta_{xy}})\bar{Y}(\frac{N - n}{Nn})(\rho_{xy}c_x c_y) , \quad (10)$$

$$mse(\bar{y}_{paa3}) = \alpha^2 (\frac{\bar{X}}{\bar{X} + \beta_{xy}})^2 \bar{Y}^2 (\frac{N-n}{Nn})[c^2{}_x + c^2{}_y + 2\rho_{xy} c_x c_y] \quad (11)$$

4.0 Data Used

The proposed product estimators, $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$ are said to be better and more efficient than the conventional product estimator, $\bar{y}_{pr}$, depending on the value of α being considered in this study. The conditions attached to these proposed product estimators are that: $0 < \alpha < 1$, $(\frac{\bar{X}}{\bar{X} + \rho_{xy}}) > 1$, $(\frac{\bar{X}}{\bar{X} + c_x}) < 1$ and $(\frac{\bar{X}}{\bar{X} + \beta_{xy}}) > 1$. Two data sets are used to justify this claim.

Table 1: Summary of the data sets used.

Population	I	II
Source	[12]	[13]
Case	$\bar{Y} > \bar{X}$	$\bar{Y} < \bar{X}$
N	20	30
n	8	6
$\bar{X}$	18.8	75.4313
$\bar{Y}$	19.55	7.6375
c_x	0.1281	0.0986
c_y	0.1445	0.2278
ρ_{xy}	-0.9199	-0.6823
s_x	2.4083	7.4375
s_y	2.8250	4.7582
β_{xy}	-1.079067	-0.4365
$\frac{\bar{X}}{\bar{X} + \rho_{xy}}$	1.051448	1.009127881
$\frac{\bar{X}}{\bar{X} + c_x}$	0.9932	0.9987
$\frac{\bar{X}}{\bar{X} + \beta_{xy}}$	1.0609	1.005820404

5.0 Results

The results obtained are shown in Tables 2.

Table 2:- Mean Square Errors obtained on $\bar{y}_{pr}$, $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$ using Populations I and II

Estimator	α	Mse (population I)	Mse (population II)
$\bar{y}_{pr}$		0.0927	0.2408

$\bar{y}_{paa1}$	1.0	0.1025	0.2452
	0.9	0.0830	0.1986
	0.8	0.0656	0.1568
	0.7	0.0502	0.1201
	0.6	0.0369	0.0883
	0.5	0.0256	0.0613
	0.4	0.0164	0.0392
	0.3	0.0092	0.0221
	0.2	0.0041	0.0098
	0.1	0.0010	0.0025
$\bar{y}_{paa2}$	1.0	0.0914	0.2402
	0.9	0.0740	0.1946
	0.8	0.0585	0.1537
	0.7	0.0448	0.1177
	0.6	0.0329	0.0865
	0.5	0.0229	0.0601
	0.4	0.0146	0.0384
	0.3	0.0082	0.0216
	0.2	0.0037	0.0096
	0.1	0.0009	0.0024
$\bar{y}_{paa3}$	1.0	0.1043	0.2436
	0.9	0.0845	0.1973
	0.8	0.0668	0.1559
	0.7	0.0513	0.1194
	0.6	0.0375	0.0880
	0.5	0.0261	0.0609
	0.4	0.0167	0.0390
	0.3	0.0094	0.0219
	0.2	0.0042	0.0097
	0.1	0.0010	0.0024

6.0 Discussion

From the estimates in Table 2

(i).when $\alpha = 1,$ the proposed product estimator, $\bar{y}_{paa2}$, has the least mse of 0.0914 and 0.2402 for populations I and II respectively and hence preferred.

(ii).when $0.1 \leq \alpha \leq 0.9$, the proposed product estimators $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$, are better and more efficient than the conventional product estimator, $\bar{y}_{pr}$ and hence preferred but

(iii) of all these proposed product estimators, $\bar{y}_{paa2}$,is the most preferred in this range.

7.0 Conclusion

Therefore, from the estimates obtained, the proposed product estimators, $\bar{y}_{paa1}$, $\bar{y}_{paa2}$ and $\bar{y}_{paa3}$ are preferred to the conventional product estimator, $\bar{y}_{pr}$, whenever $0.1 \leq \alpha \leq 0.9$ but $\bar{y}_{raa2}$ was best preferred. Hence, this proposed product estimator, $\bar{y}_{raa2}$, is recommended for usage in sample survey.

8.0 References

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