ON THE KINEMATICS OF WAVE PARAMETERS IN A MEDIUM WITH VARYING CHACTERISTICS.

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Abstract:

The study concernsthe evolutions of wave parameters in a slowly varying medium, for example,ideal fluid medium. It re-establises that the group velocity tends to modulation velocity as both the frequency and wave number spectral bandwidthstend to zero.

Further, in the slowly varying medium, it is established that both the wave length and period are governed by quasi-linear partial differential equation for the hyperbolic system.A rather more generalised concept of group velocity is also suggested.

Introduction

The parameters in this study are (i). wave length, L, related to the wave number,k, in the form, L= $\frac{2\pi}{K}$, ($\pi = {}^{22}/_{7}$); (ii) wave period T, related to the wave frequency,ω, in the form, T =$\frac{2\pi}{\omega}$. The combined forms in (one way or the other)of the two parameters (L and T), give rise to: (i)the phase velocity c, which defines the movement of each waves component briefly and (ii). the group velocity c_g, associated with envolving wave group.

The activities of these parameters give rise to the foundation on which some important marine physical processes are built. A very educative detail description of the above may be found in Kundu [1], Mei [2], and Lighthill [3].Indeed, these parameters play remarkable and commanding part in such wave activities as wave energy fluxes and breaking.

Okeke and Asor [4] and Asor and Okeke [5] introduced the theory of low wave numbers and related high phase velocity components in the energy spectrum for the gravity induced water waves. These components of gravity waves spectrum resonate identical components in the layered sea-bed, and thus,excite microseismic oscillations on seabed.[2,3].

In the previous studies, the wave parameters were assumed to be constantin space and time[1]. We intend to analyse those parameters affected by slowly envolvingsurrounding environment. Consequently both wave number and frequency will be spacial and time dependent, same applies to the related parameters (wave length, frequency, phase and groupvelocities).

The modulation velocity

Consider two dominant componentgravit**y** waves with equal amplitudes, A, but with different wave number ($k_{1,}k_2$) and frequency (ω_1,ω_2), respectively. These components η_1(x,t) and $\eta_2(x, t)$ move in the same direction along x axis at time t. It is assumed that they are unidirectional. i.e

$\eta_1(x,t) = A\,cos(k_1x - \omega_1t),\ \eta_2(x,t) = \mathrm{A}\cos(k_2x - \omega_2\mathrm{t}),$

The interference patterns

The interference patterns, η(x, t) is given by

η(x, t) = η_1(x, t) + η_2(x,t) = 2P(x,t)Q(x,t)

Where

P(x,t) = $\cos[(\Delta k/2)x - (\Delta\omega/2)t](1)$

= $\cos[(k_1 x - \omega_1 t) - (x\frac{\Delta k}{2} - t\frac{\Delta\omega}{2})]$

= $\cos[(k_2 x - \omega_2 t) - (x\frac{\Delta k}{2} - t\frac{\Delta\omega}{2})]$

As $k_1 \to k_2 \to$ k and $\omega_1 \to \omega_2 \to \omega$, then

Q = cos t [kx –ωt(2)

Because, k >>$\frac{\Delta k}{2}$, ω>>$\frac{\Delta\omega}{2}$

The term 2AP is the modulation amplitude.

$\Delta k = k_1 - k_2,\ \Delta\omega = \omega_1 - \omega_2$. Thus,

$$\eta(x,t) = 2A\,cos\,\frac{1}{2}[x\Delta k - t\Delta t]\cos(kx - \omega t) \quad (3)$$

The phase velocity is ω/k, modulation velocity = ($\frac{\Delta\omega}{\Delta k}$).

The wave length of the interacting component wave = $\frac{2\pi}{k}$ and

Period T= $\frac{2\pi}{\omega}$, that of the carrier wave (modulation) is given by $L_m = \frac{2\pi}{\Delta k}$,

$T_m = \frac{2\pi}{\Delta\omega.}$. Thus, because $k_1 \to k_2$ and $\omega_1 \to \omega_2$, L_m and T_m are much larger than L and T respectively.

Finally, the modulation velocity, $c_m = \frac{\Delta\omega}{\Delta k}$, and as $\Delta\omega \to 0$ and

$\Delta k \to 0$, $c_m \to c_g$ (the group velociy).

Thuslim$_{\Delta k \to 0} \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} = c_g$(4)

Implication of eqn (4)

From (4),

$c_g = \frac{d\omega}{dk} = \frac{d(kc)}{dk}$ = c + k$\frac{dc}{dk}$. But k = $\frac{2\pi}{L}$

Thus, c_g = c- L($\boldsymbol{\frac{dc}{dL}}$)**(5)**

In non- dispersive medium, c ≠ c(k), thus, c_g = c , for $\frac{dc}{dL}$ = 0.

In dispersive medium,

ω^2 **= k**g(tankh)R, g is the acceleration of gravity.

R = 1 + $\pi\delta^2 + \pi^2\delta^4$, δ = kh_0, h_0 = water dept scale.

Thus, kh_0 is the non-linearity parameters. In marine physics, $\delta = 0.08$, hence, δ^2 is small compared to unity, thus, R = 1 (approximately) and

$\boldsymbol{\omega^2}$ **= k**g tanh (kh_0)(6)

From (6), $2\omega\frac{d\omega}{dk}$ = g tanhkh + khg sech^2kh and

$\frac{d\omega}{dk} = \frac{g \tan hkh}{2\omega}$ +khg $sech^2$kh

$= \frac{g}{2\omega}$tanhkh [1 +$\frac{2kh}{Sinh(2kh)}$] =$\frac{C}{2}$n (7)

n = $\frac{1}{2}$ [1 + $\frac{2kh}{Sinh2kh}$] (8)

h = constant fluid depth =$h_0$in this case.

