Application of thermal conductivity/ steady state method in estimating length of metal rods

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ABSTRACT

The paper describes the application of knowledge of thermal conductivity to investigate the variation of length with quantity of heat flow in the metals. The method used is the steady-state techniques, which determines the unknown lengths of metal rods using the amount of heat-transfer in known lengths. The selected rods used in this study are Brass rods, iron rods and stainless steel rods with different thermal conductivities. Each of the metal rods was cut into three different lengths; the longest which is 22.0 cm was set as known, while the other two lengths were referred to as unknown (been the length to be determined). Estimation of lengths of the rods were made by measuring the amount of heat-transfer rate in the known length of the same type metal rod. The knowledge of heat-transfer rate and Fourier law were employed to determine the lengths of other (unknown) rods. The calculated lengths agreed with the actual values of the lengths of the rods values within 93 to 99.33 percent. The results obtained from the experiment showed that the amount of heat-transfer in the rods depend on the lengths, sizes, and the materials from which the rods were made.

Keywords: Thermal Conductivity, metal rods, heat-transfer rate, Steady State method

1.0 Introduction

Thermal conductivity is a bulk property of the material that describes the ability of the material to transfer heat to accomplish the transmission of thermal energy by molecule motion. It is the quantity of heat transmitted through a unit thickness of materials in a direction, as a result of temperature differences under steady state boundary condition [1]. Generally, methods of determining thermal conductivity of any material can be grouped into two; steady-state and transient state methods [2]. The measurement by two methods are either absolute or comparative, the major difference between these measurements are that the comparative measurement requires the property of another material in the calculation of thermal conductivity of a sample, which is a disadvantage [3]. Several publications on the thermal conductivity of metals have been made recently [4,5,6,7,8,9].

Alam *et al.*[10], studied thermal property of engineering material and analysis of insulating materials. Property of insulating materials were determined by steady state technics and compared with the existing values. Kumar and Chandra [6] in 2013 determine the thermal conductivity of engineering material with a hot rod method which used water as a coolant medium. The heat loss through the rod for the steady state of thermal conductivity of small samples were measured and used with Fourier equation to obtain thermal conductivity, the percentage error in the calculated values compared to literature values were between 23 and 53%. According to [11]the temperature at any particular point of a body remains constant with time, a condition of steady state heat flow is assumed to have established. Thermal conductivity in metals is closely related to electrical conductivity because the freely moving valence electron caries both the electric current and heat energy [12, 13]. Hence good electrical conductors are likely to be good conductors of heat energy. However, the general correlation between electrical and thermal conductance is broken in other materials such as semiconductors, due to the relative importance of phonon carriers for heat in non-metals and other related materials [14].Thermal conductivity hinge onproperties of a material such as temperature, length, pressure, density, crystalline structure and purity of material [15].The aim of this study is to quantitatively illustrate a basic concept of heat-transfer to determine the length of metal rods by carrying out an experiment on thermal conductivity on different metal rods.

2.0Theoretical Consideration

Consider an insulated metal rod of a homogeneous material (e.g. a metal) placed in contact at on end with heating element. The metal rod is heated at one of its ends and the thermal energy propagates along it towards the opposite end. If the heat source is constant in time, a stationary temperature distribution will be achievedalong the metal rod. Focusing on some infinitesimal segment of the metal rod between x and x + dx, the balance thermal energy in the steady state may be expressed in the following way: the heat $\frac{dq}{dx}(x)$ entering the segment through its end that is closest to the heat source will become $\frac{dq}{dx}(x+dx)$ at some distance $x+dx$.

Figure 1 shows conduction of heat through rod length, L from x_1 at temperature T_1 to x_2 at temperature T_2, which is measured by change in temperature. The steady flow energy one-dimensional temperature distribution in a solid cylinder plane with no energy generation such that the temperature is only a function of x-coordinate can be described by the simplified form of heat conduction equation (1).

