Chemical Reaction Effect on Natural Convective Flow between Fixed Vertical Plates with suction and injection

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KEYWORDS: Chemical Reaction, suction /injection, Natural Convection, Vertical plates.

ABSTRACT

This study investigates the effect of chemical reaction on natural convective flow between two fixed vertical porous plates. The continuity, momentum, energy and concentration equations were used as the governing equations. The dimensionless forms of the equations were solved analytically using Perturbation method in order to obtain the velocity, temperature and concentration. Expressions forthe Skin-friction, Nusselt number and Sherwood number were derived. Furthermore, Effects of assisting free convective current Grmagnetic field$M$, Prandtl number Pr, chemical reaction$R$ suction/injectionδ, sustention parameter N and Schmidt number on heat and mass transfer in the flow were discussed and presented graphically.

1.0 INTRODUCTION

Natural convection has attracted a great deal of attention from researchers because of its importance both in nature and engineering applications. In nature, convection cells formed from air rising above sunlight-warmed land or water are a major feature of all weather systems. Convection is also seen in the rising plume of hot air from fire, oceanic currents. In engineering, convection is commonly visualized in the formation of microstructures during the cooling of molten metal, and fluid flows around shrouded heat-dissipation fins, and solar ponds. A very common industrial application of natural convection is free air cooling without the aid of fans: this can happen on small scales (computer chips) to large scale process equipment. In this system, heat is transferred from a vertical plate to a fluid moving parallel to it by natural convection. The study of convection process in porous channels is a well developed field of investigation because of its importance to a variety of situation Jha *et al.* [1]. Convection flow formations were studied by many authors. For instance, Langellotto *et a.*[2] reported the numerical investigation of transient natural convection in air in a convergent vertical channel symmetrically heated at uniform heat flux. Floria and Harnoy [3] augmenting natural convection in a vertical flow path through transverse vibrations of an adiabatic wall. Jha and Ajibade [4] studied free convective flow between vertical porous plates with periodic heat input. Abdulaziz and Hashim [5]considered free convective flow between porous vertical plates with asymmetric wall temperature and concentrations and used homotopy analysis method to solve boundary value problems. Related works in natural convection research is the finding of new configuration to improve heat transfer or the analysis of standard configuration to determine optimal geometrical permutations in order to achieve a better heat transfer rate. Rees [6] studied the stability of Dersy-bernard convection. Jha*et al.* [7] studied unsteady natural convection flow between infinite vertical parallel plates with ramped temperature.Rajput and Sahu [8] studied the effect of chemical reactions on free convective (MHD) flow past an exponentially accelerated infinite vertical plate through a porous medium with variable temperature and mass diffusion. Kesavaiah *et al.* [9] analyzed the effect of chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Ajibade and Jha [10] reported transient natural convection flow between vertical parallel plates with temperature dependent heat source/sinks. Muthucumaraswamy [11] investigated the effects of a chemical reaction on a moving isothermal surface with suction. Pal and Talukdar [12] examined the perturbation analysis of unsteady magneto hydro dynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Patil and Kulkarni, [13] investigated the effect of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation. Rout *et al.* [14] studied the effect of radiation and chemical reaction on natural convective MHD flow through a porous medium with double diffusion. Song*et al.* [15] observed the steady state and oscillation in homogeneous-heterogeneous reaction system. Williams*et al.*[16] analyzed the Ignition and extinction of surface and homogeneous Oxidation of NH_3 andCH_4. Nguyen*et al.* [17] investigated unsteady non-Darcy reaction driven flow from an anisotropic cylinder in porous media. Ingham*et al.* [18] studied free convection boundary layer at a three dimensional stagnation point driven by exothermic surface reaction. Chaudhary and Merkin [19] observed free convection stagnation point boundary layers driven by catalytic surface reaction

However, recent years revealed an increased interest about fluid and thermal system where chemical reactions take place. These chemical reactions may undergo through the volume of (porous) region which is analyzed along interfaces/boundaries of the region. Real world application includes chemical engineering systems, contaminant transport in ground water systems, or geothermal processes.

Models for convective flows on reactive surfaces in porous media have been proposed by many investigators [20, 21].

This present work considers chemical reaction effect on natural convective flow between fixed vertical plates with suction and injection.

Nomenclature

B_0 External magnetic field
C Dimensionless concentration
C' Dimensional concentration of the fluid
C'_w Constant concentration at the plate
Gr Thermal Grashof number
g Acceleration due to gravity
M Magnetic parameter
N Suspension parameter
$\Pr$ Prandtl number
Q Dimensional heat generation term
R Chemical reaction parameter
S Dimensionless heat sink parameter
Sc Schmidt number
t Dimensionless time
t_0 Characteristic time
t' Dimensional time
U Dimensionless velocity of the fluid
U' Dimensional velocity of the fluid
y Dimensionless co-ordinate perpendicular to the plate
y' Dimensional co-ordinate to the plate
θ_w Constant temperature at the plate
θ' Dimensional temperature of the fluid
δ Suction
C'_0 Initial concentration of the fluid
R_c Chemical reaction
T' Dimensional fluid Temperature
C_p Specific heat at constant pressure
T'_0 Dimensional initial temperature of the fluid
h Gap between the plate
β Volumetric coefficient of thermal expansion
σ Stefan Boltzmann constant (electrical Conductivity)
Nu_0 Nusselt number
Sh_0 Sherwood number

Greek symbols

ν Kinematic viscosity
ρ Density of the fluid
θ Fluid Temperature
ν Kinematic viscosity
β Volumetric coefficient of thermal expansion
τ_0 Skin friction

2.0 Mathematical analysis

This problem considers chemical reaction effect on natural convective flow between fixed vertical plates with suction and injection. Figure 1, Shows the physical configuration of the problem. One of the plate is placed at $y' = 0$ and the other at distance $y' = h$.

