Hybrid of ARIMA-ARCH Modelling of Daily Share Price Data of Okomu Oil Plc in Nigeria

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ABSTRACT

The aim of this work is to study and develop an appropriate time series model for the residuals from the autoregressive integrated moving average (ARIMA) model derived from the daily stock data of Okomu Oil.

The autocorrelation structure of the residuals and the squared residuals were examined. The Box-Ljung test, Box-Pierce and McLeod-Li test were applied to the residuals and squared residuals from the ARIMA model. These tests revealed the presence of conditional variance (volatility) in the residuals from the ARIMA model. The autoregressive conditional heteroscedastic (ARCH) models were then applied in modelling this volatility.

Our results showed that the ARCH (5) model was best (having the smallest AIC) giving rise to a hybrid ARIMA-ARCH model. This model better explains and captures the dynamics of the daily stock price of the company being studied.

Keywords: ARCH, McLeod-Li test, Akaike Information Criterion, volatility, autocorrelation function, Okomu.

1.1 INTRODUCTION

Stock market prediction is the act of trying to determine the future value of a company stock or other financial instrument traded on a financial exchange [1]. Stock market forecasters emphasizes on developing a successful modelling approach for forecasting index values or stock prices [2]. The essential idea to successful stock market prediction is achieving best results and also to minimize the inaccurate forecast of stock price.

The nature of stock market return process is characterized as a combination of drift and volatility. Stock price volatility is an indicator that is most often used to find changes in trends in the market place. The need to maximize profit is the ultimate aim of any stock market investor. Very often, many of these investors rely on intuition, advertorials and newspaper headlines to make their investments. This has led to great losses by stock market investors [3].

When modelling stock time series, focus is usually on modelling and predicting the mean behaviour (first order moments) and rarely concerned with the conditional variance (second order moments). The increased importance played by risk and uncertainty considerations in stock price management has necessitated the development of new time series techniques that allow for the modelling of time varying variances. The autoregressive conditional heteroscedasticity (ARCH) type models give us an appropriate framework for studying this problem.

Osemwenkhae and Eguasa [4] examined and modeled the serial dependence structure of daily stock prices in Nigeria using data from Okomu Oil Palm Plc. They fitted several ARIMA models and based on the Akaike information criterion (AIC), their work revealed that the ARIMA (0,1,1) was most adequate in modelling the Okomu share price series.

The aim of this work is to examine and model the time varying variance (volatility) of the daily stock price of Okomu Oil Palm Plc which extends the work of [4].

Engle [5] introduced the ARCH model where it was pointed out that traditional econometric models assumed a constant one-period forecast variance. To this end, a regression model with disturbances following an ARCH process was developed. The test was based simply on the autocorrelation of the squared ordinary least squares (OLS) residuals and used the model to estimate the means and variances of inflation in the U.K.

The ARCH effect was found to be significant and the estimated variances increased substantially during the chaotic seventies in the U.K.

Brailsford and Robert [6] opined that the task of forecasting volatility is a difficult one. The work noted that while evidence abound in literature supporting the superiority of complex models such the ARCH class of models, there are also evidences supporting the superiority of more simple alternatives. Brailsford and Robert [6] used data from the Australian stock market to examine this issue. The results suggested that the ARCH class of models and a simple regression model provide superior forecasts of volatility. In some cases, volatility may impair the smooth functioning of the financial system and adversely affect economic performance. The increase or decrease in volatility may be attributed to changes in investors' emotions in the market place [7]. Stock price volatility according to them tends to rise when new information is released into the market; however the extent to which it rises is determined by the relevance of that new information as well as the degree in which the news surprises investors.

Bollerslev [8] noted that a generalization of the ARCH process was introduced in the work of [5]. Bollerslev [8] explained that while conventional time series and econometric models operate under the assumption of constant variance, the ARCH process introduced in [5] allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. Bollerslev [8] further pointed out that in the study of [9], the autoregressive moving average (ARMA) models with ARCH errors are found to be successful in modelling thirteen different U.S. macroeconomic time series.

3.0 METHODOLOGY

3.1 Autoregressive Conditional Heteroskedasticity (ARCH) Model

The ARIMA (p,d,q), where p is the autoregressive order, d is the order of differencing and q is the moving average order, is a method for fitting linear models to time series data. A major drawback of these models (linear stationary models) is that they fail to account for changing volatility (i.e., variance): the width of the forecast intervals remains constant even as new data become available. On the other hand however, actual financial time series often show sudden bursts of volatility.

