

# Unsteady free convective flow in a porous medium with heat generation in an infinite vertical plate

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## Abstract

This paper investigates the unsteady free convective viscous incompressible and electrically conductive fluid, between infinite vertical porous plates due to heat generation and Hartmann number. Solutions of time dependent energy, momentum and concentration equations under the relevant initial and boundary conditions were derived using perturbation technique. Selected set of line graph representing the effect of controlling parameters embedded in the problem are discussed during the course of numerical computation. It is observed that, an increase in  $Gr$  and  $Gc$  results in the thickening of the thermal boundary layer, which leads to an increase in the unsteady velocity layer. However, a raise in the magnitude of the Hartmann number  $M$ , leads to a decrease of velocity.

<b>Nomenclature</b>	$U'$ Dimensional velocity of the fluid
$U$ Dimensionless velocity of the fluid	$g$ Acceleration due to gravity
Pr Prandtl number	$t'$ Dimensional time
$C'$ Dimensional concentration of the fluid	$C$ Dimensionless concentration
$y'$ Dimensional co-ordinate perpendicular to the plate	$y$ Dimensionless co-ordinate perpendicular to the plate
$S$ Dimensionless heat sink parameter	$T_\infty$ Initial temperature
$Q$ Dimensional heat sink parameter	$Gr$ Thermal Grashof number
$K$ Permeability parameter	$Sc$ Schmidt number
$C_\infty$ Concentration of the fluid far away from the fluid	$C'_w$ Constant concentration at the plate
$\gamma$ Suction	$Gc$ Mass Grashof number
$T_\infty$ Temperature of the fluid far away from the plate	$T_w$ Temperature of the fluid near the plate
$B_0$ External magnetic field	$M$ Magnetic parameter
<b>Greek alphabets</b>	
$\beta$ Volumetric coefficient of thermal expansion	$\nu$ Kinematic viscosity
$\rho$ Density of the fluid	$\sigma$ Stefan Boltzmann constant (electrical Conductivity)

## 1.0 INTRODUCTION

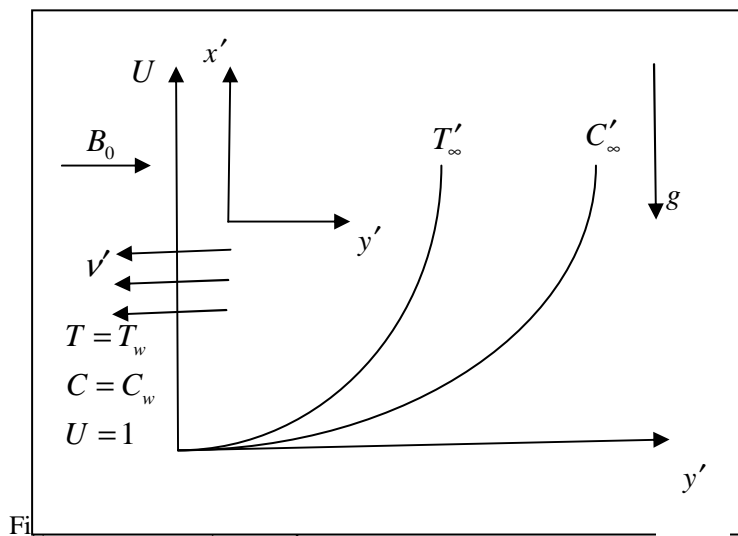
The study of convection processes in porous media has attracted significant attention in recent years because of the wide spread application of such flows in science, industry and many engineering processes. These include cooling of electronic equipment, heating of the Trombe wall system, gas-cooled nuclear reactors .Detailed review of the flow through and past porous media can be found in the book of Neild and Benjan[1].Unsteady flow investigations with porous boundaries include the works of Wang et al.[2].Jhaet al. [3] investigated the theoretical analysis of natural convection flow between infinite vertical parallel plates with ramped temperature on one of the plate using Laplace transform technique and reported that convection current due to isothermal heating of the boundary is higher than the ramped heating of the boundary. The velocity due to ramped temperature on the plate is always less than the velocity induced by isothermal temperature on plate. Elbashbeshy *et al.* [4] studied the unsteady boundary layer flow over a porous stretching surface embedded in a porous medium in presence of heat source. He observed that among other things, Nusselt number decreases with increase of porous parameter in the presence of heat source parameter and it also increases with suction. Sharma and Gupta [5] analyzed the unsteady flow and heat transfer along a hot

vertical porous plate in the presence of periodic suction and heat source. Kim [6] examined the unsteady MHD free convection flow past a moving semi-infinite vertical porous plate embedded in a porous medium with variable suction.

Theoretical/experimental investigations of convective boundary layer flow with heat and mass transfer induced due to a moving surface with a uniform or non-uniform velocity play an important role in several manufacturing processes in industry which include the boundary layer flow along material handling conveyers, extrusion of plastic sheets, cooling of an infinite metallic plate in cooling bath, glass blowing, continuous casting and levitation, design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of trees, damage of crops due to freezing common industrial sight especially in power plants e.t.c. Kamel [7] investigated the unsteady hydro magnetic convection flow due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate with temperature depended heat sources and sinks. Chamkha [8] studied the unsteady hydro magnetic two dimensional convective laminar boundary layer flow with heat and mass transfer of a viscous, incompressible, electrically conducting and temperature dependent heat absorbing fluid along a semi infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field. Aruna *et al.* [9] studied the effects of magnetic field on free convective flow of Jeffery fluid past an infinite vertical porous plate with constant heat flux. Adeniyana *et al.* [10] investigated the effects of thermal dissipation heat generation/absorption on MHD mixed convection boundary layer flow over a permeable vertical flat plate embedded in an anisotropic porous medium. Ajibade and Jha [11] analyzed the transient natural convection flow between vertical parallel plate with temperature dependent heat source/sinks. Chamkha [12] reported the effect of heat generation on g-jitter induced natural convective flow in a channel with isothermal or isoflux walls. Chamkha *et al.* [13] analyzed the radiation effects on a free convection flow past a semi infinite vertical plate with mass transfer. Chandet *et al.* [14] studied the hydromagnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and solet effect. Mahanti and Gau [15] analyzed the effect of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. Mishra *et al.* [16] studied the effects of radiation on free convection flow due to heat and mass transfer through a porous medium bounded by two vertical walls. Muthukumaraswamy and Kumar [17] reported that heat and mass transfer effect on moving vertical plate in the presence of thermal radiation. Patil and Kulkarni [18] investigated the effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation. Raptis and Perdakis [19] analyzed the unsteady flow through a porous medium in the presence of radiation. Seddeek [20] reported that heat and mass transfer on a stretching sheet with a magnetic field in a viscoelastic fluid flow in a porous medium with heat source. To the best of the author's knowledge no work seems to have addressed the time dependent of unsteady free convective flow in a porous medium with heat generation in an infinite vertical plate.