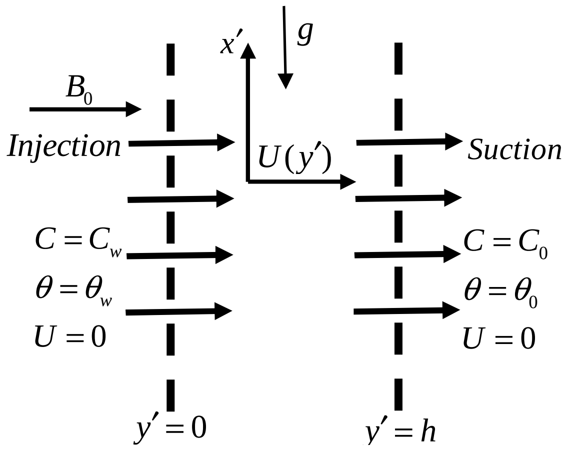

Figure 1: Schematic diagram of the problem

The equations governing the flow under the usual boundary layer and Boussineques approximations are:

$$\frac{\partial V'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial U'}{\partial t'} + V'\frac{\partial U'}{\partial y'} = \nu \frac{\partial^2 U'}{\partial y'^2} - \frac{\sigma B_0^2 U'}{\rho} + g\beta(T' - T_0') + g\beta_1(C' - C_0') \tag{2}$$

$$\frac{\partial \theta'}{\partial t'} + v'\frac{\partial \theta'}{\partial y'} = \frac{K}{\rho Cp}\frac{\partial^2 \theta'}{\partial y'^2} - \frac{Q}{\rho Cp}\left(T' - T_0'\right) \tag{3}$$

$$\frac{\partial C'}{\partial t'} + V'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - R_c\left(C' - C_0'\right) \tag{4}$$

The boundary conditions are

$$\left.\begin{array}{l} t \le 0, U' = 0, T' = 0 \;\; for \;\; 0 \le y \le H \\ t > 0, U' = 0, \theta' = T_w', C' = C_w' \; at \; y' = 0 \\ \quad U' = 0, \theta' = T_0', C' = C_0' \; at \; y' = H \end{array}\right\} \tag{5}$$

In order to write the governing equations and boundary conditions in dimensionless form the following non-dimensional quantities are introduced into equation (1) – (4)

$$\left.\begin{array}{l} U = \dfrac{U'}{U_0}, y = \dfrac{y'U_0}{H\nu}, t = \dfrac{t'}{t_0}, \theta = \dfrac{T' - T_0'}{T_w' - T_0'}, C = \dfrac{C' - C_0'}{C_w' - C_0'}, \\ M = \dfrac{\sigma B_0^2 \nu}{\rho U_0^2}, Gr = \dfrac{g\beta\nu\left(T_w' - T_0'\right)}{U_0^2}, N = \dfrac{g\beta_1\nu\left(C_w' - C_0'\right)}{U_0^2}, \\ P_r = \dfrac{K}{\mu Cp}, Sc = \dfrac{\nu}{D}, S = \dfrac{Q\nu}{\rho Cp U_0^2}, \delta = \dfrac{V_0}{U_0}, R = \dfrac{R_c \nu}{U_0^2} \end{array}\right\} \tag{6}$$

Using equations (1) and (6) in equations (2) to (4) and boundary conditions (5), the dimensionless governing equations become;

$$\frac{\partial U}{\partial t}+\delta\frac{\partial U}{\partial y}=\frac{\partial^2 U}{\partial y^2}-MU+G_r\theta+NC \quad (7)$$

$$\frac{\partial \theta}{\partial t}+\delta\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+S\theta \quad (8)$$

$$\frac{\partial C}{\partial t}+\delta\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-RC \quad (9)$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &t\le 0, U=\theta=C \quad for\ 0\le y\le 1\\ &t>0, U=0, \theta=1+\varepsilon e^{iwt}+\varepsilon^2 e^{2iwt}, C=1+\varepsilon e^{iwt}+\varepsilon^2 e^{2iwt}\ at\ y=0\\ &\quad U=0, \theta=0, C=0\ at\ y=1 \end{aligned}\right\} \quad (10)$$

To solve equations (7) to (9) subject to the boundary conditions (10), we take solutions of the form

$$U=U_0+U_1\varepsilon e^{iwt}+U_2\varepsilon^2 e^{2iwt}+0\left(\varepsilon^3\right)+\cdots=\sum_{j=0}^{\infty}\varepsilon^j U_j e^{j(iwt)} \quad (11)$$

$$\theta=\theta_0+\theta_1\varepsilon e^{iwt}+\theta_2\varepsilon^2 e^{2iwt}+0\left(\varepsilon^3\right)+\cdots=\sum_{j=0}^{\infty}\varepsilon^j \theta_j e^{j(iwt)} \quad (12)$$

$$C=C_0+C_1\varepsilon e^{iwt}+C_2\varepsilon^2 e^{2iwt}+0\left(\varepsilon^3\right)+\cdots=\sum_{j=0}^{\infty}\varepsilon^j C_j e^{j(iwt)} \quad (13)$$

Where $U_0(y), U_1(y), U_2(y), \theta_0(y), \theta_1(y), \theta_2(y), C_0(y), C_1(y)$ and $C_2(y)$ are to be determined.

Substituting equation (11) into (7), (12) into (8) and (13) into (9) and equating the harmonic and non-harmonic terms. Also neglecting the higher order terms of $0\left(\varepsilon^3\right)$ we obtained the velocity, temperature and concentration equations as

$$\begin{aligned} U(y)=&(B_{13}e^{m_{13}y}+B_{14}e^{-m_{14}y}+B_{15}e^{m_7 y}+B_{16}e^{-m_8 y}+B_{17}e^{m_1 y}+B_{18}e^{-m_2 y})\\ &+(B_{21}e^{m_{15}y}+B_{22}e^{-m_{16}y}+B_{23}e^{m_9 y}+B_{24}e^{-m_{10}y}+B_{25}e^{m_3 y}+B_{26}e^{-m_4 y})\varepsilon e^{iwt}\\ &+(B_{29}e^{m_{17}y}+B_{30}e^{-m_{18}y}+B_{31}e^{m_{11}y}+B_{32}e^{-m_{12}y}+B_{33}e^{m_5 y}+B_{34}e^{-m_6 y})\varepsilon^2 e^{2iwt} \end{aligned} \quad (14)$$

$$\theta(y)=B_7e^{m_7 y}+B_8e^{-m_8 y}+(B_9e^{m_9 y}+B_{10}e^{-m_{10}y})\varepsilon e^{iwt}+(B_{11}e^{m_{11}y}+B_{12}e^{-m_{12}y})\varepsilon^2 e^{2iwt} \quad (15)$$

$$C(y)=B_1e^{m_1 y}+B_2e^{-m_2 y}+(B_3e^{m_3 y}+Be^{-m_4 y})\varepsilon e^{iwt}+(B_5e^{m_5 y}+B_6e^{-m_6 y})\varepsilon^2 e^{2iwt} \quad (16)$$

