Radial Solution of the s-wave D-Dimensional Non-Relativistic Schrodinger Equation for Generalized Manning-Rosen plus Mie-type NucleiPotential within the framework ofNikifarov-UvarovMethod.

[1]B. I. Ita., [2]B. E. Nyong.,[1]H. Louis.,[1]T. O. Magu.,[1]N. A. Nzeata-Ibe and [1]S. Barka

[1]Physical/Theoretical Chemistry Research Group, Department of Pure and Applied Chemistry,

University of Calabar, Calabar, Cross River State, Nigeria.

[2]Department of Chemical Sciences, Cross River University of Technology, Calabar,

Cross River State, Nigeria.

Corresponding author's email: tommylife4u@yahoo.com **Phone No**: +2347067024323

ABSTRACT

We present the bound state solution of Schrodinger equation in $D-dimension$ for generalized Manning-Rosen plus Mie-typepotential using the generalized parametric Nikiforov–Uvarov method to obtain the energy levels and the corresponding un-normalizedeigenfunction in closed form. The energy eigenvalues for different conditions of potential consideration and the numerical analysis for the calculated eigen energy are also computed.

Keywords: Schrodinger equation, Mie-type plus Manning-Rosen potential, Nikifarov-Uvarov Method.

1. INTRODUCTION

Exact or approximate solutions of Schrödinger equation has been solved in 3D for various potential [1-9]. Though few cases of these potentials are exactly solvable, for example the Coulomb and harmonic potentials [1, 2]. In recent times, the study of exponential-type potential has attracted a lot of interest of different authors. [5−8] These potentials under investigation include the ManningRosen potential [9], the Hulthen potential, [10] the Eckart potential [11] the five parameter exponential-type potential [12], Hylleraas potential [13−14], and others. [15] For an arbitrary l-state, most quantum systems could only be treated by approximation methods. Nonetheless, many authors have applied different approximation to the centrifugal term and obtained analytical approximation to the l-wave solutions of the Schrodinger equation with exponential-type potentials. [16−23] Various methods have been used to obtain the exact or approximate solutions of the Schrodinger equation for exponential-type potentials. These methods include the supersymmetric (SUSY) and shape-invariance method [24], the variational[25], the standard method [26], path integral approach [27], the asymptotic interaction method AIM [28], the exact quantization rule (EQR) [29−31], the hypervirial perturbation [32], series method [33], the shifted 1/N expansion [34], the algebraic approach [35], the Nikiforov–Uvarov method (NU)[36] and others.

The purpose of this work is to attempt to study the arbitrary l-state solution of the D-dimensional Schrodinger equation with Generalized Manning-Rosen plus Mie-type potential (GMRMP),which has many applications in physics and chemistry. The organization of the paper is as follows: The Nikiforov–Uvarov method is presented in Sec. 2. We present the D-dimensional eigenvalue and the corresponding wave functions in Sec. 3. Results and discussion are given in Sec. 4. Finally, a brief conclusion is given in Sec. 5.

2. THE NIKIFOROV-UVAROV METHOD

The Nikiforov-Uvarov (NU) method is based on solving the hypergeometric-type second-order differential equations by means of the special orthogonal functions. For a given potential, the Schrodinger or Schrodinger-like equation given as

$$\psi''(x) + \big(E - V(x)\big)\psi(x) = 0 \tag{1}$$

Are reduced to a generalized equation of hypergeometric-type with an appropriate coordinate transformation $r \to s$ and then they can be solved systematically to find the exact or particular solutions. The main equation which is closely associated with the method is given in the following form (Nikiforov and Uvarov, 1988).

$$\psi''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\tau'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0 \tag{2}$$

Where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials at most second-degree, $\tilde{\tau}(s)$ is a first-degree polynomial and $\psi(s)$ is a function of the hypergeometric-type.

In order to find the exact solution to Eq. (2), we set the wave function as

$$\psi(x) = \emptyset(s)\mathcal{X}(s) \tag{3}$$

and on substituting Eq. (3) into Eq. (2), then Eq. (3) reduces to hypergeometric-type,

$$\sigma(s)\mathcal{X}''(s) + \tau(s)\mathcal{X}'(s) + \lambda\mathcal{X}(s) = 0 \tag{4}$$

where the wave function $\emptyset(s)$ is defined as the logarithmic derivative

$$\frac{\emptyset'(s)}{\emptyset(s)} = \frac{\pi(s)}{\sigma(s)'} \tag{5}$$

Where $\pi(s)$ is at most first-order polynomial?