## 2.0 MATHEMATICAL FORMULATION

The effects of unsteady convective flow in a porous medium with heat generation in an infinite vertical plate are considered. The Fluid is viscous, incompressible electrically conductive and a uniform magnetic field  $B_0$  is applied to porous plate also the axial ( $x' - direction$ ) velocity depends only on transverse coordinate,  $y'$ . The system under consideration is sketched in figure 1. The  $x'$  - axis is taken along the direction of the flow and parallel to the infinite vertical porous plate and  $y'$  - direction perpendicular to the flow  $T_\infty$  is the initial fluid and wall temperature,  $T'$  dimensional temperature,  $T_w$  is the temperature of the fluid near the plate.



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Under, bonssineques approximations the required governing equations are

$$\frac{\partial V'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial U'}{\partial t'} + V' \frac{\partial U'}{\partial y'} = \nu \frac{\partial^2 U'}{\partial y'^2} - \frac{\sigma B_0^2 U'}{\rho} - \frac{\nu}{K_1} U' + g \beta_r (T' - T'_h) + g \beta_c (C' - C'_h) \quad (2)$$

$$\frac{\partial T'}{\partial t'} + \nu' \frac{\partial T'}{\partial y'} = K \frac{\partial^2 T'}{\partial y'^2} + \frac{Q_r}{\rho C_p} (T' - T'_h) \quad (3)$$

$$\frac{\partial C'}{\partial t'} + V' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

The boundary conditions are

$$\left. \begin{aligned} y' = 0 : U' = 1, T' = T'_h, C' = C'_h \\ y' = 1 : U' = 0, T' = T'_w, C' = C'_w \end{aligned} \right\} \quad (5)$$

The non dimensional quantities are introduced, to obtain the governing equations and the boundary conditions in dimensionless form.

$$\left. \begin{aligned} U = \frac{U'}{L}, y = \frac{y'L}{\nu}, t = \frac{t'}{t_0}, T = \frac{T' - T'_h}{T'_w - T'_h}, C = \frac{C' - C'_h}{C'_w - C'_h} \\ P_r = \frac{K}{\nu \rho C_p}, M = \frac{\sigma B_0^2 \nu}{\rho L^2}, Gr = \frac{g \beta_r \nu (T'_w - T'_h)}{L^2}, \\ Gc = \frac{g \beta_c \nu (C'_w - C'_h)}{L^2}, Sc = \frac{\nu}{D}, S = \frac{Q_r \nu}{\rho C_p L^2}, \gamma = \frac{V_0}{U_0} \end{aligned} \right\} \quad (6)$$

Substituting equation (6) in equations (1-5) to obtain the dimensionless form

$$\frac{\partial U}{\partial t} - \gamma \frac{\partial U}{\partial y} = \frac{\partial^2 U}{\partial y^2} - MU - KU + GrT + GcC \quad (7)$$

$$\frac{\partial T}{\partial t} - \gamma \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + ST \quad (8)$$

$$\frac{\partial C}{\partial t} - \gamma \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad (9)$$

with boundary conditions

$$\left. \begin{aligned} U = 0, T = 0, C = 0 \text{ at } y = 0 \\ U \rightarrow 1, T \rightarrow 1, C \rightarrow 1 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (10)$$

The solution to the dimensionless partial differential equations set in equations (7) to (10) can be obtained by representing concentration, temperature and velocity as follows:

$$C(y) = C_0 + C_1 \varepsilon e^{i\alpha x} + C_2 \varepsilon^2 e^{2i\alpha x} + C_3 \varepsilon^3 e^{3i\alpha x} + \dots = \sum_{j=0}^{\infty} \varepsilon^j C_j e^{j(i\alpha x)} \quad (11)$$

$$T(y) = T_0 + T_1 \varepsilon e^{i\omega t} + T_2 \varepsilon^2 e^{2i\omega t} + T_3 \varepsilon^3 e^{3i\omega t} + \dots = \sum_{j=0}^{\infty} \varepsilon^j T_j e^{j(i\omega t)} \quad (12)$$

$$U(y) = U_0 + U_1 \varepsilon e^{i\omega t} + U_2 \varepsilon^2 e^{2i\omega t} + U_3 \varepsilon^3 e^{3i\omega t} + \dots = \sum_{j=0}^{\infty} \varepsilon^j U_j e^{j(i\omega t)} \quad (13)$$

The corresponding boundary conditions are:

When  $y = 0$ ,

$$C_0 = 1, C_1 = 1, C_2 = 1, C_3 = 1, T_0 = 1, T_1 = 1, T_2 = 1, T_3 = 1, U_0 = 0, U_1 = 0, U_2 = 0, U_3 = 0 \quad (14)$$

When  $y = \infty$ ,

$$C_0 = 0, C_1 = 0, C_2 = 0, C_3 = 0, T_0 = 0, T_1 = 0, T_2 = 0, T_3 = 0, U_0 = 0, U_1 = 0, U_2 = 0, U_3 = 0 \quad (15)$$