Using equation (14) the skin friction on the plates (y=0) is

$$\tau_0 = \frac{dU}{dy}\Big|_{y=0} = (m_{13}B_{13} - m_{14}B_{14} + m_7B_{15} - m_8B_{16} + m_1B_{17} - m_2B_{18})$$

$$+(m_{15}B_{21} - m_{16}B_{22} + m_9B_{23} - m_{10}B_{24} + m_3B_{25} - m_4B_{26})\varepsilon e^{iwt}$$

$$+(m_{17}B_{29} - m_{18}B_{30} + m_{11}B_{31} - m_{12}B_{32} + m_5B_{33} - m_6B_{34})\varepsilon^2 e^{2iwt} \quad (17)$$

Using equation (15) the rate of heat transfer on the plates (y=0) is:

$$Nu_0 = \frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{e^{-m_8} - e^{m_7}}\left(m_7 e^{-m_8} + m_8 e^{m_7}\right) + \frac{\varepsilon}{e^{-m_{10}} - e^{m_9}}\left(m_9 e^{-m_{10}} + m_{10} e^{m9}\right) e^{2iwt}$$
$$+ \frac{\varepsilon^2}{e^{-m_{12}} - e^{m_{11}}}\left(m_{11} e^{-m_{12}} + m_{12} e^{m_{11}}\right) e^{2iwt} \quad (18)$$

Using equation (16) the Sherwood number on the plates (y=0) is obtained as:

$$Sh = \frac{dC}{dy}\Big|_{y=0} = \frac{1}{e^{-m_2} - e^{m_1}}\left(m_1 e^{-m_2} + m_2 e^{m_3}\right) + \frac{\varepsilon}{e^{-m_4} - e^{m_3}}\left(m_3 e^{-m_4} + m_4 e^{m_3}\right) e^{2iwt}$$
$$+ \frac{\varepsilon^2}{e^{-m_5} - e^{m_6}}\left(m_5 e^{-m_6} + m_6 e^{m_5}\right) e^{2iwt} \quad (19)$$

3.0 Result and Discussion

In order to study the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters under study. The value of Prandtl number (Pr) are chosen to be (Pr = 0.71) for air, (Pr = 7.0) for water and (Pr = 3) for saturated liquid Freon. The values for Schmidt number are chosen (Sc = 0.22) for hydrogen, (Sc = 0.62) for water vapour, (Sc = 0.8) for ammonia and (Sc= 2.01) for Ethyl Benzene [8]. The values of thermal Grashof number (Gr) indicate the state of the plates. Since Gr depends on the plates, it can take positive zero and negative values depending on the temperature of the plates.

The behaviors of the fluid velocity for different values of parameters are presented in Figures 2, 3, 4 and 5. Figure 2 shows the effect of thermal Grashof number (Gr), on fluid velocity profile. It is observed that the velocity increases due to the increase in the values of thermal Grashof number. Increase in Gr, has the tendency to increase the thermal buoyancy effect and this give rise to an increase in the induced flow. Figure 2a reflects that velocity profile is higher in water than in the air this is physically possible because there is greater density in water than in the air due to closeness of molecules in water which makes velocity to be higher. The positive values of Gr indicates the cooling of the plate. When $(Gr > 0)$ the velocity increases, the reverse effect is observed in figure 2b which represent heating of the plate $(Gr < 0)$ increasing Gr implies heat is removed from the channel walls which result in thickening of the thermal boundary layer which leads to an elevation in the transient velocity similarly. Figure 3 shows effect of sustention parameter (N) in the velocity profile. It is evident that the velocity profile is higher in case of water (Pr = 7.0) than that of air (Pr = 0.71). Figure 4 reveal the effect of magnetic field on velocity profile it is seen from this figure that the velocity decreases with the increase in the magnetic field. It is interesting to note that the effect of magnetic field is to decrease the value of velocity profile throughout the boundary layer presence of the magnetic field in an electrically conducting fluid introduce Lorentz force which act against the fluid flow. Figure 5 demonstrate the effect of suction/injection parameter on velocity profile, it is observed that velocity increases with the increase of injection for cooling of the plate and decreases for the heating of the plate. It is also clear that suction stabilizes the boundary layer.