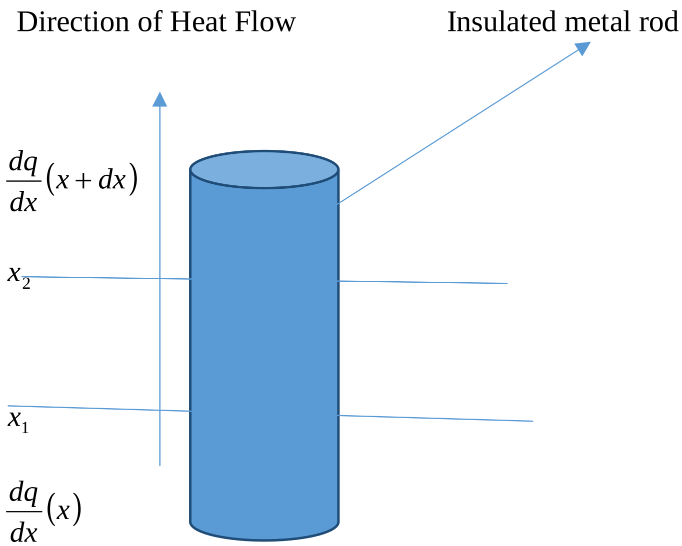

Figure1: One-dimensional heat Conduction

$$\frac{d^2T}{dx^2}=0 \quad (1)$$

Since the metal rod is insulated such that no transfer of heat on the sides (Figure 1).

Solving equation (1) in terms of two constants of integration **a** and **b** will yield:

$$T(x)=ax+b \quad (2)$$

Equation (2) is an expression for the temperature field where **a** and **b** are constants of integration. For a second order equation, two boundary conditions are required to determine a and b. This is achieved by specifying the temperatures at the two extreme surfaces of the rod as shown in the figure.

$$T(0)=T_1$$
$$T(L)=T_2$$

Applying these two boundary conditions give:

The condition $T(0)=T_1$ implies that $b=T_1$ and the condition $T_2=T(L)$ implies that $T_2=aL+T_1$, or $a=(T_2-T_1)/L$. Putting a and b in equation (2) will yield

$$T(x) = \frac{(T_2 - T_1)}{L}x + T_1 \qquad (3)$$

Which can be rearranged to give

$$\frac{T(x) - T_1}{T_2 - T_1} = \frac{x}{L} \, (4)$$

Equation (4) shows linearity of temperature distribution with x. the heat-transfer rate through the rod as determined by the Fourier law is

$$q = -kA\frac{dT}{dx} = kA\frac{T_1 - T_2}{L} \qquad (5)$$

or $q = -kA\frac{dT}{dx} = kA\frac{T_1 - T_2}{x_2 - x_1}$

Which is the rate of heat removed from the sample rod and expressed by the first law of thermodynamics [16].

The heat flux, which is heat-transfer rate per unit area is written as

$$q^{"} = \frac{q}{A} = \frac{k(T_1 - T_2)}{L} = \text{Constant} \qquad (6)$$

Where:

q = the heat-transfer rate in Watts (W),

A = the cross-sectional area ($\pi*d^2/4\ m^2$),

k = the thermal conductivity of the rods in $(W/m-K)$,

$x_2 - x_1$ = the length of the rods in meters (m),

$T_1 - T_2$ = the temperatures of the immersed end subtracted from the free end respectively (k).

3.0 Materials and Method

3.1 Materials

Heat transfer rates for different metals were determine using steady state method. Three types of metal rods were used in the experiment; brass rods, stainless steel rods, and iron rods. Each of these rods were cut into different sizes uniformly across the types of rods. Two LCD Digital Thermometers (Mextech) with resolution 0.1°, accuracy ±1°c in the range of -30°c to +150°cwere used to measure the temperature of the rods, an electric heater (with thermostat) was used as the source of heat and was set to maintain constant temperature for the water baths, other materials used for the experiments include retort stand with clamp, stop watch, magnetic stirrer to make the temperature of the water bath uniform, and cylindrical Styrofoam for insulation to prevent heat loss.

3.2 Experimental Setup

500ml of water was poured into the 600ml beaker, the beaker was placed on the electric heater and the water in it was heated to a desired temperature (75^0C). The water temperature was recorded.Holes were drilled into the Styrofoam cylinders and the holes were sized so that the metal rod fits snugly within the Styrofoam insulation(Figure 2). The Styrofoam cylinders have thickness of 7cm each with lengths and width that depend on the length and diameter of the rods which were to be inserted into Styrofoam.