The complete solution of concentration, temperature and velocity equations are obtained as:

$$C(y) = e^{-(Sc\gamma)y} + \varepsilon e^{i\omega t - m_2 y} + \varepsilon^2 e^{2i\omega t - m_4 y} + \varepsilon^3 e^{3i\omega t - m_6 y} \quad (16)$$

$$T(y) = e^{-m_8 y} + \varepsilon e^{i\omega t - m_{10} y} + \varepsilon^2 e^{2i\omega t - m_{12} y} + \varepsilon^3 e^{3i\omega t - m_{14} y} \quad (17)$$

$$U(y) = A_{18} e^{-m_{16} y} + A_{19} e^{-m_8 y} + A_{20} e^{-(Sc\gamma)y} + (A_{22} e^{-m_{18} y} + A_{23} e^{-m_{10} y} + A_{24} e^{-m_2 y}) \varepsilon e^{i\omega t} \\ + (A_{26} e^{-m_{20} y} + A_{27} e^{-m_{12} y} + A_{28} e^{-m_4 y}) \varepsilon^2 e^{2i\omega t} + (A_{30} e^{-m_{22} y} + A_{31} e^{-m_{14} y} + A_{32} e^{-m_6 y}) \varepsilon^3 e^{3i\omega t} \quad (18)$$

While the Sherwood number, Nusselt number and Skin friction at  $y=0$  and  $y=1$  are:

$$Sh_0 = \left. \frac{dc}{dy} \right|_{y=0} = -Sc\gamma - \varepsilon m_2 e^{i\omega t} - \varepsilon^2 m_4 e^{2i\omega t} - \varepsilon^3 m_6 e^{3i\omega t} \quad (19)$$

$$Sh_1 = \left. \frac{dc}{dy} \right|_{y=1} = -Sc\gamma e^{-Sc\gamma} - \varepsilon m_2 e^{-m_2} e^{i\omega t} - \varepsilon^2 m_4 e^{-m_4} e^{2i\omega t} - \varepsilon^3 m_6 e^{-m_6} e^{3i\omega t} \quad (20)$$

$$Nu_0 = \left. \frac{dT}{dy} \right|_{y=0} = -m_8 - \varepsilon m_{10} e^{i\omega t} - \varepsilon^2 m_{12} e^{2i\omega t} - \varepsilon^3 m_{14} e^{3i\omega t} \quad (21)$$

$$Nu_1 = \left. \frac{dT}{dy} \right|_{y=1} = -m_8 e^{-m_8} - \varepsilon m_{10} e^{-m_{10}} e^{i\omega t} - \varepsilon^2 m_{12} e^{-m_{12}} e^{2i\omega t} - \varepsilon^3 m_{14} e^{-m_{14}} e^{3i\omega t} \quad (22)$$

$$\begin{aligned}
\tau_0 = \frac{du}{dy} \Big|_{y=0} = & -m_{16}A_{18} - m_8A_{19} - Sc\gamma A_{20} - m_{18}A_{22}\epsilon e^{i\omega t} - m_{10}A_{23}\epsilon e^{i\omega t} - m_2A_{24}\epsilon e^{i\omega t} \\
& -m_{20}A_{26}\epsilon^2 e^{2i\omega t} - m_{12}A_{27}\epsilon^2 e^{2i\omega t} - m_4A_{28}\epsilon^2 e^{2i\omega t} - m_{22}A_{30}\epsilon^3 e^{3i\omega t} \\
& -m_4A_{31}\epsilon^3 e^{3i\omega t} - m_6A_{32}\epsilon^3 e^{3i\omega t}
\end{aligned} \tag{23}$$

$$\begin{aligned}
\tau_1 = \frac{du}{dy} \Big|_{y=1} = & -m_{16}A_{18}e^{-m_6} - m_8A_{19}e^{-m_8} - Sc\gamma A_{20}e^{-(Sc\gamma)} - m_{18}A_{22}e^{-m_{18}}\epsilon e^{i\omega t} - m_{10}A_{23}e^{-m_{10}}\epsilon e^{i\omega t} \\
& -m_2A_{24}e^{-m_2}\epsilon e^{i\omega t} - m_{20}A_{26}e^{-m_{20}}\epsilon^2 e^{i\omega t} - m_{12}A_{27}e^{-m_{12}}\epsilon^2 e^{i\omega t} - m_4A_{28}e^{-m_4}\epsilon^2 e^{i\omega t} \\
& -m_{22}A_{30}e^{-m_{22}}\epsilon^3 e^{3i\omega t} - m_4A_{31}e^{-m_4}\epsilon^3 e^{3i\omega t} - m_6A_{32}e^{-m_6}\epsilon^3 e^{3i\omega t}
\end{aligned} \tag{24}$$

### 3.0 Results and Discussion

The basic parameters that governed this flow are the suction parameter ( $\gamma$ ), Hartmann number(M), permeability parameter (K), thermal Grashof number (Gr), mass Grashof number (Gc), Prandlt number (Pr),heat source parameter (S) and Schmidt number (Sc).The velocity of the flow field is found to change more or less with the variation of the flow parameters. The major parameters affecting the velocity of the flow field are suction parameter ( $\gamma$ ), Hartmann number(M), permeability parameter (K), thermal Grashof number (Gr) as well as mass Grashof number (Gc).The effects of those parameters on velocity profile have been analyzed with the aid of line graph seen in Figures 2-6.The different values of suction parameter ( $\gamma$ ) is presented in Figure2 while keeping the Hartmann number(M),permeability parameter (K),thermal Grashof number (Gr) and mass Grashof number (Gc) constant. It shows that as the suction parameter ( $\gamma$ ) increases the velocity also decreases.

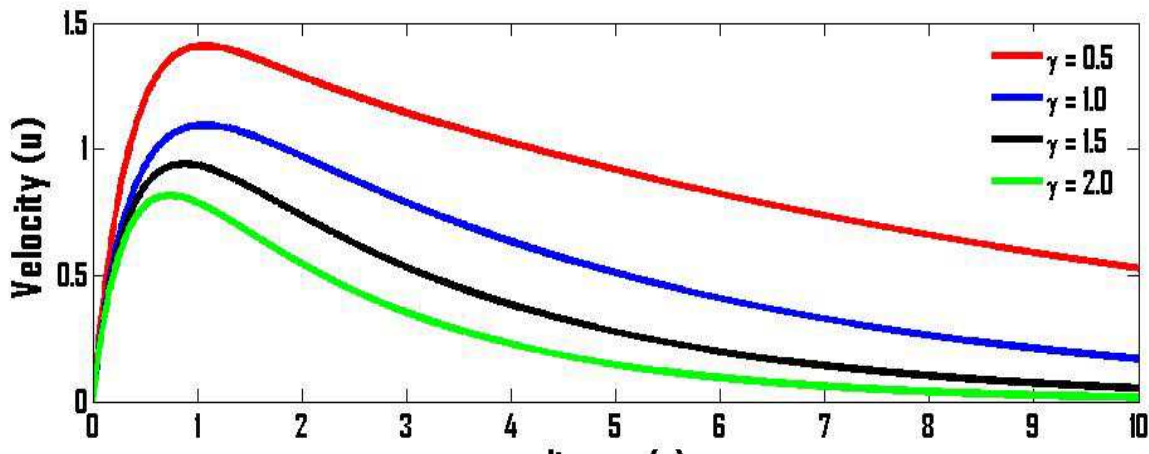


Figure 2: Effects of suction parameter ( $\gamma$ ) on velocity.