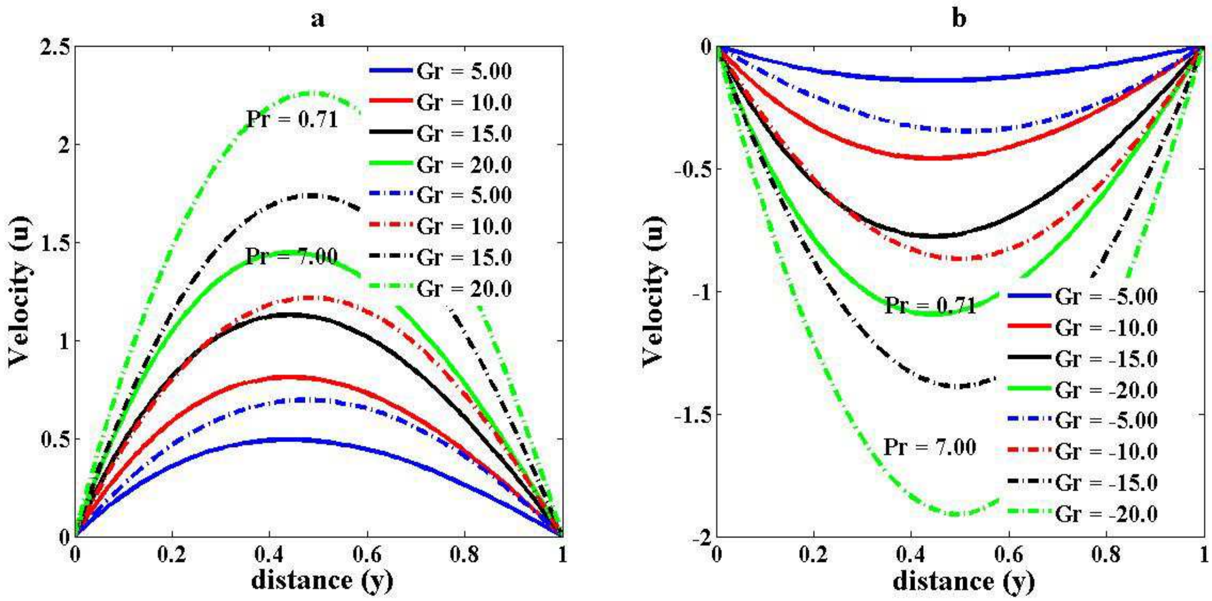

Figure 2: Effect of thermal Grashof number (Gr) on velocity (u)

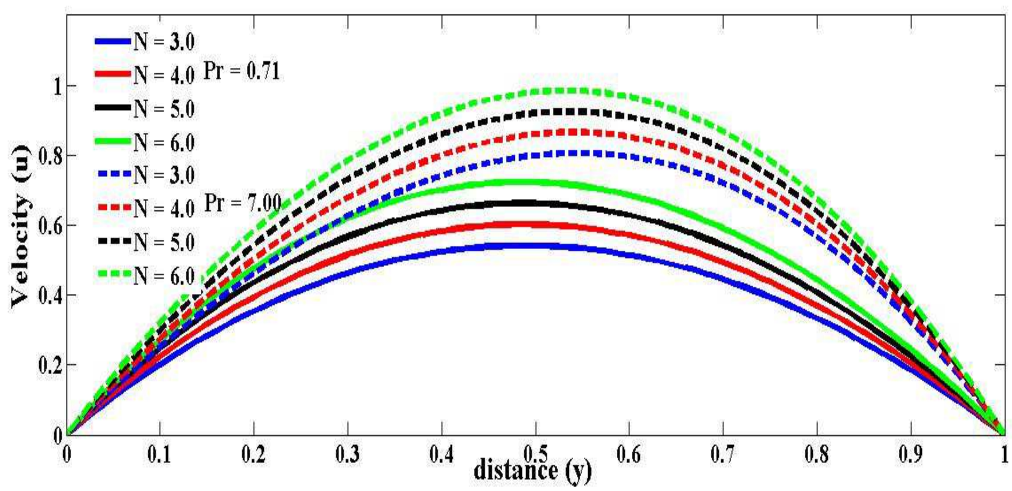

Figure 3: Effect of sustention parameter (N) on Velocity (u)

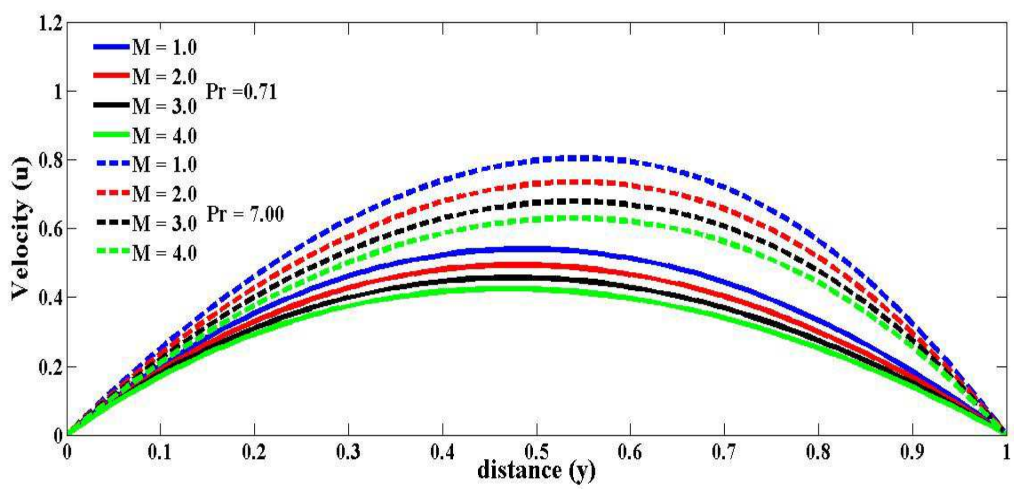

Figure 4: Effect of magnetic field (M) on velocity (u)