The hypergeometric-type function $\emptyset(s)$ whose polynomial solutions are given by the Rodrigues relation

$$\emptyset(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}\left[\sigma^n(s)\rho(s)\right] \tag{6}$$

Where B_n is the Normalization constant and the weight function $\rho(s)$ most satisfy the condition

$$\frac{d}{ds}[\sigma^n(s)\rho(s)] = \tau(s)\rho(s) \tag{7}$$

Where

$$\tau(s) = \check{\tau}(s) + 2\pi(s) \tag{8}$$

In order to accomplish the condition imposed on the weight function $\rho(s)$, it is necessary that the classical or polynomials $\tau(s)$ be equal to zero to some point of an interval (a, b) and its derivative at this interval at $\sigma(s) > 0$ will be negative, that is

$$\frac{d\tau(s)}{ds} < 0 \tag{9}$$

Therefore, the function $\pi(s)$ and the parameters λrequired for the NU method are defined as follows:

$$\pi(s) = \frac{\sigma' - \check{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \check{\tau}}{2}\right)^2 - \tilde{\sigma} + k\sigma} \tag{10}$$

Where $\lambda = k + \pi'(s)$

The parametric generalization of the NU method is given by the generalized hypergeometric-type equation as

$$\psi''(s) + \left(\frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\right)\psi'(s) + \left(\frac{-\xi_1 s^2 + \xi_2 s - \xi_3}{s^2(1 - c_3 s)^2}\right)\psi(s) = 0 \tag{11}$$

Equation (11) is solved by comparing it with Eq. (2) and the following polynomials are obtained:

$$\check{\tau}(s) = (c_1 - c_2 s),\ \sigma(s) = s(1 - c_3 s),\ \tilde{\sigma}(s) = -\xi_1 s^2 + \xi_2 s - \xi_3 \tag{12}$$

Now substituting Eq. (12) into Eq. (11), we find

$$\bar{\sigma}(s) = c_4 + c_5 s \pm \sqrt{[(c_6 - c_3 k_\pm)s^2 + (c_7 + k_\pm)s + c_8]} \tag{13}$$

Where $c_4 = \frac{1}{2}(1 - c_1)$, $c_5 = \frac{1}{2}(c_2 - 2c_3)$, $c_6 = c_5^2 + \xi_1$, $c_7 = 2c_4 c_5 - \xi_2$, $c_8 = c_4^2 + \xi_3$, $c_9 = c_3 c_7 + c_3^2 c_8 + c_6$, $c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}$, $c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)$, $c_{12} = c_4 + \sqrt{c_8}$, $c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right)$ (14)

The resulting value of k in Eq. (13) is obtained from the condition that the function under the square root be square of a polynomials and it yields,

$$k_\pm = -(c_7 + 2c_3 c_8) \pm 2\sqrt{c_9 c_8} \tag{15}$$

Where $c_9 = c_3 c_7 + c_3^2 c_8 + c_6$

The new $\pi(s)$ for k becomes

$$\pi(s) = c_4 + c_5 s - \left[\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)s - \sqrt{c_8}\right] \tag{16}$$

Using Eq. (8), we obtain

$$\tau(s) = c_1 + 2c_4 - (c_2 - 2c_5)s - 2\left[\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)s - \sqrt{c_8}\right] \tag{17}$$

We obtain the energy equation as

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3c_8 + 2\sqrt{c_8c_9} = 0 \tag{18}$$

While the wave function is given as

$$\Psi_n(s) = N_{n,l} S^{c_{12}} (1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10} - 1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s) \tag{19}$$

Where P_n is the orthogonal polynomials.