In Figure 3 different values of Hartmann number (M) is presented and keeping the suction parameter ( $\gamma$ ), permeability parameter (K),thermal Grashof number (Gr) and mass Grashof number (Gc) constant. It also shows that as Hartmann number (M) increases the velocity also decreases, because the presence of a Hartmann number (M) in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow in the normal direction.

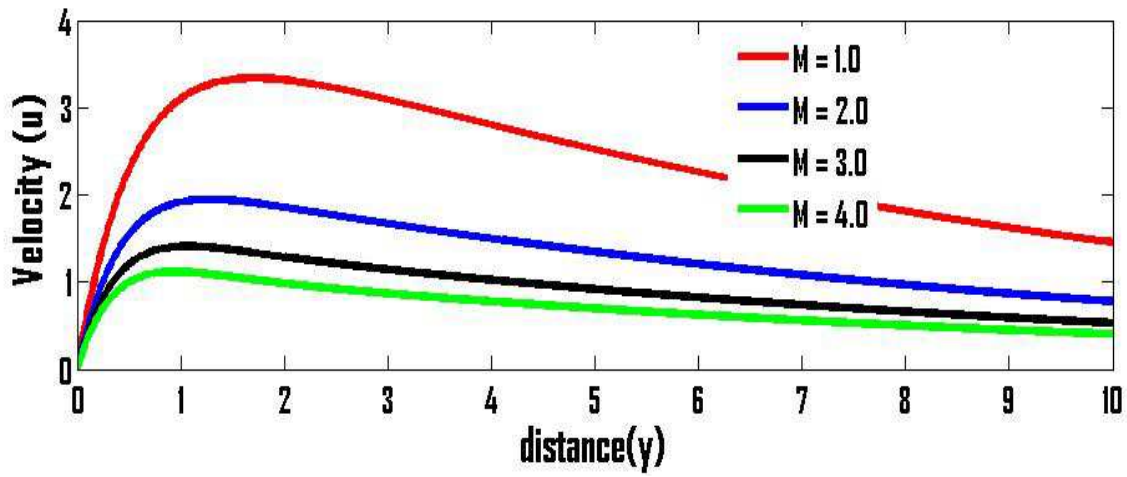


Figure 3: Effects of Hartmann number ( $M$ ) on velocity.

The increase in permeability parameter ( $K$ ) is observed in Figure 4 when other parameters are kept constant. It is observed that the velocity decreases with the increasing dimensionless porous medium parameter.

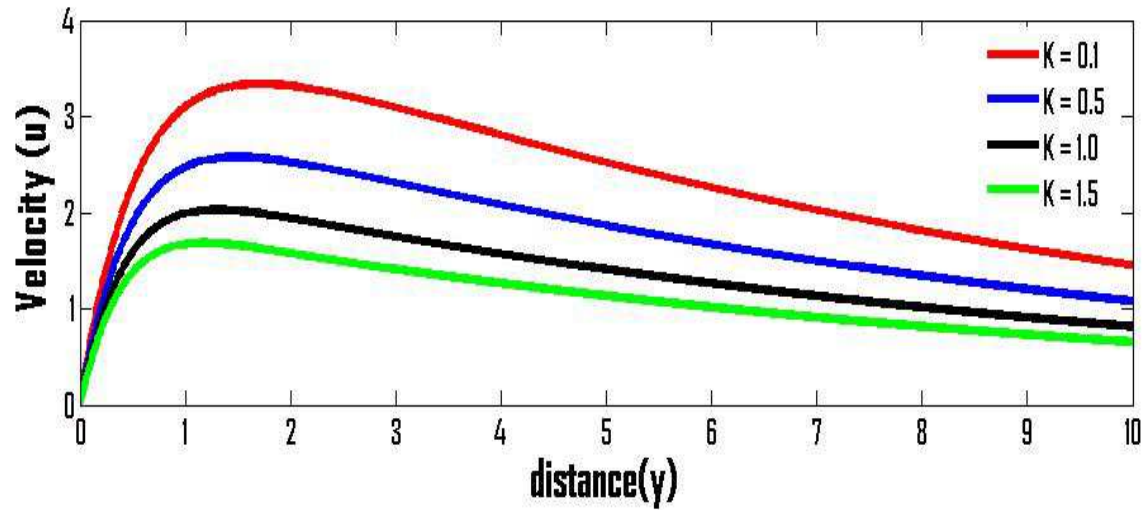


Figure 4: Effects of permeability parameter ( $K$ ) on velocity.

In Figure 5 different values of mass Grashof number ( $G_c$ ) is presented and keeping the suction parameter ( $\gamma$ ), Hartmann number ( $M$ ), permeability parameter ( $K$ ) and thermal Grashof number ( $Gr$ ) constant. It shows that as mass Grashof number ( $G_c$ ) increases the velocity also increases due to increase in the species buoyancy force.