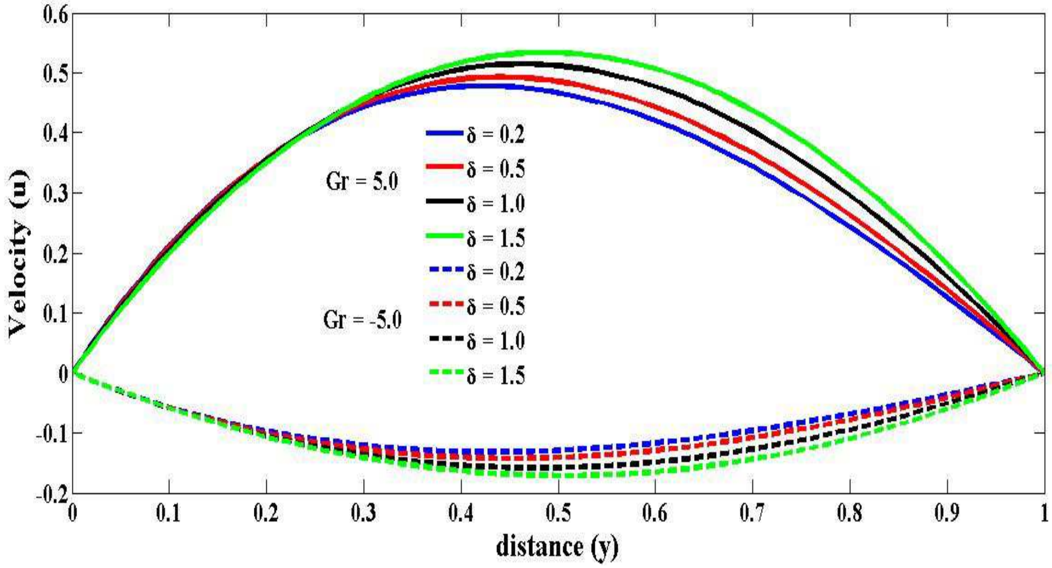

Figure 5: Effect of suction/injection on velocity (u).

The effects of Prandtl number, heat sink and suction/injection parameters on the fluid temperature are illustrated in Figures 6, 7 and 8. It is observed from Figure 6 that the temperature increases with an increase of heat sink parameter (S). It is seen in Figure 7 the temperature decreases with an increase in Prandtl number (Pr) due to suction $(\delta < 0)$. Also an increase in Prandtl number (Pr) increases the temperature due to injection $(\delta > 0)$, higher Pr fluid transfer heat less effectively than the lower Pr fluid and consequently a decrease in temperature take place which in turn reduces the velocity in the region Figure 8, reveals that the temperature increases with the increase of the suction/injection parameter.

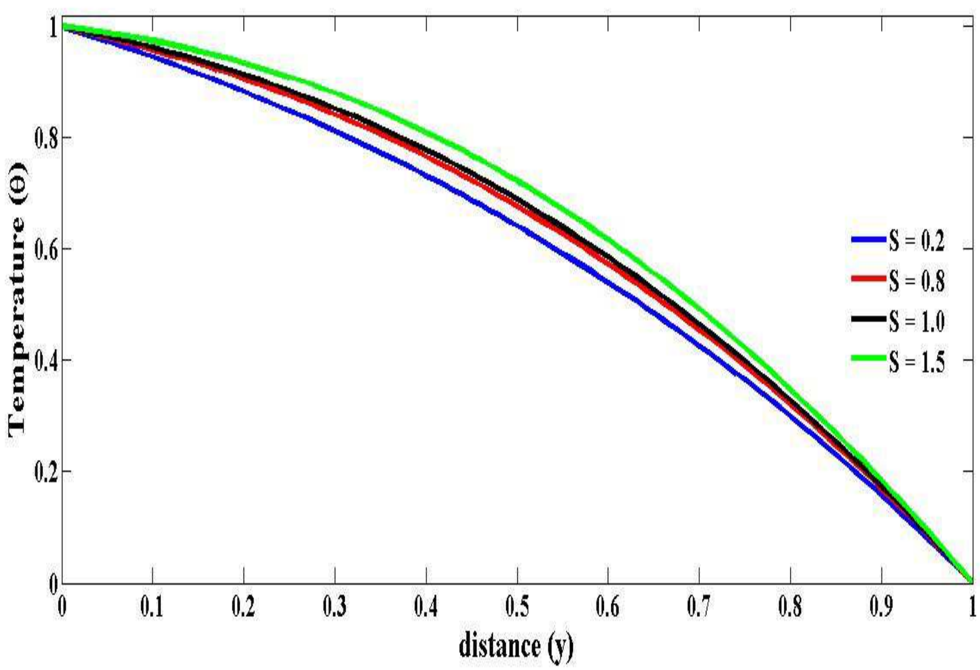

Figure 6: Effect of heat sink parameter (S) on temperature (θ).

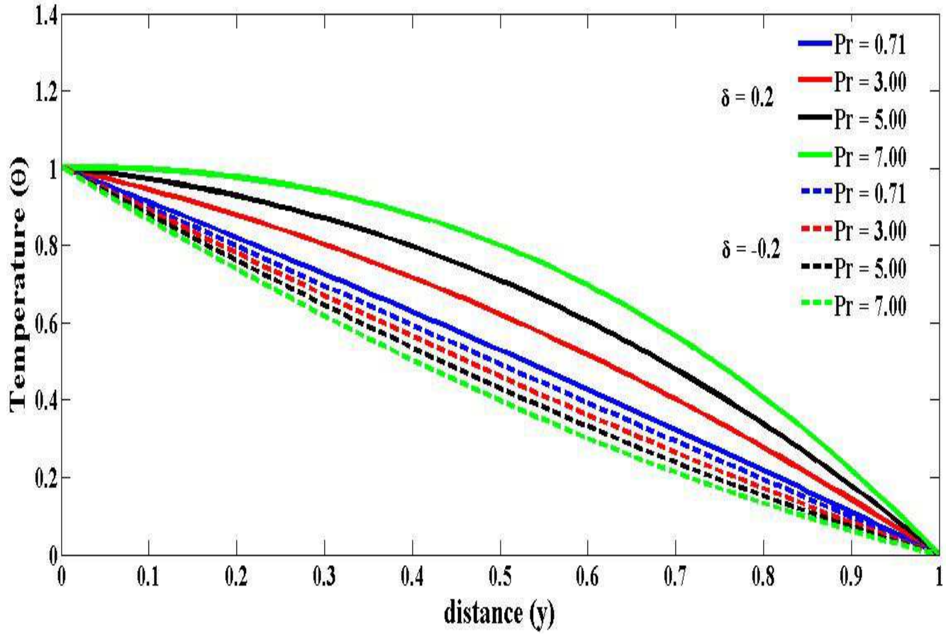

Figure 7: effect of Prandtl number (Pr) on temperature (θ).