Given that $P_n^{(\alpha,\beta)} = \sum_{r=0}^{n} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(\alpha+r+1)\Gamma(n+\beta-r+1)(n-r)!r!}\left(\frac{x-1}{2}\right)^r \left(\frac{x+1}{2}\right)^{n-r}$ (20)

This can also be expressed in terms of the Rodriguez's formula

$$P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n n!}(x-1)^{-\alpha}(x+1)^{-\beta}\left(\frac{d}{dx}\right)^n \left((x-1)^{n+\alpha}(x+1)^{n+\beta}\right) \tag{21}$$

3. FACTORIZATION METHOD

In spherical coordinate, the Schrödinger equation with the potential $V(r)$ is given as

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{\partial}{\partial \mathrm{r}}\left(r^2\frac{\partial}{\partial \mathrm{r}}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)\Psi(r,\theta,\varphi) + V(r)\Psi(r,\theta,\varphi) = E\Psi(r,\theta,\varphi) \tag{22}$$

Using the common ansatz for the wave function as

$$\Psi(r,\theta,\varphi) = \frac{R(r)}{r}Y_{l,m}(\theta,\varphi) \tag{23}$$

And substituting eq. (22) into eq. (23), we obtain the following set of equation

$$\frac{d^2R_{nl}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[(E - V) - \frac{\lambda\hbar^2}{2\mu r^2}\right]R_{nl} = 0 \tag{24}$$

$$\frac{d^2\Theta(\theta)}{d\theta^2} + cot\theta\frac{d\Theta(\theta)}{d\theta}\left(\lambda - \frac{m^2}{sin^2\theta}\right)\Theta(\theta) = 0 \tag{25}$$

$$\frac{d^2\Phi(\varphi)}{d\theta^2}+m^2\Phi(\varphi)=0 \tag{26}$$

SOLUTION OF THE D-DIMENSIONAL SCHRODINGER EQUATION

The s-wave d-dimensional Schrodinger Equation with vector V(r), potential is given as

$$\frac{d^2\,R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[(E-V(r))-\left(\frac{(D+2l+1)(D+2l+3)}{4r^2}\right)\right]R(r)=0 \tag{27}$$

For s-wave, $l=0$ the equation above reduces to

$$\frac{d^2\,R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[(E-V(r))-\left(\frac{(D+1)(D+3)}{4r^2}\right)\right]R(r)=0 \tag{28}$$

The generalized Manning-Rosen Potential is given as

$$V(r)=-\left(\frac{Ce^{-\alpha r}+De^{-2\alpha r}}{(1-e^{-2\alpha r})^2}\right) \tag{29}$$

The Mie-type Potential, $V(r)=-\frac{A}{r}+\frac{B}{r^2}+C$ (30)

in this paper, we assume $C=0$

Applying the transformation $S=e^{-\alpha r}$ and pekeris-type approximation [37] given as $\frac{1}{r^2}=\frac{4\alpha^2}{(1-s)^2}$

The superposed potential can be represented as GMRMMP

$$V(s)=-\left(\frac{Ds^2+Cs}{(1-s)^2}\right)-\frac{2A\alpha}{(1-s)}+\frac{4B\alpha^2}{(1-s)^2} \tag{31}$$

Applying the pekeris-type approximation and after lengthy algebra, we have

$$\frac{d^2R(s)}{ds^2}+\frac{(1-s)}{(1-s)s}\frac{dR(s)}{ds}+\frac{1}{(1-s)^2s^2}\left[(2\beta^2+Q)s^2+(-H+P-4\beta^2)s+(2\beta^2+H-2R-\lambda)\right]R(s)=0 \tag{32}$$

Where

$$-\beta^2=\left(\frac{\mu E}{4\alpha^2\hbar^2}\right),\ \lambda=\left(\frac{(D+1)(D+2)}{4}\right),\ P=\left(\frac{\mu}{2\alpha^2\hbar^2}\right)C,\ Q=\left(\frac{\mu}{2\alpha^2\hbar^2}\right)D,\ H=\left(\frac{\mu}{\alpha\hbar^2}\right)A,\ R=\left(\frac{\mu}{\hbar^2}\right)B,$$

$$c_1 = c_2 = c_3 = 1, c_4 = 0, c_5 = -\frac{1}{2}, c_6 = \frac{1}{4} + 2\beta^2 + Q, c_7 = -4\beta^2 + P - H,$$