The ARIMA (p,d,q) model cannot capture the heteroskedastic effects of a time series process, typically observed in the form of high kurtosis or "clustering of volatilities"

[10,11]. In the presence of heteroskedasticity, the regression coefficients for an ordinary least squares regression are still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. However, instead of considering this as a problem to be corrected, ARCH models treat heteroskedasticity as a variance to be modeled. The term conditional implies the level of association on the past sequence of observations and the autoregressive describes the feedback mechanism that incorporates past observations into the present [12].

In the ARCH model, the forecast intervals are able to adjust immediately to account for sudden changes in volatility, without changing the parameters of the model. This feature has made the ARCH (and other related) models to become a very key tool in the analysis of economic time series. The ARCH (n) model for the series $\{w_t\}$ is defined by specifying the conditional distribution of w_t, given the information available up to time t-1. Let ψ_{t-1} denote this information. A process $\{w_t\}$ is ARCH (n) if the conditional distribution of w_t given the available information ψ_{t-1} is

$$w_t \mid \psi_{t-1} \sim N(0, h_t), \tag{1}$$

$$h_t = \omega + \sum_{i=1}^{n} \alpha_i w_{t-i}^2 \tag{2}$$

with $\omega > 0, \alpha_i \geq 0 \; \forall \; i, \sum_{i=1}^{n} \alpha_i < 1$, where ω, α are constants to be estimated and n is the number of lags [13].

The time varying variance (i.e. heteroskedasticity or volatility) which depends on the observations of the immediate past is called the conditional variance [14,15]. The advantage of the ARCH models lies in their ability to describe the time-varying stochastic conditional volatility, which can then be used to improve the reliability of interval forecasts and to help in understanding the process.

3.2 ARIMA-ARCH Model (Testing and Order Selection)

The ARIMA-ARCH model is one model in which the variance of the error term of the ARIMA model follows an ARCH process. Chand et al. [16] explained that the main tests before actually estimating the conditional volatility are Engle's test (LM Test) and portmanteau tests (Box-Ljung, Box-Pierce).

3.2.1 McLeod-Li Test

This test for ARCH effects was proposed by McLeod and Li, see [10]. "It examines the autocorrelation function of the squares of the pre-whitened data and test whether the correlation (x_t^2, x_{t-j}^2) is non-zero for some j".

3.2.2 ARCH Lagrange Multiplier (LM) Test

Engle [4] proposed the Lagrange multiplier method for testing ARCH effect. This procedure simply involves obtaining the squares of the residual from fitted model $\hat{w}^2$ and regress them on a constant and n lagged values:

$$\hat{w}_t^2 = \hat{\alpha}_0 + \sum_{i=1}^{n} \hat{\alpha}_i \hat{w}_{t-i}^2 \qquad (3)$$

where n is the length of ARCH lags. The hypothesis is that, in the absence of ARCH components, we have $\alpha_i = 0$ for all i=1, 2, …,n, against the alternative that, in the presence of ARCH components, at least one of the estimated α_i coefficients must be significant [10].

4.0 MODELLING AND ANALYSIS OF DATA

4.1 ARIMA (0,1,1) Model [4]

Here, the fitted ARIMA(0,1,1) model for the Okomu series from 2010-2014 is presented. This model in [4] is stated in equation (4);

$$\hat{y}_t = y_{t-1} + w_t + 0.2456 \ w_{t-1} \qquad (4)$$

where $\hat{y}_t$ is the estimate (forecast) of the series at time t, y_{t-1} is the value of the observed series at time t-1, w_t is the model residual, 0.2456 is the coefficient of the moving average term of order 1 (w_{t-1}).

4.2 NON-LINEARITY TEST OF ARIMA MODEL RESIDUALS

In this section, the Box-Pierce, Box-Ljung, and McLeod-Li non-linearity tests are carried out on the ARIMA (0,1,1) model residual and squared residual. This is done to test the presence of volatility in the residual as follows:

Table 1: Non-Linearity Test.

	Model Residual			**Squared Model Residual**		
	Chi-squared	**df**	**p-value**	**Chi-squared**	**df**	**p-value**
Box-Pierce test	0.0162	1	0.8987	126.0998	1	< 2.2e-16
Box-Ljung test	0.0162	1	0.8986	126.4059	1	< 2.2e-16

In Table 1, the first section shows the values for the Box-Pierce and Box-Ljung tests for the model residual while the second section shows the values for the Box-Pierce and Box-Ljung tests for the squared model residual. A cursory look at Table 1 shows that the p-values for the Box-Pierce and Box-Ljung test are greater than 0.05 (5% level of significance) for the model residual and respectively smaller than 0.05 for the squared model residual. This indicates that the linear ARIMA (0,1,1) model do not adequately capture the behaviour of Okomu stock series. Consequently, these results show that the residuals are uncorrelated but the squared residuals are correlated. This is a clear indication of the presence of volatility in the series.