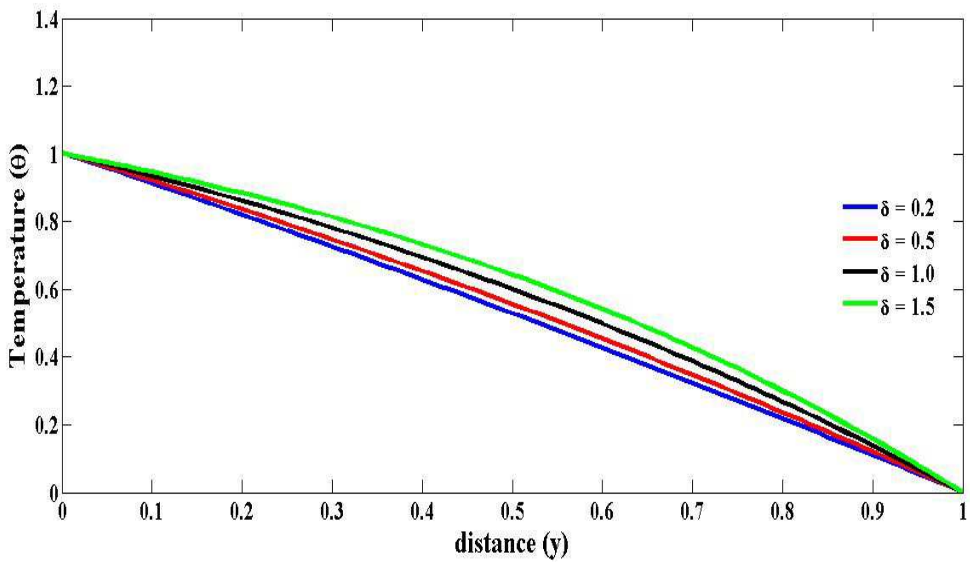

Figure 8: Effect of suction/injection on temperature (θ)

From Figure 9 it is observed that increase in Schmidt number decreases the concentration of the fluid this causes the concentration buoyancy effects to decreases, thus to say the reduction in concentration profile are accompanied by reduction in the concentration boundary layer. Figure 10 shows the effect of chemical reaction parameter (R) on the concentration of the fluid. It is observed that the concentration of fluid increases with the decrease of chemical reaction parameter (R).

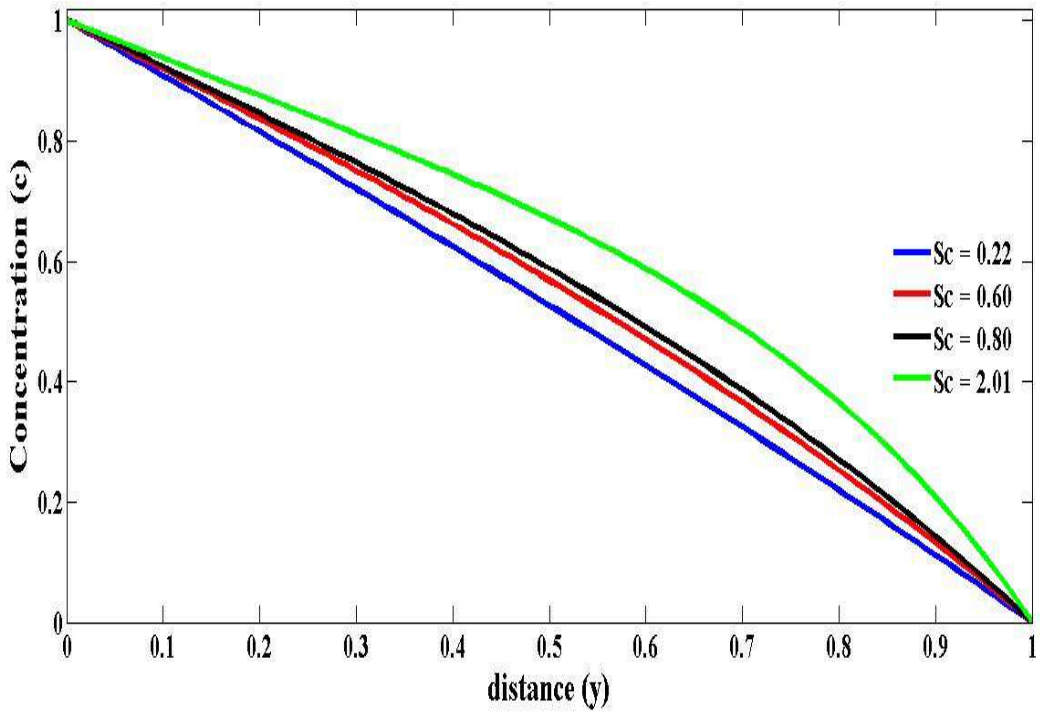

Figure 9: Effect of Schmidt number (Sc) on concentration (c).