$$c_8 = 2\beta^2 - 2R + H - \lambda, c_9 = \frac{1}{4} - 2R - \lambda + P + Q, c_{10} = 1 + 2\sqrt{2\beta^2 - 2R + H - \lambda},\ c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - 2R - \lambda + P + Q} + \sqrt{2\beta^2 - 2R + H - \lambda}\right), c_{12} = \sqrt{2\beta^2 - 2R + H - \lambda}, c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - 2R - \lambda + P + Q} + \sqrt{2\beta^2 - 2R + H - \lambda}\right), \varepsilon_1 = 2\beta^2 + B + P + K, \varepsilon_2 = 4\beta^2 - \emptyset + B + H, \varepsilon_3 = 2\beta^2 - 2J - K + H \qquad (33)$$

Using the eigenvalue equation, the energy eigen spectrum of GMRMMP is found to be

$$\beta^2 = \left[\frac{(4R + 2\lambda) - (P + H) - \left(n^2 + n - \frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - 2R - \lambda + P + Q}}{\left(n + \frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - 2R - \lambda + P + Q}}\right]^2 - (2R + \lambda - H) \qquad (34)$$

The above equation can be solved explicitly and the energy eigen spectrum of GMRMMP becomes

$$E = \frac{2\alpha^2\hbar^2}{\mu}\left\{\left[\frac{\left(4\left(\frac{\mu}{\hbar^2}\right)B + 2\left(\frac{(D+1)(D+2)}{4}\right)\right) - \left(\left(\frac{\mu}{2\alpha^2\hbar^2}\right)C + \left(\frac{\mu}{\alpha\hbar^2}\right)A\right) - \left(n^2 + n + \frac{1}{2}\right) - (2n+1)\sqrt{\frac{q^2}{4} - 2\left(\frac{\mu}{\hbar^2}\right)B - \left(\frac{(D+1)(D+2)}{4}\right) + \left(\frac{\mu}{2\alpha^2\hbar^2}\right)C + \left(\frac{\mu}{2\alpha^2\hbar^2}\right)D}}{\left(n + \frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - 2\left(\frac{\mu}{\hbar^2}\right)B - \left(\frac{(D+1)(D+2)}{4}\right) + \left(\frac{\mu}{2\alpha^2\hbar^2}\right)C + \left(\frac{\mu}{2\alpha^2\hbar^2}\right)D}}\right]^2\right\} - \left(2\left(\frac{\mu}{\hbar^2}\right)B + \left(\frac{(D+1)(D+2)}{4}\right) - \left(\frac{\mu}{\alpha\hbar^2}\right)A\right) \qquad (35)$$

Eigen function consideration

The wave function $\rho(s)$ is given as

$$\rho(s) = s^{c_{10}-1}(1 - c_3 s)^{\frac{c_{11}}{c_3} - c_{10} - 1} \qquad (36)$$

Using equation (28) we get the weight function as

$$\rho(s) = s^u (1 - s)^v \qquad (37)$$

Where $u = 2\beta^2 - 2R + H - \lambda$, and $v = 2\sqrt{\frac{1}{4} - 2R - \lambda + P + Q}$

Also we obtain the wave function $X_n(s)$ as

$$X_n(s) = p_n^{(u,v)}(1 - 2s), \qquad (38)$$

Where$p_n^{(u,v)}$are Jacobi polynomials

Lastly, $\varphi(s) = s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}}$ (39)

Using equation (39) we get

$$\varphi(s) = s^{u/2}(1-s)^{1+v/2} \tag{40}$$

We obtain Radial wavefunctionfrom

$$R_n(s) = N_n\varphi(s)X_n(s) \tag{41}$$

as

$$R_n(s) = N_n s^{u/2}(1-s)^{1+v/2}P_n^{(u,v)}(1-2s) \tag{42}$$

Where n is a positive integer and N_n is the normalization constant.