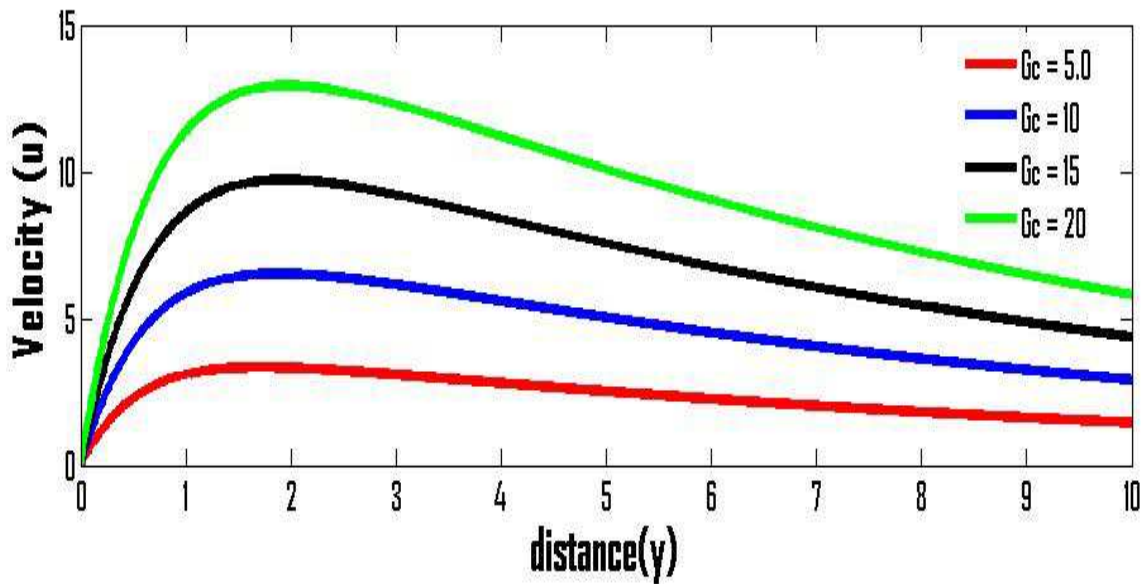


Figure 5: Effects of mass Grashof number ( $G_c$ ) on velocity.

In Figure 6 the effect of thermal Grashof number ( $Gr$ ) on velocity is presented for fixed values of suction parameter ( $\gamma$ ), Hartmann number ( $M$ ), permeability parameter ( $K$ ) and mass Grashof number ( $G_c$ ). It shows that as thermal Grashof number ( $Gr$ ) increases the velocity also increases due to the enhancement of thermal buoyancy force.

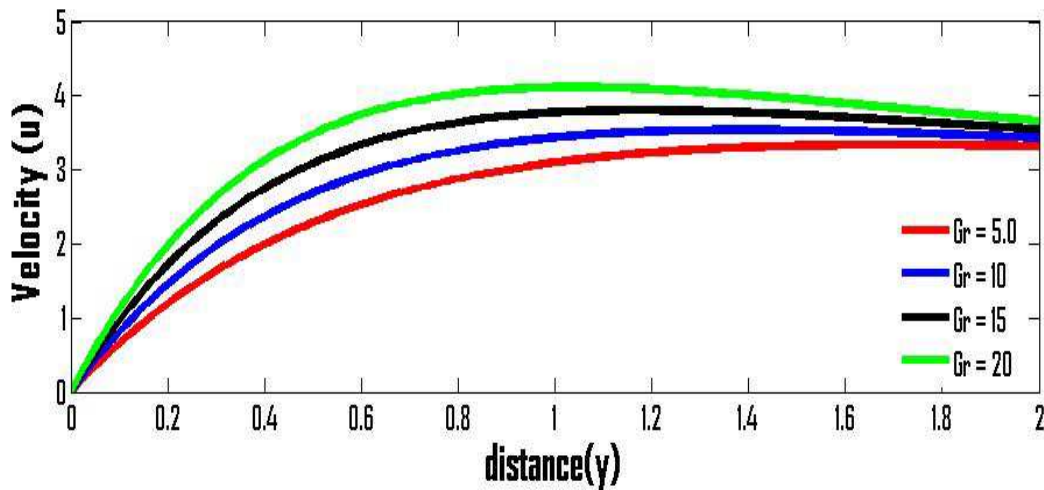


Figure 6: Effects of thermal Grashof number ( $Gr$ ) on velocity

The temperature of the flow changes with the variation of the flow parameters such as suction parameter ( $\gamma$ ), Prandtl number ( $Pr$ ) and heat source parameter ( $S$ ). In Figure 7 It shown that when suction parameter ( $\gamma$ ) increases the temperature also decreases because in the presence of higher suction parameter ( $\gamma$ ) more amount of fluid is pushed into the flow field through the plate due to which the flow field suffers a decrease in temperature at all points.

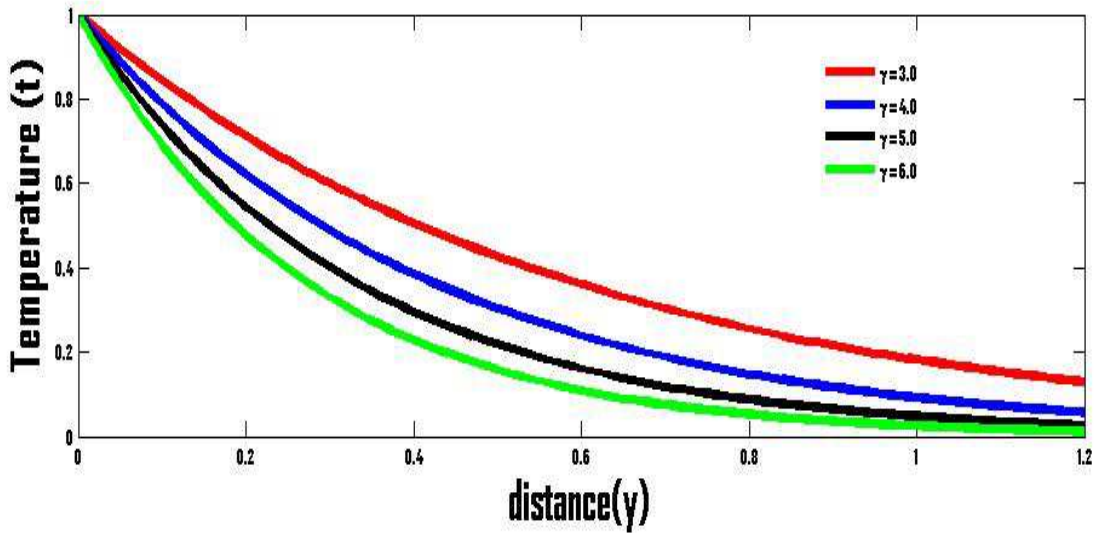


Figure 7: Effects of suction parameter ( $\gamma$ ) on temperature.