Fig. 2: Insulated metal rod samples in Styrofoam cylinder

The metal rods were inserted into the center of the Styrofoam cylinders and approximately 10mm of the metals were exposed beneath the Styrofoam cylinders. The digital thermometer was used to record the (initial) temperatures at the top of the metal rods.The rods were inserted into the Styrofoam, which was carefully placed into the boiling water bath so that 10mm of the cylinder is submerged beneath the water line. The cylinders were held in place using the clamp and the retort stand (Figure 3).

Fig. 3: Experimental setup

One of the digital thermometers was placed on the top of the metal rods and the other in the hot water bath. The temperature readings at 5 minutes were taken.After each experiment, the rods were allowed to cool for several minutes by removing them from the Styrofoam cylinders before other trials were carried out on them. The water content reduces a little after each experiment due to the temperature at which the experiment is being carried out and the foam might have also absorbed some water. The water is then topped again to 500ml before the start of other experiment.

The idea is to have a hot water bath with the experiment carefully monitored and immerse insulated metal rods (the rods used in this experiment are iron, stainless steel and brass rods, but can be any type) for a specified amount of time (five minutes was used in this experiment) in hot water bath and analyze the temperature change of the metal rods over time. The temperatures at the water and the free end of the metal rods were measured with digital thermometers accurate to one decimal place. Holes were machined out of the ends of the rods by the Colchester Lathe Machine. The holes are of equivalent depth and diameter in all the rods (they were machined to hold a thermometer that was used in the experiment with the rods hole having a depth of 4.6 cm and diameter of 3mm each). The rods were subjected to the 75^0C bath for exactly 5.00 minutes in all of the experiments in order to ensure that the same amount of heat energy is transferred to the rods in the experiments and “q” the heat transfer rate for the metal is determined using equation 5 for a rod of known length.

Insulated rods of unknown length were then subjected to the same hot water bath and the same constraints (i.e. held in the bath at the same depth, time, etc. as in the initial run). The length of the rod is instantly calculated using the **q** calculated from the rods of known length for the same type of rods. The calculated lengths of the unknown rods are then compared with the actual lengths measured using meter rule. The expected outcome is to have the calculated lengths of the rods to be equivalent to the actual lengths.

K-values for the brass rod is 120 W/m-K, stainless steel rod is 15.9 W/m-K, and for iron rod is 80 W/m-K[17].The diameter of the brass rod is 1.5cm and the diameter for stainless steel and iron rods is 2.0cm.

4.0 Results and Discussion

4.1 Brass rod

The length of the control brass rod 0.22m, the diameter 0.015m, thermal conductivity 120 W/m-K, and the difference in temperature calculated to be 36.7^{0}C was put in the equation (5)

The heat-transfer rate is:

$$q = \pi \frac{0.015^2}{4} * \frac{k}{0.22} * (36.7)$$

$$q = 3.537504728 Watts$$

Putting the values constants q, A and K into the Equation(5) we have

$$3.54 = 0.00017679 \frac{120}{x_2 - x_1}(T_1 - T_2)$$

$$x_2 - x_1 = 0.00017679 \frac{120}{3.54}(T_1 - T_2) \quad (7)$$

The general equation (7) was then used to find the estimated length of the other three brass rods using the value of their individual change in temperature. Table 1 shows the results.

Table1: Experimental Results for Brass

Brass	Control Rod			Unknown Length Rod A			Unknown Length Rod B			Unknown Length Rod C		
Trials	1	2	3	1	2	3	1	2	3	1	2	3
Initial Temperature ^{0}C (Water) (T1)	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
Final Temperature ^{0}C (Rod) (T2)	38.1	38.5	38.3	50.1	50.2	50.3	58.9	58.6	59.2	46.7	47.1	46.9
Temp Difference (T1-T2) ^{0}C	36.9	36.5	36.7	24.9	24.8	24.7	16.1	16.4	15.8	28.3	27.9	28.1
Average Temp difference	36.70			24.80			16.10			28.10		
Length cm	22.00			14.90			9.70			29.90		
Heat Transfer rate (q)	3.547604			3.547604			3.547604			3.547604		

4.2 Steel Rods

The length of the control steel rod 0.22m, the diameter 0.02m, thermal conductivity 15.9 W/m-K, and the difference in temperature calculated to be 45.6^{0}C was put in the Equation (5).