4. DISCUSSION

In this subsection, we discuss some special potential under consideration

CASE I: if $C = D = 0$, equation (35) reduces to Mie-type eigen energy spectrum equation of the form

$$E = \frac{2\alpha^2\hbar^2}{\mu}\left\{\left[\frac{\left(4\left(\frac{\mu}{\hbar^2}\right)B+2\left(\frac{(D+1)(D+2)}{4}\right)\right)-\left(\left(\frac{\mu}{\alpha\hbar^2}\right)A\right)-\left(n^2+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{q^2}{4}-2\left(\frac{\mu}{\hbar^2}\right)B-\left(\frac{(D+1)(D+2)}{4}\right)}}{\left(n+\frac{1}{2}\right)+2\sqrt{\frac{1}{4}-2\left(\frac{\mu}{\hbar^2}\right)B-\left(\frac{(D+1)(D+2)}{4}\right)}}\right]^2\right\} - \left(2\left(\frac{\mu}{\hbar^2}\right)B + \left(\frac{(D+1)(D+2)}{4}\right) - \left(\frac{\mu}{\alpha\hbar^2}\right)A\right) \tag{43}$$

Where D is the dimension of the schrodinger equation

CASE II: $If\ A = B = 0$, equation (35) reduces to generalized Manning-Rosen eigen energy spectrum equation of the form

$$E = \frac{2\alpha^2\hbar^2}{\mu}\left\{\left[\frac{\left(2\left(\frac{(D+1)(D+2)}{4}\right)\right)-\left(\left(\frac{\mu}{2\alpha^2\hbar^2}\right)C\right)-\left(n^2+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{1}{4}--\left(\frac{(D+1)(D+2)}{4}\right)+\left(\frac{\mu}{2\alpha^2\hbar^2}\right)C+\left(\frac{\mu}{2\alpha^2\hbar^2}\right)D}}{\left(n+\frac{1}{2}\right)+2\sqrt{\frac{1}{4}-\left(\frac{(D+1)(D+2)}{4}\right)+\left(\frac{\mu}{2\alpha^2\hbar^2}\right)C+\left(\frac{\mu}{2\alpha^2\hbar^2}\right)D}}\right]^2\right\} - \left(\left(\frac{(D+1)(D+2)}{4}\right)\right) \tag{44}$$

CASE III: For Schrodinger equation in zero-dimension, equation (35) reduces to

$$E=\frac{2\alpha^{2}\hbar^{2}}{\mu}\left\{\left[\frac{\left(4\left(\frac{\mu}{\hbar^{2}}\right)B\right)-\left(\left(\frac{\mu}{2\alpha^{2}\hbar^{2}}\right)C+\left(\frac{\mu}{\alpha\hbar^{2}}\right)A\right)-\left(n^{2}+n+\frac{1}{2}\right)-(2n+1)\sqrt{\frac{1}{4}-2\left(\frac{\mu}{\hbar^{2}}\right)B-\left(\frac{\mu}{2\alpha^{2}\hbar^{2}}\right)C+\left(\frac{\mu}{2\alpha^{2}\hbar^{2}}\right)D}}{\left(n+\frac{1}{2}\right)+2\sqrt{\frac{1}{4}-2\left(\frac{\mu}{\hbar^{2}}\right)B-\left(\frac{\mu}{2\alpha^{2}\hbar^{2}}\right)C+\left(\frac{\mu}{2\alpha^{2}\hbar^{2}}\right)D}}\right]^{2}\right\}-\left(2\left(\frac{\mu}{\hbar^{2}}\right)B-\left(\frac{\mu}{\alpha\hbar^{2}}\right)A\right) \tag{45}$$

4.1 NUMERICAL ANALYSIS

In this subsection, we tested the accuracy of our work by studying the behavior of the superposed potential and the calculated eigen energy as shown in **Figures I and II.**The potential plot in **Figure I** depicts an attractive behavior and increases as the screening parameter increases. The behavior of the potential also fit perfectly with the superposed potential in equation[31]. The variation of the eigen energy with principal quantum number is presented in **Figure II.** It can be observe from the plot that the calculatedeigen energy values increases to a maximum value and decays exponentially with increasing principal quantum number.