In addition, the plot of p-values for the McLeod-Li test is given in Figure 1.

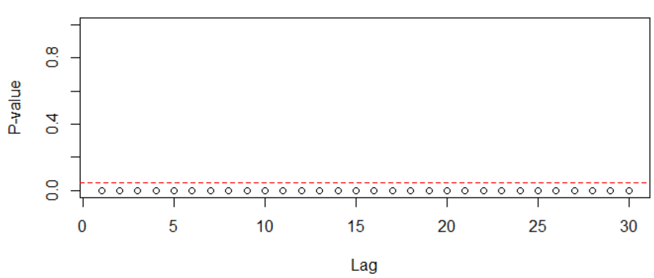

Figure 1: Plot of McLeod-Li Test

This plot (Figure 1) points to the rejection of the null hypothesis of no ARCH effect since all the p-values for the various lags (up to lag 30) are less than 0.05.

We therefore proceed to model the residuals using the ARCH approach in order to account for the presence of volatility. So the next step is to determine the significant order of the ARCH model.

4.3 SELECTION OF ARCH LAG ORDER

In order to determine the order of the ARCH model, we examine the autocorrelation function and partial autocorrelation functions. The order of the model is indicated by the significant lags in the plots.

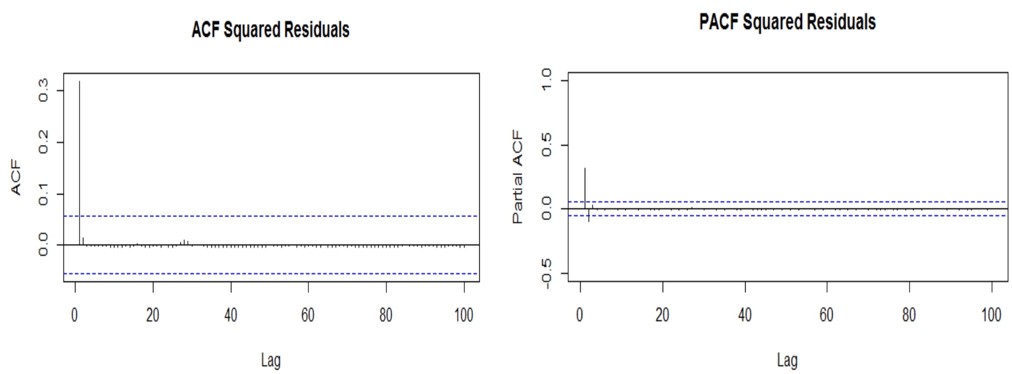

Figure 2: ACF and PACF Plots of ARIMA Squared Residuals.

From the plots, the ACF do not indicate any significant lag but the PACF is significant at lag 1 indicating autoregressive term. We therefore proceed to estimate the AICs of adequate ARCH models and select the best model based on these AICs.

Table 2: AIC Values for different ARCH Models

ARCH Model	AIC
ARCH (1)	-4407.506
ARCH (2)	-4398.848
ARCH (3)	-4388.297
ARCH (4)	-4377.934
ARCH (5)	-4423.234
ARCH (6)	-4409.672

Table 2 shows the AIC values for different ARCH models. From the table, the ARCH model with five (5) lag is most adequate since it is the least AIC value. This model thus serves as the measure of volatility of the Okomu stock price. Hence, we apply this model to the residuals of the best ARIMA model (ARIMA (0,1,1)) to derive the combined ARIMA (0,1,1)-ARCH(5) model for the Okomu stock price data.

4.4 PARAMETER ESTIMATES AND DIAGNOSTICS FOR ARCH (5) MODEL.

In this Section, the estimates and diagnostic of the ARCH (5) model is presented.

Table 3: PARAMETER ESTIMATES FOR ARCH (5) MODEL.

ARCH lag	Estimate	Std. Error	t value	Pr(>|t|)
0	1.399e-03	5.294e-06	264.269	< 2e-16***
1	7.868e-02	2.354e-02	3.342	0.000832***
2	7.864e-03	2.347e-02	0.335	0.737618
3	1.928e-15	5.959e-03	0.000	1.000000
4	1.426e-02	2.402e-02	0.594	0.552757
5	1.314e-01	4.323e-03	30.388	< 2e-16 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Table 3 gives the parameter estimates, standard errors, t values with their corresponding p-values for the five ARCH lags. In Table 3, the p-values for the first, second and sixth parameters (ARCH lags) are less than 0.05 indicating that they are statistically significant.