Figure 8 depicts the effect of Prandtl number (Pr) on temperature field when suction parameter ( $\gamma$ ) and heat source parameter (S) remain unchanged. It is interesting to observe that an increase in the Prandtl number (Pr) decreases the temperature of the flow field. This is physically true since (Pr) is inversely proportional to the thermal diffusivity of the working fluid.

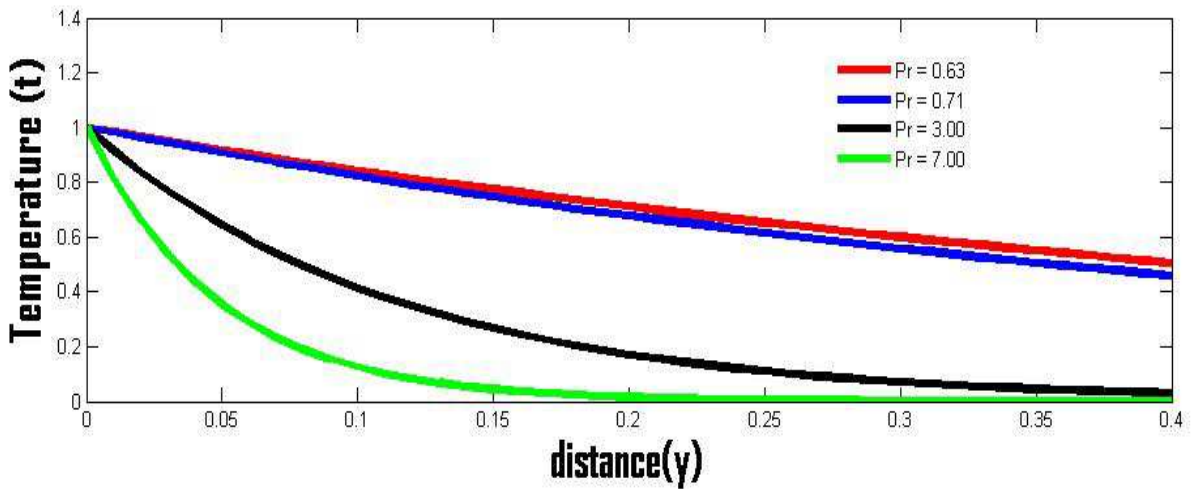


Figure 8: Effects of Prandtl number (Pr) on temperature.

In Figure 9 it is observed that when there is increase in values of heat source parameter (S) the temperature of the flow field also increases.

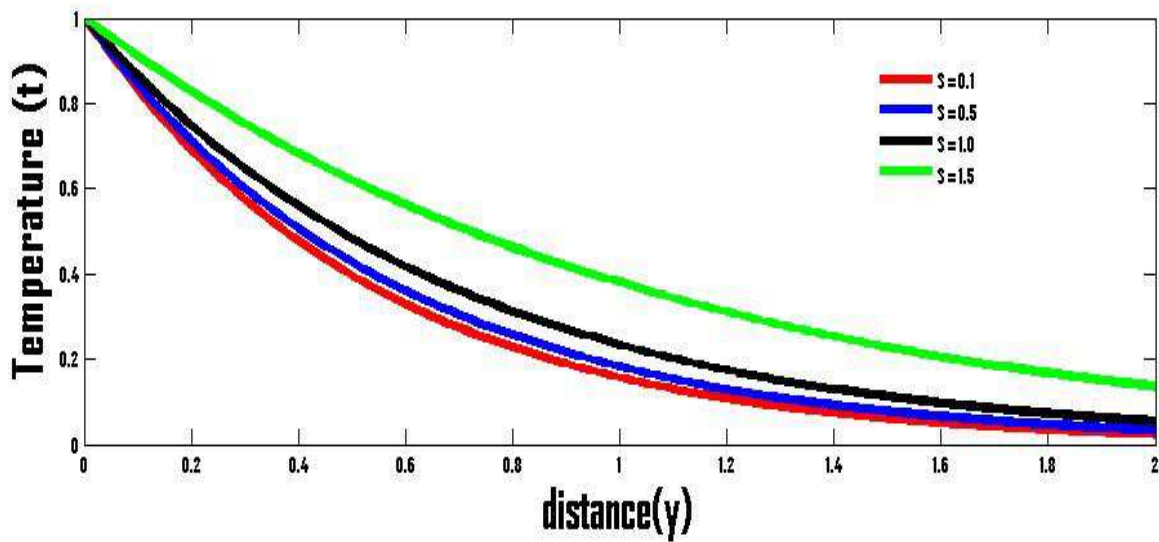


Figure 9: Effects of heat source parameter (S) on temperature.

The Concentration of the flow suffers a substantial change with the variation of the flow parameters such as suction parameter ( $\gamma$ ) and Schmidt number (Sc). It is shown that when suction parameter ( $\gamma$ ) increases the concentration decreases. In other words, cooling of the plate is faster as the suction parameter ( $\gamma$ ) becomes larger. Thus it may be concluded that larger suction parameter ( $\gamma$ ) leads to faster cooling of the plate.