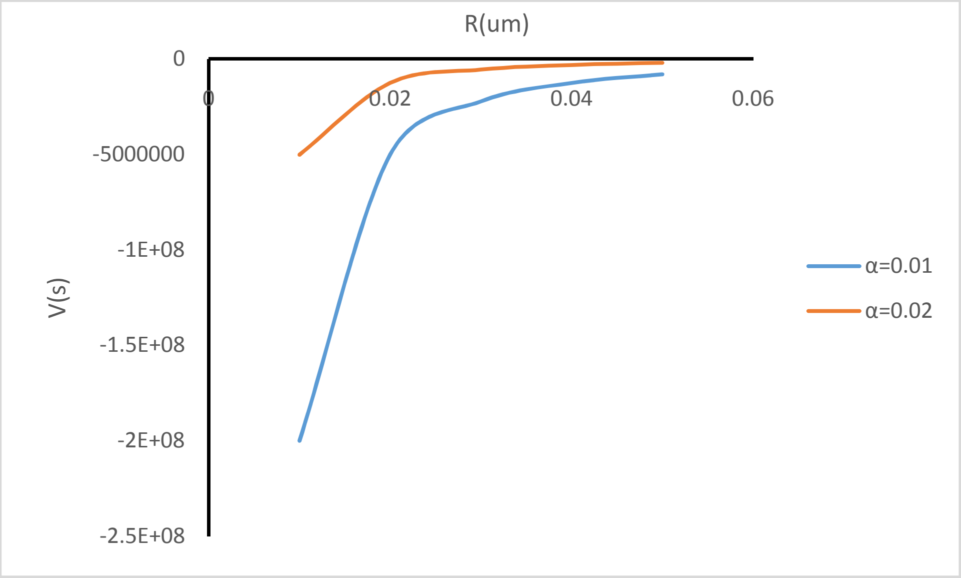

Figure I: Variation of potential with internuclear distance, r

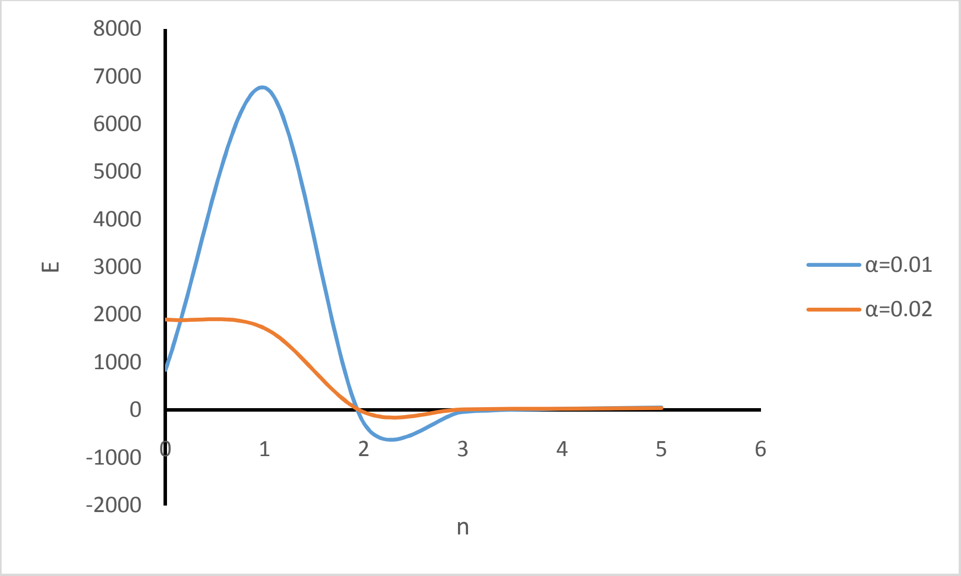

Figure II: Variation of calculated eigen energy with Principal quantum number, n

5. CONCLUTION

In this paper, we investigate the bound state solutions to the Schrodinger equation for Generalized Manning-Rosen plus Mie-type potential. The eigen energy spectrum and the corresponding un-normalized eigen function were computed in terms of Jacobi polynomials. Special conditions of potentials under study viz$A = B = 0\ and\ C = D = 0$ was also investigated.