Table 4: DIAGNOSTICS FOR ARCH (5) MODEL.

	Chi-squared	**df**	**p-value**
Box-Ljung Test	0.085	**1**	0.7706

In Table 4, the p-value of the Box-Ljung test is greater than 0.05. This means that the hypothesis of no serial correlation cannot be rejected. Therefore the model is accurate at 95% significant levels and fully captured the ARCH effect.

ARCH Model (volatility estimate) for the Okomu stock price data spanning 2010 to 2014 is thus expressed as:

$$h_t = 1.399e-03 + 7.868e-02w_{t-1}^2 + 7.864e-03w_{t-2}^2 + 1.928e-15w_{t-3}^2 + 1.426e - 02w_{t-4}^2 + 1.314e-01w_{t-5}^2 \quad (5)$$

Our final model of the combined ARIMA (0,1,1)-ARCH (5) model is

$$\hat{y}_t = y_{t-1} + 0.2456w_{t-1} + 1.399e-03 + 7.868e-02w_{t-1}^2 + 7.864e - 03w_{t-2}^2 + 1.928e-15w_{t-3}^2 + 1.426e-02w_{t-4}^2 + 1.314e-01w_{t-5}^2 \quad (6)$$

4.5 Non-Linearity Test on Final Model (ARIMA (0,1,1)-ARCH (5)) Residuals.

In this section, diagnostics of our final model is carried out. This is to enable us check if the final model adequately represent the characteristic behaviour of the Okomu stock price series.

The Box-Pierce, Box-Ljung and McLeod-Li test were performed on the residuals and squared residuals of the series.

Table 5: Non-Linearity Test for final model

	Final model residual			Final model squared residual		
	Chi-squared	Df	p-value	Chi-squared	df	p-value
Box-Pierce test	12.8165	1	0.0003436	0.0848	1	0.7709
Box-Ljung test	12.8478	1	0.0003379	0.085	1	0.7706

In Table 5, the first section shows the values for the Box-Pierce and Box-Ljung tests for the model residual while the second section shows the values for the Box-Pierce and Box-Ljung tests for the squared model residual of the final model. A look at Table 5 shows that the p-values for the Box-Pierce and Box-Ljung test are smaller than 0.05 (5% level of significance) for the model residual and respectively greater than 0.05 for the squared model residual. These results show that the residuals are correlated but the squared residuals are uncorrelated. This indicate that the hybrid model (final model) adequately capture the behaviour of Okomu stock series. This clearly indicates that the presence of volatility in the series has been accounted for. So equation (6) is in order.

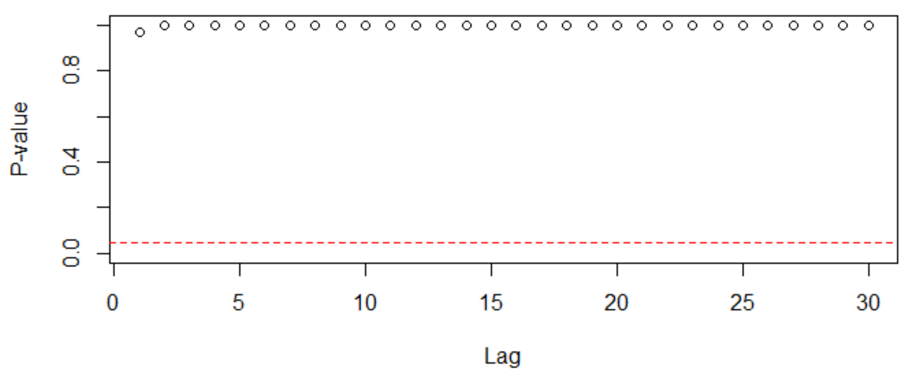

Figure 3: Plot of McLeod Li Test on the ARIMA-ARCH residuals.

The plot of p-values for the McLeod-Li test is given in Figure 3. This plot (Figure 3) confirms the adequacy of the hybrid ARIMA(0,1,1)-ARCH(5) since all the p-values are greater than 0.05.

5.0 DISCUSSION OF RESULTS

In Section 4.2, the structures of the residuals and squared residuals of the best ARIMA model in Section 4.1 were examined. Some non-linearity tests were performed such as Box-Ljung test, Box-Pierce test and McLeod-Li test. The results from these tests indicated the presence of heteroscedasticity (volatility) in the model residuals. This variance was modeled using the ARCH approach in Section 4.4.