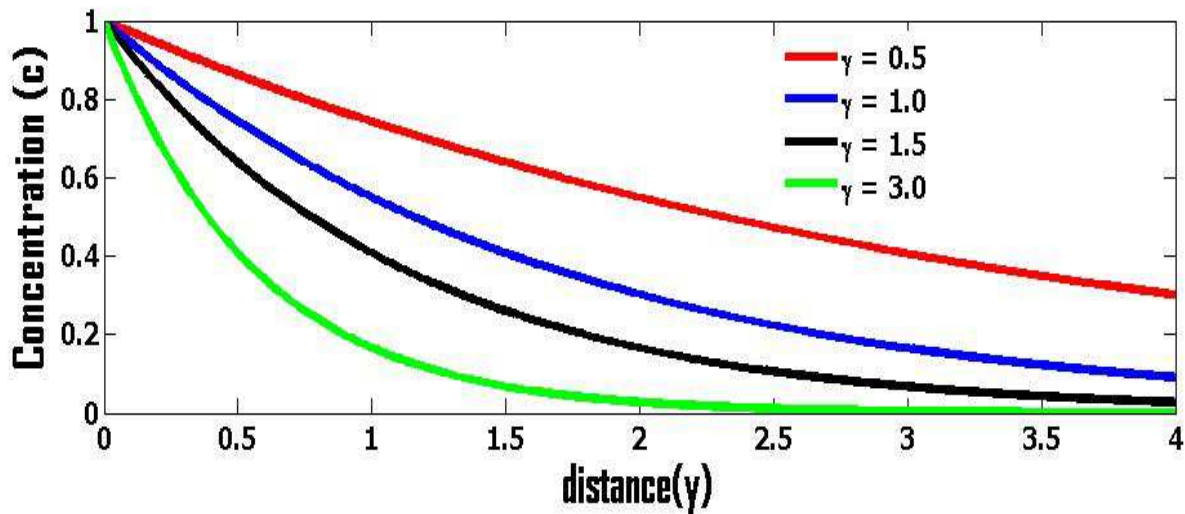


Figure 10: Effects of suction parameter ( $\gamma$ ) on concentration.

The concentration distribution is found to decrease faster as the diffusing foreign species becomes heavier. Higher Schmidt number (Sc) leads to a faster decrease in the concentration of the flow field seen in Figure 11.

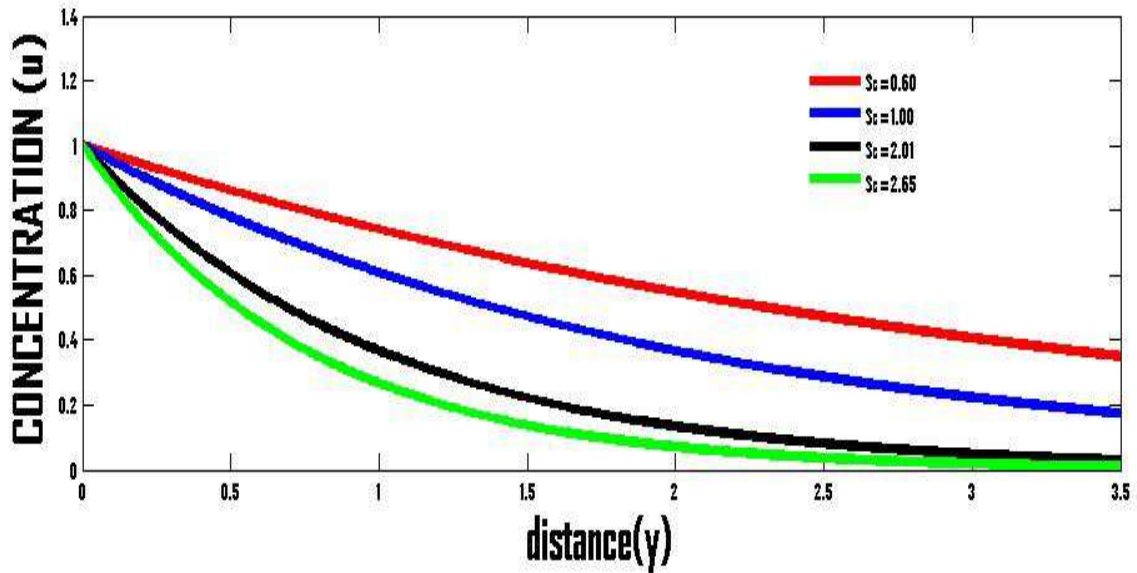


Figure 11: Effects of Schmidt number ( $Sc$ ) on concentration.

Numerical computation are carried out for skin friction coefficient ( $\tau$ ), Nusselt number ( $Nu$ ) and Sherwood number ( $Sh$ ) at  $y = 0$  and  $y = 1$ , for various values of thermal Grashof number ( $Gr$ ), mass Grashof number ( $Gc$ ), permeability parameter ( $k$ ), Hartmann number ( $M$ ), suction parameter ( $\gamma$ ), Prandtl number ( $Pr$ ), heat source parameter ( $S$ ) and Schmidt number ( $Sc$ ).

**Table 1:** values of Skin friction coefficient ( $\tau$ ) at  $y = 0$  and  $y = 1$

Gr	Gc	K	M	$\gamma$	Pr	S	Sc	$\tau_0$	$\tau_1$
5	5	0.1	1	3	0.63	0.1	0.6	4.6377	16.7843
10	5	0.1	1	3	0.63	0.1	0.6	6.9265	17.0497
5	10	0.1	1	3	0.63	0.1	0.6	6.9867	33.3032
5	5	0.5	1	3	0.63	0.1	0.6	4.4202	14.9733
5	5	0.1	2	3	0.63	0.1	0.6	4.1469	12.8904
5	5	0.1	1	4	0.63	0.1	0.6	3.7000	26.8662
5	5	0.1	1	3	0.71	0.1	0.6	4.4113	16.7504
5	5	0.1	1	3	0.63	0.5	0.6	4.8072	16.8075
5	5	0.1	1	3	0.63	0.1	1	3.7913	$2.7362 \times 10^2$

**Table 2:** Nusselt number ( $Nu$ ) at  $y = 0$  and  $y = 1$

$\gamma$	Pr	S	$Nu_0$	$Nu_1$
3	0.63	0.1	1.8572	0.2903
4	0.63	0.1	2.4964	0.2060
3	0.71	0.1	2.0974	0.2579
3	0.63	0.5	1.7063	0.3101

**Table 3:** Sherwood number (Sh) at  $y = 0$  and  $y = 1$ 

$\gamma$	Sc	$Sh_0$	$Sh_1$
3	0.6	1.8010	10.8896
4	0.6	2.4015	26.4558
3	1	3.0019	60.2567

From Table 1, it is observed that, an increase in thermal Grashof number (Gr), mass Grashof number (Gc) and heat source parameter (S) at  $y = 0$  and  $y = 1$  enhances the values of skin friction ( $\tau$ ). Also at  $y = 0$  and  $y = 1$  the skin friction ( $\tau$ ) suffers due to the increase in Hartmann number (M), permeability parameter (k), Prandtl number (Pr) and Schmidt number (Sc). Similarly at  $y = 0$  the skin friction ( $\tau$ ) decreases and at  $y = 1$  the skin friction increase due to increase of suction parameter ( $\gamma$ ). In Table 2, an increase in suction parameter ( $\gamma$ ) and Prandtl number (Pr) at  $y = 0$  enhance the rate of heat transfer (Nu) while at  $y = 1$  decrease the rate of heat transfer (Nu). Also at  $y = 0$  the rate of heat transfer decreases and at  $y = 1$  the rate of heat transfer increases due to increase heat source parameter (S). It is observed in Table 3 that, an increase in suction parameter ( $\gamma$ ) and Schmidt number (Sc) at  $y = 0$  and  $y = 1$  enhances the value of Sherwood number (Sh).