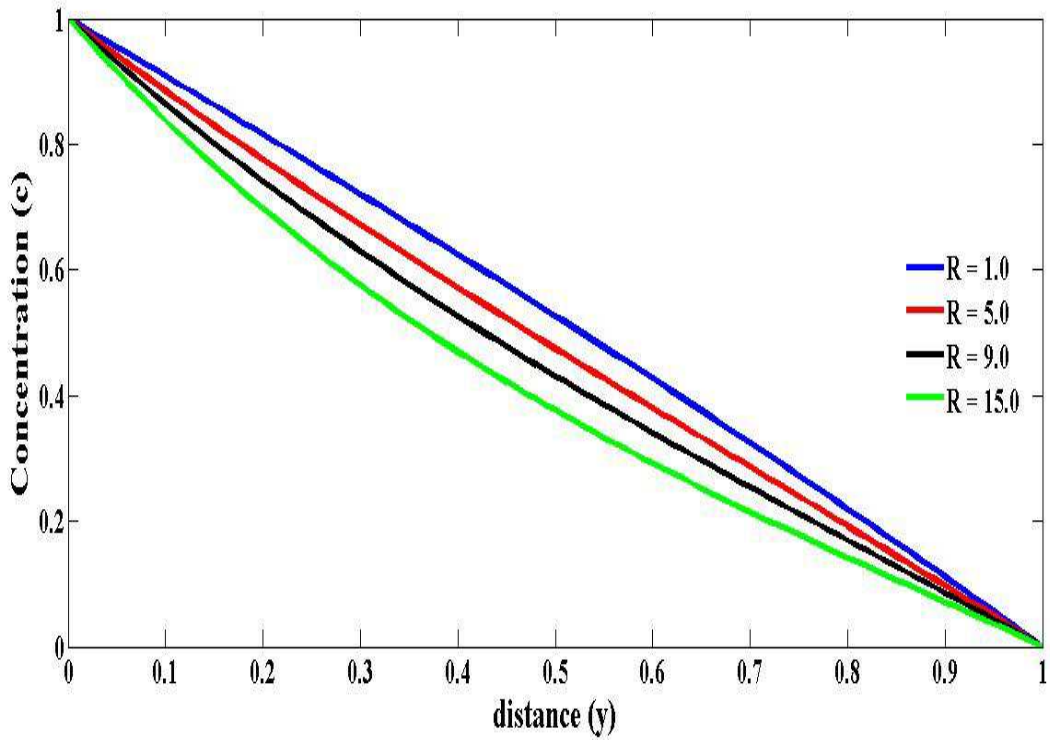

Figure 10: Effect of radiation (R) on concentration (c)

Table 1: For Skin friction, Nusselt number and Sherwood number.

Pr	Gr	M	R	Sc	δ	N	τ_0	Nu_0	Sh_0
0.71	5	1	1	0.22	0.2	3	2.8972	0.8838	1.0515
7.00	5	1	1	0.22	0.2	3	3.1910	-0.0233	-
0.71	10	1	1	0.22	0.2	3	4.2604	-	-
0.71	5	2	1	0.22	0.2	3	2.4045	-	-
0.71	5	1	2	0.22	0.2	3	4.3003	-	1.1218
0.71	5	1	1	0.60	0.2	3	5.8859	-	1.1347
0.71	5	1	1	0.22	0.5	3	2.9539	0.7862	1.0193
0.71	5	1	1	0.22	0.2	4	3.4084	-	-

The comparison of variation of Skin friction $\left(\tau_0 = \left(U'\right)\right)$, Nusselt number $\left(Nu_0 = \left(\theta'\right)\right)$ and Sherwood number $\left(Sh_0 = \left(C'\right)\right)$ at $y=0$ is shown in table 1. Increase in Grashof number Gr and sustention parameter N, increases the skin friction. Increase in the suction/injection parameter increases the skin friction and decreases rate of heat transfer and Sherwood number. For increasing magnetic parameter M, the skin friction decreases. Increase in the value of Prandtl number (Pr), increases the skin friction but has reverse effect on rate of heat transfer. Also the skin friction and Sherwood number increases as Schmidt number increases.

3.0 Conclusion.

In this paper we have studied the chemical reaction effect on natural convective flow between fixed vertical porous plates. In the absence of mass transfer equation [4] our present result agrees with available work [1, 7]. From the study conducted, weconclude that:

i. The velocity increases with the increase of thermal Grashof number, increasing Gr implies heat is removed from the channel walls which result in thickening of the thermal boundary layer which leads to an elevation in the transient velocity similarly,sustention parameter and suction/injection parameter for cooling of the plate, also decreases with an increase in magnetic field parameter and suction/injection parameter for heating of the plate**,.**

ii. 666The temperature profile increases with an increase in suction/injection, heat sink parameters and Prandtl number due to injection, similarly decreases with the increase in Prandtl number due to suction, higher Pr fluid transfer heat less effectively than the lower Pr fluid and consequently a decrease in temperature take place which in turn reduces the velocity in the region

iii. 777The concentration increases with an increase in Schmidt number and decreases with an increase in radiation parameter.

iv. 888An increase in thermal Grashof number $\left(Gr\right)$ and sustention parameter $\left(N\right)$ increases skin friction and an increase in magnetic field parameter (M) decreases skin friction, magnetic field decrease the value of velocity profile throughout the boundary layer presence of the magnetic field in an electrically conducting fluid introduce Lorentz force which act against the fluid flow. Also, an increase in suction/injection parameter $\left(\delta\right)$ increases skin friction but decreases the rate of heat transfer and Sherwood number. Similarly, an increase in radiation parameter (R) and Schmidt number (Sc) increases skin friction and Sherwood number. Moreover, an increase in Prandtl number (Pr) increases skin friction but reverse effect is observed on rate of heat transfer.

Acknowledgement

The author would like to thank Usmanu Danfodiyo University, Sokoto and Federal University Birnin Kebbi, for their support.

Appendix

$$P_1=\sqrt{(\delta Sc)^2+4RSc}\;\; m_1=\frac{\delta Sc+P_1}{2}\quad m_2=\frac{-\delta Sc+P_1}{2}\;\; B_1=1-B_2\;\; B_2=\frac{-1}{e^{-m_2}-e^{m_1}}e^{m_1}$$

$$C_0=\frac{1}{e^{-m_2}-e^{m_1}}\left(e^{-m_2}e^{m_1 y}-e^{m_1}e^{-m_2 y}\right)\;\; P_2=\sqrt{(\delta Sc)^2+4Sc(R+iw)},\quad m_3=\frac{\delta Sc+P_2}{2}\;\; m_4=\frac{-\delta Sc+P_2}{2},$$

$$B_3=1-B_4\,,\; B_4=\frac{-1}{e^{-m_4}-e^{m_3}}e^{m_3}\,,\; C_1=\frac{1}{e^{-m_4}-e^{m_3}}\left(e^{-m4}e^{m_3 y}-e^{m_3}e^{-m_4 y}\right)$$

$$P_3=\sqrt{(\delta Sc)^2+4Sc(R+2iw)}\,,\quad m_5=\frac{\delta Sc+P_3}{2},\quad m_6=\frac{-\delta Sc+P_3}{2},\quad B_5=1-B_6\,,\quad B_6=\frac{-1}{e^{-m_6}-e^{m_5}}e^{m_5}$$