However, the Box-Pierce, Box-Ljung and McLeod-Li tests revealed the presence of volatility (heteroskedasticity) in the model residuals. Furthermore, a hybrid ARIMA-ARCH model was developed to take into account both the serial dependence and the volatility in the Okomu stock price data.

From the analysis carried out in Section 4.3, the ARCH (5) model best captured the volatility in the series since it had the least Akaike information criterion value.

Finally, tests of heteroskedasticity were performed on the final model (Section 4.5), and this revealed that the ARIMA (0,1,1)-ARCH(5) is adequate both in accounting for serial dependence and volatility.

Thus, we recommend the use of the ARIMA (0,1,1)-ARCH(5) in forecasting the values of the Okomu oil stock prices in the Nigerian Stock Exchange.

6.0 CONCLUSION

It is shown in this work that the daily stock price data of Okomu Oil Palm Plc is affected by nonlinear characteristics of the variance often referred to as volatility in which large changes often follow large changes and small changes often follow small changes.

In this paper, a hybrid ARIMA-ARCH model was developed (based on the ARIMA (0,1,1) in [4] to take into account the volatility in the daily stock price series.

This model can provide better information that will guide decision makers and investors in understanding the dynamic behavior of the stock price data of Okomu Oil Palm Plc.

REFERENCES

[1] Sisodia, D., Kumar, B., and Gupta, J. K. (2014) *Efficient Prediction of Close Value using Genetic Algorithm Based Horizontal Partition Decision Tree in Stock Market* International Journal of Advance Research in Computer Science and Management Studies vol. 2 Isssue 1, pp. 412-418

[2] Ghezelbash, A. and Keynia, F. (2014) *Design and Implementation of Artificial Neural Network for Stock Exchange Prediction* African Journal of Computing and ICT vol. 7. No. 1, pp. 153-160

[3] Sharma S. and Chouhan, M. (2014) *Stock Market Forecasting Techniques: A Survey* International Journal of Advances in Science and Technology, vol. 1 Issue 3, pp. 36-40.

[4] Osemwenkhae J. E. and Eguasa E. B. (2016) *On Modelling Daily Share Price Data in Nigeria using ARIMA Modelling Approach* Journal of the Nigerian Association of Mathematical Physics, vol. 35, pp. 185-192.

[5] Engle, R. (1982) *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation* Econometrica, vol. 50, No. 4, pp. 987-1008

[6] Brailsford and Robert T. J., Robert W. F. (1996) *An evaluation of volatility forecasting techniques* Journal of Banking and Finance Volume 20, Issue 3, pp. 419–438

[7] Gabriel, A. M., and Wemambu, M. U. (2012) *Volatility Estimation and Stock Prediction in the Nigerian Stock Market* International Journal of Financial Research, vol. 3, No. 1, pp 2-14.

[8] Bollerslev, T. (1986) *Generalized Autoregressive Conditional Heteroscedasticity*, Journal of Econometrics 31, pp. 307-327.

[9] Weiss, A. A. (1984) *ARMA models with ARCH errors* Journal of Time Series Analysis 5, pp. 129-143.

[10] Yusof, F., Kane, I. L., Yusop, Z. (2013) *Hybrid of ARIMA-GARCH Modelling in Rainfall Time Series* Jurnal Teknologi 63:2, pp. 27-34.

[11] Engle, R. (2001) *Garch 101: The Use of ARCH/GARCH Models in Applied Econometrrics* Journal of Economic Perspectives, vol. 15, No. 4 pp. 157-168.

[12] Laux, P., Vogl, S., Qiu, W., Knoche, H.R. and Kunstmann. H. (2011) *Copula-based statistical refinement of precipitation in RCMsimulations over complex terrain*. Hydrol. Earth Syst. Sci. 15: pp. 2401–2419.

[13] Cowpertwait, P. S. P. and Metcalfe, A. V. (2009) *Introductory Time Series with R* Springer Science, London.

[14] Zheng, Y., Zhang, L Zhang, X., Wang, K., and Zheng, Y., (2015) *Forecast Model Analysis for the Morbidity of Tuberculosis in Xinjiang, China* PLOS one 10(3): e0116832. doi:10.1371/journal pp. 1-13.

[15] Xu, K., Zhou, J., Zhang, Y., and Gu, R. (2012) *The long and Short Term Volatility Modelling for Load Series based on ARCH Type Models* 2nd International conference on Information and Multimedia Technology (ICIMT 2010) IPCSIT vol. 42

[16] Chand, S., Kamal, S., and Ali, I., (2012) *Modelling and Volatility Analysis of Share Prices using ARCH and GARCH Models* World Applied Sciences Journal 19 (1): 77-82