### Conclusion

Unsteady free convective flow in a porous medium with heat generation in an infinite vertical plate is presented. The governing equations are transformed into dimensionless form and solved numerically using finite difference scheme. The results are obtained and presented graphically and on tables to illustrate the details of the characteristics parameters within the flow. When  $M = K = Gc = 0$  and  $Gr = 1$ , the result of the present work are in excellent agreement with [3] respectively. From the present investigation it is concluded that:

- 1 Concentration decreases with increase in parameters  $\gamma$  and Sc.
- 2 The increase of variable parameters  $\gamma$  and Pr, results to the decrease in temperature and increase with S.
- 3 The results show that the velocity increases with increase in material parameters Gr and Gc and decrease with  $\gamma$ , M and K.
- 4 Table 1 shows that Skin friction coefficient ( $\tau$ ) increases with the increase in material parameters Gr, Gc and S and decreases in material parameters K, M, Pr, and Sc respectively.
- 5 Nusselt number (Nu) in Table 2 increase in the material parameter  $\gamma$  and Pr at  $y = 0$  and decreases at  $y = 1$  but S decreases at  $y = 0$  and increases at  $y = 1$ .
- 6 Also in Table 3 shows that the sherwood number (Sh) increases with the increase of material parameters ( $\gamma$ ) and (Sc).

### Appendix

$$m = 0, \quad m = -Sc\gamma \quad A_2 = 1 \quad P = \sqrt{(Sc\gamma)^2 + 4i\omega} \quad m_1 = \frac{-Sc\gamma + P}{2}, m_2 = \frac{Sc\gamma + P}{2}$$

$$A_4 = 1 \quad p_1 = \sqrt{(Sc\gamma)^2 + 8Sci\omega} \quad m_3 = \frac{-Sc\gamma + p_1}{2}, m_4 = \frac{Sc\gamma + p_1}{2}$$

$$A_6 = 1$$

$$P_2 = \sqrt{(Sc\gamma)^2 + 12Sci\omega} \quad m_5 = \frac{-Sc\gamma + P_2}{2}, m_6 = \frac{Sc\gamma + P_2}{2} \quad A_8 = 1$$

$$P_3 = \sqrt{(Pr\gamma)^2 - 4PrS} \quad m_7 = \frac{-Pr\gamma + P_3}{2}, m_8 = \frac{Pr\gamma + P_3}{2} \quad A_{10} = 1$$

$$P_4 = \sqrt{\text{Pr} \gamma^2 - 4(\text{Pr}(S - i\omega))} \quad m_9 = \frac{-\text{Pr} \gamma + P_4}{2}, m_{10} = \frac{\text{Pr} \gamma + P_4}{2} \quad A_{12} = 1$$

$$P_5 = \sqrt{(\text{Pr} \gamma)^2 - (8i\omega + 4S)} \quad m_{11} = \frac{-\text{Pr} \gamma + P_5}{2}, m_{12} = \frac{\text{Pr} \gamma + P_5}{2} \quad A_{14} = 1$$

$$P_6 = \sqrt{(\text{Pr} \gamma)^2 - 4(\text{Pr}(S - 3i\omega))} \quad m_{13} = \frac{-\text{Pr} \gamma + P_6}{2}, m_{14} = \frac{\text{Pr} \gamma + P_6}{2} \quad A_{16} = 1$$

$$P_7 = \sqrt{\gamma^2 + 4(M + K)} \quad m_{15} = \frac{-\gamma + P_7}{2}, m_{16} = \frac{\gamma + P_7}{2} \quad A_{18} = 1 - A_{19} - A_{20}$$

$$A_{19} = \frac{-Gr}{m_8^2 - \gamma m_8 - (M + K)}$$

$$A_{20} = \frac{-Gc}{(Sc\gamma)^2 - \gamma(Sc\gamma) - (M + k)}$$

$$P_{10} = \sqrt{\gamma^2 + 4(M + K + i\omega)} \quad m_{17} = \frac{-\gamma + P_{10}}{2}, m_{18} = \frac{\gamma + P_{10}}{2}$$

$$A_{22} = -1 - (A_{23} + A_{24})$$

$$A_{23} = \frac{-Gr}{m_{10}^2 - \gamma m_{10} - (M + k + i\omega)}$$

$$A_{24} = \frac{-Gc}{m_2^2 - \gamma m_2 - (M + K + i\omega)}$$

$$P_{13} = \sqrt{\gamma^2 + 4(M + K + 2i\omega)}$$

$$m_{19} = \frac{-\gamma + P_{13}}{2}, m_{20} = \frac{\gamma + P_{13}}{2} \quad A_{26} = -1 - (A_{27} + A_{28}) \quad P_{16} = \sqrt{\gamma^2 + 4(M + k + 3i\omega)}$$

$$A_{27} = \frac{-Gr}{m_{10}^2 - \gamma m_{10} - (M + K + 2i\omega)}$$

$$A_{28} = \frac{-Gc}{m_4^2 - \gamma m_4 - (M + K + 2i\omega)}$$

$$m_{21} = \frac{-\gamma + P_{16}}{2}, m_{22} = \frac{\gamma + P_{16}}{2} \quad A_{30} = -1 - (A_{31} + A_{32})$$

$$A_{31} = \frac{-Gr}{m_4^2 - \gamma m_4 - (M + K + 3i\omega)}$$

$$A_{32} = \frac{-Gc}{m_6^2 - \gamma m_6 - (M + K + 3i\omega)}$$

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