REFERENCES

[1] O. Gomul and M. Kocak, Quant-Ph/0106144(2001)
[2] K. J. Oyewumi, F. O. Akinpelu and A. D. Agboola, *Int. J. Theor. Phys***47**(2008) 1039
[3] A. N. Ikot and L. E. Akpabio,*Appl.Phys.Res*.**2**(2010)202.
[4] A. N. Ikot, L. E. Akpabio and E. B. Umoren, *J. Sci. Res* **3**(1) (2011) 25
[5] H. Hassanabadi, S. Zurrinkamar and H. Rahimov, *Commun. Theor. Phys*. **56** (2011) 423
[6] R. L. Greene and C. Aldrich, *Phys.Rev.A***14** (1976)2363
[7] A.N. Ikot, *Afr.Phys.Rev*.**6**(2011) 221
[8] A. P. Zhang, W. C. Qiang, Y. W. Ling, *Chin.Phys.Lett*. **26**(10) (2006) 100302
[9] A.D. Antia, A.N. Ikot, and L.E. Akpabio, Euro. J. Sci. Res. 46 (2010) 107.
[10] A.N. Ikot, L.E. Akpabio, and E.J. Uwah, Electr. J. Theor. Phys. 8 (2011) 225.
[11] W.C. Qiang and S.H. Dong, Phys. Lett. A 368 (2007) 13.
[12] A.N. Ikot, L.E. Akpabio, and J.A. Obu, J. Vect. Relat. 6 (2011) 1.
[13] A.N. Ikot, O.A. Awoga, and B.I. Ita, Few-body Syst.Doi:10.1007/s00601-012-0434-y (2012).
[14] A.N. Ikot, Chin. Phys. Lett. 29 (2012) 060307.
[15] A.N. Ikot, O.A. Awoga, and A.D. Antia, Chin. Phys. B 22 (2013) 020304.
[16] S.H. Dong, W.C. Qiang, G.H. Sun, and V.B. Bezerra, J. Phys. A 40 (2007) 10535.
[17] G.F. Wei, C.Y. Long, and S.H. Dong, Phys. Lett. A 372 (2008) 2592.
[18] S. Dong, J. Garcıia-Ravelo, and S.H. Dong, Phys. Scr. 76 (2007) 393.
[19] R.L. Greene and C. Aldrich, Phys. Rev. A 14 (1976) 2363.
[20] S.H. Dong and X.Y. Gu, J. Phys.: Conf. Ser. 96 (2008) 012109.

[21] G.F. Wei, C.Y. Long, X.Y. Duan, and S.H. Dong, Phys. Scr. 77 (2008) 035001.
[22] S H. Dong, W.C. Qiang, and J. Garcıia-Ravelo, Int. J. Mod. Phys. A 23 (2008) 1537.
[23] C.S. Jia, T. Chen, and L.G. Cui, Phys. Lett. A 373 (2009) 1621.
[24] B. G¨onul and I. Zorba, Phys. Lett. A 269 (2000) 83.
[25] E.D. Ficho and R.M. Ricotta, Phys. Lett. A 269 (2000) 269.
[26] A.N. Ikot, L.E. Akpabio, and A.D. Antia, Arab. J. Sci. Eng. Doi:10.1007/S.13369-011-0160-7 (2011).
[27] S.M. Ikhadair and R. Sever, Annalen der Physik (Berlin) 17(11) (2008) 897.
[28] O. Bayrak and I. Boztosun, J. Phys. A 39 (2006) 6955.
[29] Z.Q. Ma and B.W. Xu, Euro. Phys. Lett. 69 (2005) 685.
[30] F.A. Serrano, X.Y. Gu, S.H. Dong, and Q. Dong, J. Math. Phys. 51 (2010) 082103.
[31] W.C. Qiang and S.H. Dong, EPL 89 (2010) 10003.
[32] J.P. Killingbeck, A. Grojean, and G. Jolicard, J. Chem. Phys. 116 (2002) 447.
[33] J. Yu, S.H. Dong, and G.H. Sun, Phys. Lett. A 322 (2004) 290.
[34] S.M. Ikhdair and R. Sever, Int. J. Mod. Phys. A 24 (2009) 5341.
[35] M.R. Setare and E. Karimi, Phys. Scr. 75 (2007) 90.
[36] A.F. Nikiforov and V.B. Uvarov, Special Functions of Mathematical Physics, Birkhauser, Basel (1988).
[37] Qiang and S. H. Dong, Phys. Lett. A 368 (2007) 13; S. M. Ikhdair and R. Sever, Ann. Phys. (Berlin) 17 (2008)