$$C_2=\frac{1}{e^{-m_6}-e^{m_5}}\left(e^{-m_6}e^{m_5 y}-e^{m_5}e^{-m_6 y}\right),\; P_4=\sqrt{(\delta\,\mathrm{Pr})^2+4S\,\mathrm{Pr}}\,,\; m_7=\frac{\delta\,\mathrm{Pr}+P_4}{2}\;,\quad m_8=\frac{-\delta\,\mathrm{Pr}+P_4}{2}\;\; B_7=1-B_8\,,$$

$$B_7=1-B_8\,,\;\theta_0=\frac{1}{e^{-m_8}-e^{m_7}}\left(e^{-m_8}e^{m_7 y}-e^{m_7}e^{-m_8 y}\right)\quad,\quad P_5=\sqrt{(\delta\,\mathrm{Pr})^2+4\,\mathrm{Pr}}\,(S-iw)\,, m_9=\frac{\delta\,\mathrm{Pr}+P_5}{2},$$

$$m_{10}=\frac{-\delta\,\mathrm{Pr}+P_5}{2}\,,\quad B_9=1-B_{10}\,,\quad B_{10}=\frac{-1}{e^{-m_{10}}-e^{m_9}}e^{m_9}\;\;\theta_1=\frac{1}{e^{-m_{10}}-e^{m_9}}\left(e^{-m_{10}}e^{m_9 y}-e^{m_9}e^{-m_{10} y}\right),$$

$$P_6=\sqrt{(\delta\,\mathrm{Pr})^2+4\,\mathrm{Pr}(S-2iw)}\quad,\quad m_{12}=\frac{-\delta\,\mathrm{Pr}+P_6}{2}\,, B_{11}=1-B_{12}\,,\quad B_{12}=\frac{-1}{e^{-m_{12}}-e^{m_{11}}}e^{m_{11}}\,,$$

$$\theta_2=\frac{1}{e^{-m_{12}}-e^{m_{11}}}\left(e^{-m_{12}}e^{m_{11} y}-e^{m_{11}}e^{-m_{12} y}\right),\quad P_7=\sqrt{\delta^2+4M}\;\; m_{13}=\frac{\delta+P_7}{2},\quad m_{14}=\frac{-\delta+P_7}{2},\quad B_{13}=-B_{14}-B_{19}\,,$$

$$B_{14}=\frac{1}{e^{-m_{14}}-e^{m_{13}}}\left(B_{20}e^{m_{13}}-B_{19}\right)$$

$$B_{15}=\frac{-GrB_7}{m_7^2-\delta m_7-M}\,,\; B_{16}=\frac{-GrB_8}{m_8^2+\delta m_8-M}\,,\; B_{17}=\frac{NB_1}{m_1^2-\delta m_1-M}$$

$$B_{18}=\frac{-NB_2}{m_2^2+\delta m_2-M}\,,\; B_{19}=B_{15}+B_{16}+B_{17}+B_{18}\,,\;\; B_{20}=B_{15}e^{m_{17}}+B_{16}e^{-m_8}+B_{17}e^{m_1}+B_{18}e^{-m_2}$$

$$U_0=B_{13}e^{m_{13}y}+B_{14}e^{-m_{14}y}+B_{15}e^{m_7 y}+B_{16}e^{-m_8 y}+B_{17}e^{m_1 y}+B_{18}e^{-m_2 y}\,,\; P_8=\sqrt{\delta^2+4(M-iw)}$$

$$m_{15}=\frac{\delta+P_8}{2},\; m_{16}=\frac{-\delta+P_8}{2},\; B_{21}=-B_{22}-B_{27}\,,\; B_{22}=\frac{1}{e^{-m_{16}}-e^{m_{15}}}\left(B_{27}e^{m_{15}}-B_{28}\right), B_{23}=\frac{-GrB_9}{m_9^2-\delta m_9-(M+iw)}\,,$$

$$B_{24}=\frac{-GrB_{10}}{m_{10}^2+\delta m_{10}-(M+iw)}\,,\quad B_{25}=\frac{-NB_3}{m_3^2-\delta m_3-(M+iw)}\;\; B_{26}=\frac{-NB_4}{m_4^2+\delta m_4-(M+iw)}\,,$$

$$B_{27}=B_{23}+B_{24}+B_{25}+B_{26}\,,\; B_{28}=B_{23}e^{m_9}+B_{24}e^{-m_{10}}+B_{25}e^{m_3}+B_{26}e^{-m_4}$$

$$U_1=B_{21}e^{m_{15}y}+B_{22}e^{-m_{16}y}+B_{23}e^{m_9 y}+B_{24}e^{-m_{10}y}+B_{25}e^{m_3 y}+B_{26}e^{-m_4 y}\,,\; P_9=\sqrt{\delta^2+4(M-2iw)}$$

$$m_{17}=\frac{\delta+P_9}{2},\; m_{18}=\frac{-\delta+P_9}{2},\; B_{29}=-B_{30}-B_{35}\,,\; B_{31}=\frac{-GrB_{11}}{m_{11}^2-\delta m_{11}-(M+2iw)}\;\; B_{32}=\frac{-GrB_{12}}{m_{12}^2+\delta m_{12}-(M+2iw)}\,,$$

$$B_{33}=\frac{-NB_5}{m_5^2-\delta m_5-(M+2iw)}\;\;,\; B_{34}=\frac{-NB_6}{m_6^2+\delta m_6-(M+2iw)}$$

$$B_{35}=B_{31}+B_{32}+B_{33}+B_{34}\,,\;\; B_{36}=B_{31}e^{m_{11}}+B_{32}e^{-m_{12}}+B_{33}e^{m_5}+B_{34}e^{-m_6}$$

$$U_2 = B_{29}e^{m_{17}y} + B_{30}e^{-m_{18}y} + B_{31}e^{m_{11}y} + B_{32}e^{-m_{12}y} + B_{33}e^{m_5 y} + B_{34}e^{-m_6 y}$$

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