The slowly envolving wave parameters

In this consideration, k =k(x, t),ω = ω(x, t), c = c(x,t). so with other parameters related to them such as wave length, L and period, T . In the slowly varying environment, higher differential coefficient of c(x,t) will be neglected. i,e.

$\frac{\partial^2 c}{\partial x^2} << \frac{1}{L}\frac{\partial c}{\partial L}$,

If follows that if the velocity at (x,t) is c = c(x,t), and this defines the spacial position of a wave crest height and (x + L, t), the position of the adjacent crest height corresponding to the phase velocity is c + L$\frac{\partial c}{\partial x}$, consequently,

$\frac{DL}{Dt}$ = L$\frac{\partial C}{\partial x}$ + 0($\frac{\partial^2 L}{\partial x^2}$= L$\frac{\partial C}{\partial x}$(9)

But $\frac{\partial C}{\partial x}$ = $\frac{\partial C}{\partial L}\frac{\partial L}{\partial x}$ and, the total derivative provides that

$\frac{DL}{Dt}$ = $\frac{\partial L}{\partial t}$ + c$\frac{\partial L}{\partial x}$ = L$\frac{\partial C}{\partial x}$= L$\frac{\partial C}{\partial L}\frac{\partial L}{\partial x}$, from (9)

Thus

$$\frac{\partial L}{\partial t} + \frac{\partial L}{\partial x}\left[c - L\frac{\partial C}{\partial L}\right] = 0 \qquad (10)$$

Let V_g = (c - L$\frac{\partial c}{\partial L}$) (11)

And, thus,

$$\frac{\partial L}{\partial t} + V_g\frac{\partial L}{\partial x} = 0 \qquad (12)$$

Thus,the present model, suggest that the total change of L is constant in any frame moving with velocity. V_g. From (5), and (11), c_g = V_g.

Further, L = cT and eliminate L from (12), Weobtain,

$\frac{\partial}{\partial t}$(cT) + $V_g \frac{\partial}{\partial x}$(cT) = 0, this gives

T$\frac{\partial c}{\partial t}$ + c$\frac{\partial T}{\partial t}$ + V_g[c$\frac{\partial T}{\partial x}$ + T$\frac{\partial c}{\partial x}$]

Thus,

$$T\,[\frac{\partial c}{\partial t} + V_g \frac{\partial c}{\partial x}] + c[\frac{\partial T}{\partial L} + V_g \frac{\partial T}{\partial x}] = 0 \qquad (13)$$

But, $\frac{\partial c}{\partial t} = \frac{\partial c}{\partial T}\frac{\partial T}{\partial t}$, $\frac{\partial c}{\partial x} = \frac{\partial c}{\partial T}\frac{\partial T}{\partial x}$

$$\text{Then,} \frac{\partial c}{\partial t} + V_g \frac{\partial c}{\partial x} = \frac{\partial c}{\partial T}\frac{\partial T}{\partial t} + V_g \frac{\partial T}{\partial x} = \frac{\partial c}{\partial T}\,[\frac{\partial T}{\partial t} + V_g \frac{\partial T}{\partial x}] \qquad (14)$$

(13), reduces to

$$[T\frac{\partial c}{\partial T} + c][\frac{\partial T}{\partial t} + V_g \frac{\partial T}{\partial x}] = 0$$

If,T$\frac{\partial c}{\partial T}$ = -c, or$\frac{\partial c}{c}$ = -$\frac{\partial T}{T}$, then $\log_e(cT)$ =$\log_e(L)$ = constant or L = constant. This contradicts the physical meaning of slowly envolving wave form in time and space, because L = L(x, t). Thus

$$\frac{\partial T}{\partial t} = V_g \frac{\partial T}{\partial x} = 0 \qquad (15)$$

Equations (12) and (15) appear to suggest that marine physicsts sailing with speed V_g will record no change in wave length, L and period, T. This is a remarkable result, mathematically.

Further Developments

It is recalled that c=ω/k , L = $\frac{2\pi}{\omega}$, consequently,$\frac{dc}{dL} = \frac{d(\omega/k)}{d(2\pi/k)} = \frac{1}{2\pi}\frac{d(\omega/k)}{d(1/k)}$

$$= -\frac{1}{2\pi}\left(k\frac{d\omega}{dk} - \omega\right) = \frac{k}{2\pi}\left(c - \frac{d\omega}{dk}\right)$$

From (7),

$\frac{dc}{dL} = \frac{k}{2\pi}$[c-cn] = $\frac{kc}{2\pi}$[1-n] = (1-n)$\frac{\omega}{2\pi}$(16)

From (5),

V_g = c - L$\frac{dc}{dL}$

And (16) is here introduced (5) to give

V_g =c – L(1 - n)$\frac{\omega}{2\pi}$ = c -$\frac{L\omega}{2\pi}$ + n$\frac{L\omega}{2\pi}$

But c = ω/k = $\frac{\omega L}{2\pi}$. Thus, V_g = nc = c_g

This is also a remarkable result. In brief, it states that varying wave parameters in a slowly and continuously changing environment provides new definition to the group velocity concept.

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Conclusion

This investigation suggests strongly that in a slowly varying medium, both wave length and period belong to the quasi linear hyperbolic wave group. Thus, in such a medium such as ideal fluid, they may be analysed using the partial differential equation governing the evolution of such group. The physical implication is that an observer moving in the medium with group velocity will observe no change in wave length and period as already stated. Further, a more general concept of group velocity is provided by the model ofspacial and time varying wave parameters and this development does not depend on the number of wave components in the group..

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