The heat-transfer rate is:

$$q = \pi \frac{0.02^2}{4} * \frac{k}{0.22} * (45.6)$$

$$q = 1.03535 Watts$$

Putting the values constants q, A and K into the Equation(7) we have

$$1.04 = 0.00031429 \frac{15.9}{x_2 - x_1} (T_1 - T_2)$$

$$x_2 - x_1 = 0.00031429\ \frac{15.9}{1{,}04}(T_1 - T_2) \quad (8)$$

The general equation (8) was then used to find the estimated length of the other three steel rods using the value of their individual change in temperature. Table 2 shows the results.

Table 2: Experimental Results for Steel

Steel	Control Rod			Unknown Length Rod A			Unknown Length Rod B			Unknown Length Rod C		
Trials	1	2	3	1	2	3	1	2	3	1	2	3
Initial Temperature ^{0}C (Water) (T1)	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
Final Temperature ^{0}C (Rod) (T2)	29.4	29.3	29.5	44.5	44.7	44.3	55.2	54.6	54.9	13.5	12.9	13.2
Temp Difference (T1-T2) ^{0}C	45.6	45.7	45.5	30.5	30.3	30.7	19.8	20.4	20.1	61.5	62.1	61.8
Average Temp. difference	45.60			30.50			20.10			61.80		
Length cm	22.00			14.70			9.70			29.80		
Heat Transfer rate (q)	1.03535			1.03535			1.03535			1.03535		

4.3 Iron Rods

The length of the control Iron rod 0.22m, the diameter 0.02m, thermal conductivity 80 W/m-K, and the difference in temperature calculated to be 36.7 was put in the equation (5).

The heat-transfer rate is:

$$q = \pi\ \frac{0.02^2}{4} * \frac{k}{0.22} * (75.0 - 38.3)$$

$$q = 5.1065 Watts$$

Putting the values constants q, A and K into the Eqn. 7 we have

$$5.11 = 0.00031429 \frac{80}{x_2 - x_1}(T_1 - T_2)$$

$$x_2 - x_1 = 0.00031429\ \frac{80}{5.11}(T_1 - T_2)(9)$$

The general equation (9) was then used to find the estimated lengths(Table 3) of the other three iron rods using the value of their individual change in temperature.

Table 3: Experimental Results for Iron

Iron	Control Rod			Unknown Length Rod A			Unknown Length Rod B			Unknown Length Rod C		
Trials	1	2	3	1	2	3	1	2	3	1	2	3
Initial Temp. ^{0}C (Water) (T1)	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
Final Temp. ^{0}C (Rod) (T2)	30.5	30.3	30.1	46.1	46.3	46.2	56.2	55.9	56.5	15.4	15.3	15.2
Temp. Difference (T1-T2) ^{0}C	44.5	44.7	44.9	28.9	28.7	28.8	18.8	19.1	18.5	59.6	59.7	59.8
Average Temp. difference	44.70			28.80			18.80			59.70		
Length cm	22.00			14.20			9.30			29.40		
Heat Transfer rate (q)	5.1065			5.1065			5.1065			5.1065		

4.4 Comparison

Comparison of the calculated length and actual length for each rod (Table 4) show that the percentage of accuracy was between 93 to 99.33%. The average difference between the actual and calculated lengths for brass, steel and iron rod are 0.17cm, 0.27 cm and 0.7 cm respectively.

Table 4: Comparison of calculated lengths and actual lengths

	Brass Rod A	Brass Rod B	Brass Rod C	Steel Rod A	Steel Rod B	Steel Rod C	Iron Rod A	Iron Rod B	Iron Rod C
Calculated Length (cm)	14.90	9.70	29.90	14.70	9.70	29.80	14.20	9.30	29.40
Actual Length (cm)	15.00	10.00	30.00	15.00	10.00	30.00	15.00	10.00	30.00
Difference (cm)	0.10	0.30	0.10	0.30	0.30	0.20	0.80	0.70	0.60
% of Accuracy	99.33	97.00	99.67	98.00	97.00	93.13	94.67	93.00	98.00

5.0 Conclusion

From the study, it was established that knowledge of heat-transfer in steady state could be used to estimate lengths of metal rods. The heat–transfer rate (Watts) in brass, steel and iron were determined using equation with known length metal rod. The knowledge was applied to estimate unknown lengths of brass, steel and iron rods. The calculated lengths agree well with the actual values of the lengths of the rods values within 93 to 99.33 percent.

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